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INTEGRATION OF LOGICAL AND INTUITIVE STUDENT EXPERIENCE AS A CONDITION FOR UNDERSTANDING MATHEMATICS

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Abstract. Recent studies in psychology and pedagogy explore the duality of human thought, where two opposing systems – heuristic and analytical – interact in a dialectical unity. This leads to two corresponding ways of understanding the world: logical and intuitive. Focusing solely on developing students' logical abilities may result in them being able to solve only a small number of typical problems and fail to promote their overall personal development. Therefore, more emphasis is now placed on promoting a deeper understanding of topics by moving away from formal approaches. This is done by encouraging active learning and integrating logical and intuitive thinking, which helps students absorb the information at a deeper level. This article explores how these two styles of thinking can be combined in the mathematics classroom and the impact this has on student learning.

Keywords: *intelligence, cognitive thinking styles, intentional experience, conscious learning*

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Unlike other sciences, mathematics relies on logic and evidence rather than observation to derive conclusions from true premises that lead to new knowledge. Academician A. Kolmogorov [1] attributes this distinction to the influence of the ancient Greeks, who viewed nature as something rational, systematic, and ultimately mathematical. The early Greek scholars systematically organized the first mathematical theories that arose from solving practical problems shaped by the needs of everyday life. In this way, they established the unique status of mathematics.

However, scientific breakthroughs are not achieved through logic alone. Galileo Galilei, a physicist, philosopher, and mathematician, pointed out that although logic is useful for verifying the correctness of conclusions drawn from logic, it does not produce logic itself [2]. The French mathematician Henri Poincaré [3] similarly argued that logic is necessary for proof, but intuition is essential for invention. The Russian mathematician V. Steklov believed that intuition is the only method for making discoveries and inventions, as no one has ever achieved a breakthrough through purely logical thinking.

In the 18th century, the foundations of differential and integral calculus, which were developed by the physicist Isaac Newton and the mathematician Gottfried Leibniz to solve important practical problems of their time, were based on the concept of infinitesimal calculus, which at the time could only be explained intuitively. There are many such examples in history.

Recent psychological research has confirmed the existence of two opposing but interacting systems by which the human brain processes information – known as dual process theory – consisting of a heuristic (intuitive) system and an analytical (logical) system [4].

D. Zavalishina states that according to modern psychological viewpoints, "human experience is no longer considered a secondary component of intelligence... but becomes its leading component, a potential reservoir of new operational and substantive knowledge that often emerges in difficult situations as non-instrumental signals and intuitive mechanisms" [5].

The topic of intuition in mathematics, as well as in scientific knowledge in general, has not yet been sufficiently explored. Intuition manifests itself in various aspects of life and professional activity. In jurisprudence, for example, a judge relies not only on the letter of the law but also on its spirit, with the judge's conviction playing an important role in the decision-making process. In medicine, a doctor may correctly diagnose an illness immediately but may have difficulty articulating its reasons. In linguistics, specialists develop, among other things, the so-called feeling for language. Experts usually base their problem-solving processes on fundamental principles and generalized, often implicit knowledge that presents itself as complex, intuitive representations that are not always clearly verbalized. Intuition is therefore seen as "the ability to unconsciously arrive at an intellectual result based on the emergence of a subjective feeling of the unconditional correctness of a particular solution" (M. Kholodnaya) [6].

Intelligence is usually associated with logical, rational, and analytical thinking. However, it is important not to overlook the forms of mental experience that underlie conceptual knowledge, metacognition, intuition, and similar phenomena (M. Kholodnaya) [6].

In her research, psychologist M. Kholodnaya proposes a new approach to the study of the nature of intelligence by analyzing the characteristics of an individual's mental experience, focusing in particular on such components as cognitive, metacognitive, and intentional experience (intentions – individual intellectual inclinations and preferences). In this context, intentional experience is seen as a source of intuition.

M. Kholodnaya writes: "A legitimate question arises as to what happens to a child's intellectual development if their existing intentional experiences are ignored or rejected altogether? What happens is what actually occurs with children in the context of traditional school education: the intellectual development of school-age children slows down considerably compared to preschoolers, and, perhaps most regrettably, the child's creative potential declines. This is not surprising, as intentional experiences are probably one of the most powerful sources of intuition" [6].

In the applied part of M. Kholodnaya's research, the goals of stimulating the intellectual development of students are defined within the framework of an innovative enrichment model for teaching, as shown by the example of a school math course. This approach is implemented with the help of math textbooks for students in grades 5–9, which are supported by the foundation "Mathematics. Psychology. Intelligence" project team (led by E. Gelfman and M. Kholodnaya) based on the enrichment model. The results of these studies and the psycho-didactic approach to creating teaching texts in these textbooks are intended for teaching in various academic subjects.

In mathematics, intuition helps connect the whole with its parts before making logical considerations. Logic plays a crucial role in the phase of proof and analysis, but the integration of the parts is usually achieved through intuition. Attempts to model human thinking by computers cannot surpass humans in their intuitive abilities, which are based on synthesizing the whole and its parts. An intuitive hypothesis cannot be logically derived from facts; it is a product of creative imagination.

Therefore, the nature of argumentation and proof in mathematics is not limited to logical analysis alone. It is always complemented by a synthesis rooted in intellectual intuition, and both aspects are equally important.

The work of the Dutch mathematician, logician, and methodologist L. Brouwer on the role and importance of intuition in mathematics led to the movement known as intuitionism. This school of thought drew attention to the problem of intuition in mathematics and inspired philosophical investigations on the subject, particularly on the role of intuition in significant mathematical discoveries.

Let us consider the main directions of research in intuitionism:

– The a priori intuition in mathematics (as explored by I. Kant, A. Schopenhauer, and L. Brouwer). In his justification of mathematics, L. Brouwer relied on praxeological intuition – a concept of numbers that differs from empirical intuition and has an indisputable certainty.

– Development of the methodological and philosophical aspects of intuition in mathematical understanding. According to the phenomenological description of the philosopher E. Husserl, the idea of order in number theory is an essential feature of intuition as a process.

– The development of methodological ideas among various scientists. Thus, George Pólya, who distinguishes between two types of mathematical reasoning – demonstrative and plausible – writes: "The difference between the two kinds of reasoning is great and manifold. Demonstrative reasoning is safe, beyond controversy, and final. Plausible reasoning is hazardous, controversial, and provisional. Demonstrative reasoning penetrates the sciences just as far as mathematics does, but it is in itself (as mathematics is in itself) incapable of yielding essentially new knowledge about the world around us. Anything new we learn about the world involves plausible reasoning, which is the only kind of reasoning we care about in everyday affairs." [7].

This duality also affects how students understand and absorb educational material, including mathematics, during the learning process. Consequently, it should be reflected in the concept of modern mathematics education.

The current technocratic approach in this area will eventually lead to a civilizational crisis. The shift towards a more humanistic approach is still often interpreted in a narrow and literal way, often leading to a reduction in mathematics teaching in favor of humanities subjects. In practice, this frequently results in merely transmitting knowledge in a ready-made form rather than promoting deep understanding. This was due to the views of mathematics as an abstract science, the study of which exclusively develops logical thinking, as promoted by the Bourbaki group concept. In this view, mathematical education is primarily reduced to the mastery of formal logic.

In modern times, many renowned scientists such as Henri Poincaré, Morris Kline, and Vladimir Arnold, among others, have recognized the living nature of mathematics. They believe it is only natural to give logic and intuition their rightful place in mathematics. Poincaré said: "Logic, which alone can give certainty, is the instrument of demonstration; intuition is the instrument of invention" [3]. He also stated that "the mechanism of mathematical creativity... is not fundamentally different from the mechanism of any other form of creativity" [3], with the only difference being that mathematical creativity is validated not by experimentation but by deductive proof.

Other scientists' research supports Henri Poincaré's conclusions. Jacques Hadamard says: "There are practically no purely logical discoveries. Activating the unconscious is necessary, at least as a starting point for logical work" [8].

As Nassim Nicholas Taleb puts it, a creative individual who retains antifragile qualities [9] in every domain – especially in light of the increasingly common phenomenon of 'black swans' [10] – is a crucial necessity in modern society. Taleb suggests that we can better recognize talent if we break away from conventional logical frameworks. Today's education system aims to produce well-educated human individuals.

Therefore, integrating logical and intuitive cognitive styles and applying them to learning math is essential. "Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. Though different traditions may emphasize different aspects, it is only the interplay of these antithetic forces and the struggle for their synthesis that constitute mathematical science's life, usefulness, and supreme value." [11].

Mathematics has long been considered the best discipline for developing rational thinking. However, without a true understanding of the subject by students, they cannot fully achieve this goal. It is crucial to investigate whether all students can engage with mathematics, as this issue is of great importance to the topic at hand. The popular belief is that 'humanities-oriented children lack mathematical ability' and that 'girls are less gifted in math than boys'. However, research has shown that everyone has the potential to succeed in math. According to

G. Hardy, this ability is innate in most people, much like the ability to enjoy music. While not all students are expected to pursue a career in math, every math teacher can use the humanistic potential inherent in the subject to develop the abilities and talents of their students. Human psychology suggests that a weakness in one area does not preclude the possibility of success in activities that depend on the same ability [2].

From a psychological point of view, three types of understanding of teaching material can be distinguished:

1. Rationalistic understanding (understanding as knowledge and explanation)
2. Hermeneutic understanding (understanding as interpretation)
3. Existential understanding (understanding as comprehension) [12].

The first type is concerned with understanding both the symbolic form and the actual content of the information. In this phase of grasping a mathematical concept, students rely on verbal definitions and personal experience. When they encounter a new concept, it is important to facilitate understanding through verbal explanations and symbolic representations such as diagrams, drawings, sketches, and tables. This includes comparing the new concept with other concepts, contrasting it with alternatives, and analyzing a range of examples and counterexamples. In addition, informal verbal descriptions and vivid metaphors can help to create a deep inner picture and encourage personal associations with the concept.

When introducing new material, there is also a shift towards the second type of understanding (understanding as interpretation) when the information is presented in different ways. Here, the term 'interpretation' is used not only in the sense of translation but also as an 'enrichment' of formulas and symbols to make connections between abstract knowledge and objective reality. Through this process of interpretation, students arrive at a more profound and more precise understanding. This expands interpretative understanding, which can be achieved by applying learned rules and drawing on personal experience, associations, and intuition, considering each student's unique way of thinking.

Understanding is linked to grasping the essence of a mathematical concept or object in the hermeneutic phase. Explanations from teachers or classmates cannot facilitate this process, as it depends on students' inner activity, intuition, creativity, and self-awareness. Students develop the need to internalize the information they receive, to make it personal by translating it into their own inner language and relating it to their own experiences. The teacher's role is to suggest options and guide students on their individual journey by creating the conditions for the transition from hermeneutic understanding (interpretive understanding) to existential understanding (understanding as apprehension).

This transition to a deeper understanding of the material is characterized by the student asking higher-order questions such as: Why? For what purpose? With what aim? What follows from this? From what is this derived? On what basis is this assertion made? And others that he asks himself and the teacher. Teachers should create an appropriate (creative and reflective) research environment in the classroom. Such an environment fosters personal growth, the development of critical thinking, the mindset of a researcher, and the cultivation of intellectual intuition.

A meaningful environment is critical to developing a deeper understanding of academic material. In this environment, learning goes beyond simple explanations and the transmission of knowledge. The emphasis is on developing new ideas and enriching understanding through new meanings. This approach is based on the student's personal experiences (both logical and intuitive) and their unique perception of information. It promotes mastery of the meanings inherent in the material and encourages students to develop their own personal meanings.

The understanding of the mathematical teaching material is achieved through the application of the students' logical and intuitive experiences. This process is based on a previously acquired system of knowledge, skills, and the ability to extrapolate this knowledge, which is often initiated by conjecture, insight, and intuition.

The integration of logical and intuitive experiences is particularly important for the teaching of geometry in school. Traditionally, geometry has been the source of many mathematical discoveries. The study of geometry instills a sense of beauty, develops intuition, and encourages analytical thinking. However, it can also present students with great challenges regarding understanding.

It is important to specifically teach students the ability to conjecture and hypothesize, beginning with learning new theoretical knowledge – definitions of mathematical concepts, theorems, rules, algorithms, and methods for solving important examples.

The main stages of the methodology for studying theorems related to the integration of the student's logical and intuitive experience are described below:

I. Preparation for perception (activation and motivation)

II. Perception (acquisition of new knowledge)

III. Comprehension and reflection on information

IV. Consolidation and application

V. Evaluation (summarizing the results)

Following these stages, students become engaged in the process of formulating hypotheses and proving them, which integrates their intuitive and logical experiences, ultimately leading to a deeper understanding of the educational material.

Let us illustrate these phases by using the example of deriving the formula for the area of a trapezoid. The teacher leads a guided discussion with the class and gradually fills the board with notes.

I. Preparation for perception

This stage begins with repeating the previous material, motivating, and creating a relevant problem context.

Teacher: "What is the area of a polygon? What are its properties?"

Students: "The area is always a positive number and has the following properties:

1. Equal polygons have equal areas.
2. The area of a polygon that consists of several polygons is equal to the sum of the areas of these polygons.
3. The unit of measurement for the area is the unit square, a square with a side length of one unit.

Teacher: "What area of a polygon could we calculate with these properties?"

Students: "A square"

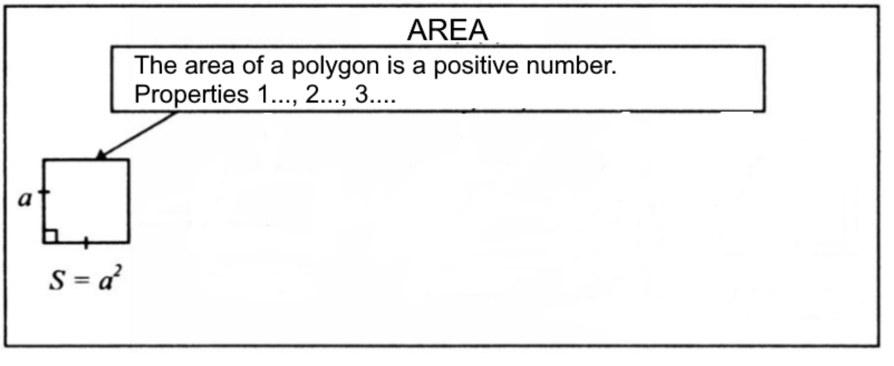


Fig. 1.

Teacher: "What other shapes do we know how to find the area of? "

Students: "A rectangle"

Teacher: "How did we find the formula to determine the area of a rectangle with sides a and b?"

Students: "We completed it to a square with side length (a + b) and then divided it into squares and rectangles."

Teacher: "What is this method called? "

Students: "Completing the shape to a square with a known area and dividing it into squares and rectangles."

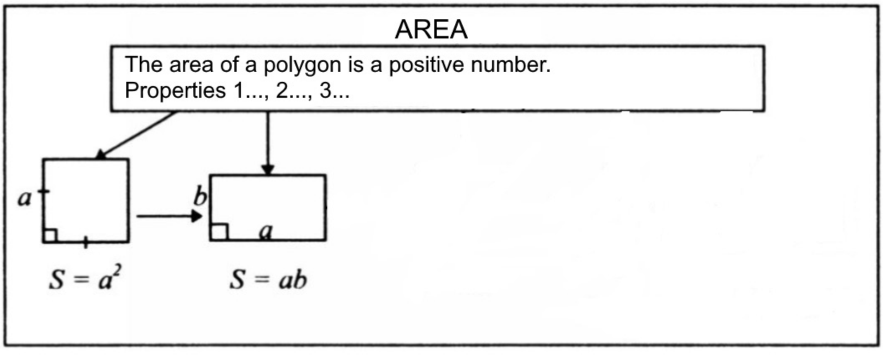


Fig. 2

Teacher: "What other shapes do we know how to find the area of?"

Students: "A parallelogram"

Teacher: "And how did we derive this formula?"

Students: "Using the same method – by completing it to a known shape, a rectangle, and dividing it into a rectangle and triangles."

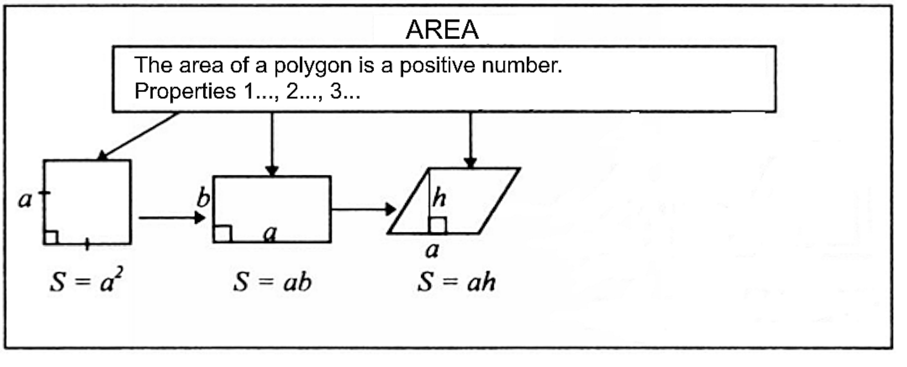


Fig. 3

Teacher: "What other shapes do we know how to find the area of?"

Students: "The area of a triangle."

Teacher: "And how did we figure that out?"

Students: "By completing it into a parallelogram and dividing it into triangles."

II. Perception

This is the stage in which you discover a theorem, formulate it, and seek and present the proof.

Teacher: “What conclusion can you draw from this writing on the board?”

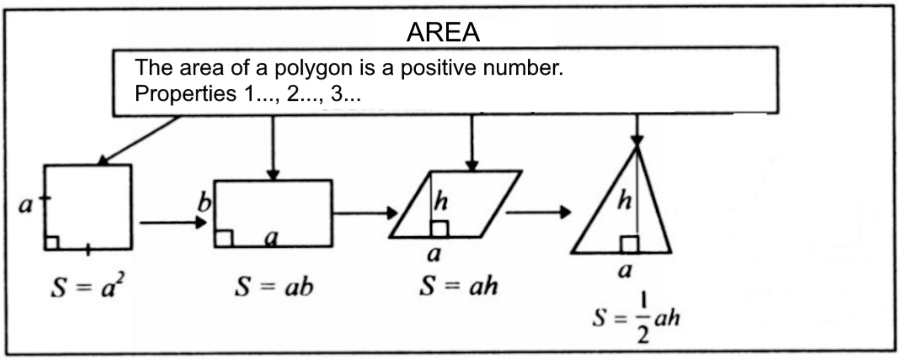


Fig. 4.

Students: “The areas of all the figures are determined by completing them to a shape whose area we already know, and they all refer to one side and the height drawn on that side.”

Teacher: “Which geometric figure did we cover in the last lesson?”

Students: “A trapezoid”

Teacher: “Can you guess which elements are used to express the area of a trapezoid?”

Students: “Maybe it’s a side and a height?”

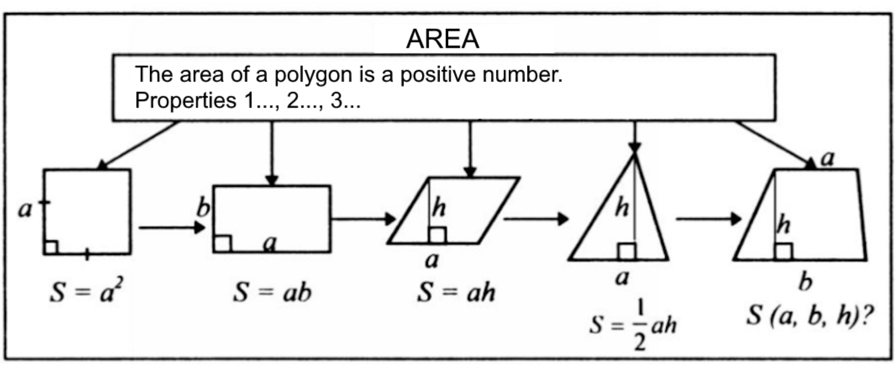


Fig. 5

Teacher: “Now, we must find this connection and derive the formula accordingly. How can we do that?”

Students: “Maybe we can use completion and partitioning again?”

Teacher: “Please let us know your suggestions.”

Students: (offer different options; see Fig. 6)

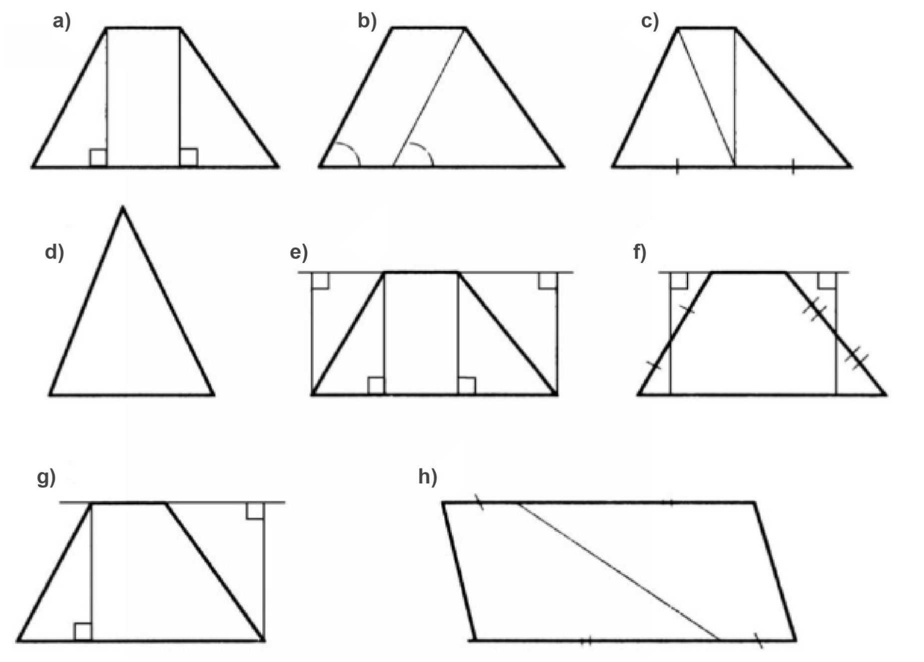


Fig. 6

The teacher can divide the class into groups and assign each group a different version of the diagram and a task: Find the area of the trapezoid using a, b, and h. In this way, several different proofs of the theorem can be achieved. The remaining unsolved methods can be assigned as homework and additional exercises to help students apply the theorem in practice.

III. Comprehension and reflection

In this stage, the teacher encourages the students to think about the process. He helps them understand the theorem's logical structure, the techniques for discovering new facts and checking their validity, and the methods and steps involved in developing proofs.

Here, it is important to design a system of questions to deepen understanding. For example, the following tasks can be formulated:

● "State the theorem we have proved."

● "Identify the condition and the conclusion."

● "Is the statement correct?" (the teacher changes the wording of the theorem slightly, which may or may not change the meaning)

● "Create a new diagram with different labels and prove the theorem."

● "Formulate the inverse statement (inverse theorem)."

- “What is the main idea of the proof?”
- “Which techniques were used in the proof?”
- “Which other theorems can be proved using this technique?”
- “Name the main steps of the proof.”
- “What prior knowledge was used in the proof?”
- “Are there other ways to prove this theorem?”
- “Solve problems using this theorem.”
- “What kinds of problems can be solved with this theorem?”
- “Create problems that can be solved using this theorem.”
- “Reconstruct the logical process of formulating and proving the theorem.”

IV. Consolidation

You must apply the theorem to solve problems and/or prove related theorems at this stage.

V. Evaluation (Summarizing results)

At this stage, conclusions are drawn about the students’ understanding if they can:

- Create a diagram and symbolic notation related to the theorem and state the conclusion correctly;
- Carry out the proof using alternative terms;
- Recognize the main idea of the proof;
- Recognize other theorems that have been proven with similar techniques
- Outline the main steps of the proof;
- Distinguish the prerequisites for the proof
- Know practical applications of the theorem;
- Understand the logical process used to discover connections and develop the proof.

However, not all stages are fully applied in learning each theorem.

It is important to note that for middle school students learning a new subject – geometry – it is not only the theoretical knowledge that is new. They are also confronted with methodological knowledge for the first time, e.g., the concept of a theorem and the nature of a proof. It is, therefore, essential to introduce seventh graders to the concept of a theorem in the first lesson using a simple example.

It, therefore, makes sense for a teacher to integrate intuitive and logical components into their teaching practice. Despite possible biases against intuition in a mathematical context, this approach promotes a learner-centered method and helps students better understand the material.

As an example of integrating logical and intuitive student experiences in the final stage – the assessment phase – of learning, we present a task system on 'Sequences and Progressions' for 9th-grade students. This system consists of three sections that are designed not only to assess but also to motivate, educate, and enrich students. It allows them to effectively demonstrate their skills in one or more areas while developing their metacognition and intellectual intuition.

The first part of the proposed assessment on 'Sequences and Progressions' is motivational and informative. Students are encouraged to share their own experiences with the topic: their ideas about sequences, finite and infinite sequences, progressions, and the summation of sequences (Tasks 1, 3, 5, 6). They are also asked to share the knowledge they have already acquired, e.g., methods for defining sequences, characteristics and distinguishing features of sequences and progressions, and types of tasks related to progressions (Tasks 2, 4, 7). They are also asked to formulate hypotheses (Tasks 4, 7), outline directions for the study of progressions, and organize their knowledge according to the coefficients of progression (Task 7).

The first section brings to the surface the students' existing knowledge, concepts, and facts on the topic of 'sequences and progressions' and thus activates their semantic field on this topic. At the same time, it provides the teacher with valuable feedback: based on the student's performance in Part I, the teacher can gauge personal commitment to the topic, the depth of their acquired knowledge and methods, and the variety of approaches they use in their learning process.

The second part is structured as a test with two variants, which, in addition to the traditional assessment test on 'sequences and progressions,' also fulfills informative and systematizing functions. Students learn about different types of problems related to progressions, such as formulating the general concept of a progression, determining the position of a concept based on its value, and formulating relationships between concepts in mathematical language. They also practice applying the properties of progressions, calculating the sum of a finite number of terms, and solving word problems with progressions.

In working on these topics, students relate the information to the different areas of knowledge and make a connection to the work on Task 7 in Part I.

The third part of the proposed assessment represents a stage of reflection and self-assessment (Tasks 1, 4), creation of new knowledge

(Task 2), and encouragement to further expand and enrich the semantic field on the topic of "sequences and progressions" (Tasks 2, 3) and the manifestation of students' creative abilities (Tasks 2, 3). In this stage, the teacher has the opportunity not only to assess the subject-related learning outcomes on the topic of "sequences and progressions" but also to draw conclusions about the creativity and self-regulation of the individual students.

Here is the content of this final assessment's first and third parts.

SEQUENCES AND PROGRESSIONS

Part I

1. From the following list, choose at most three suitable associative words that you think have to do with the concept 'order' and write them down:

● after
● in succession
● last
● follow one after another
● infinite
● investigation
● consequence
● suspects
● investigate
● intermediaries
● queue
● numbered
● unlimited

Try to find your own association with the word 'sequence.'

2. Order the following sequences: a) - c) according to how they are set. To do this, fill in the table by indicating the letter of the corresponding sequence:

Analytical Method	*Recursive Method*	*Verbal Method*

a) The Fibonacci sequence, the first two terms of which are equal to 1, and each subsequent term, starting from the third, is the sum of the two previous terms;

b) The factorial sequence, where the n-th term is equal to the product of natural numbers from 1 to n, that is, $1 \cdot 2 \cdot 3 \cdot ... \cdot (n–1) \cdot n$, denoted by $n!$;

c) The sequence of prime numbers contained in the natural range from 1 to 50.

3. Rephrase the following statements:

“The smallest of all natural numbers is equal to one.”

“Among the numbers that are the opposite of the natural numbers, there is no number 0.”

4. Determine whether each statement is true or false. Try to give a suitable example or counterexample.

a) A functional dependency defined on the set of natural numbers is an infinite numerical sequence.

b) The graph of a numerical sequence is a set of isolated points in the plane.

c) Every sequence is a progression.

d) An infinite numerical sequence is a numerical function defined on the set of all natural numbers.

e) Every recurrence relation defines a progression.

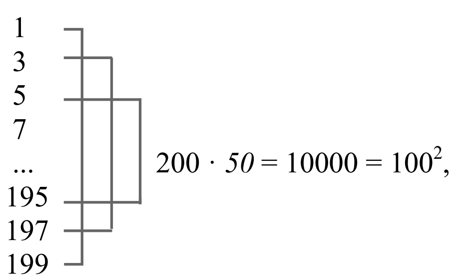
f) Every progression is a monotone sequence.

g) Every constant sequence is a geometric progression.

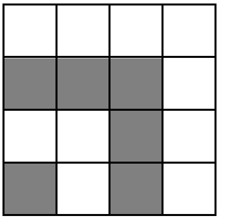
Try to create a diagram that establishes the relationships between the concepts of ‘function,’ ‘numerical sequence,’ ‘sequence,’ ‘progression,’ ‘monotonic sequence,’ ‘arithmetic progression,’ ‘geometric progression,’ and ‘stationary sequence.’

5. Think of as many situations as possible (everyday, unusual, traditional) where progressions occur.

6. Analyze the following forms of representing the sum 1 + 3 + 5 + … + 197 + 199..



1.	**1** □	+	9□□□□□□□□□	=	**10**
2.	**3** □□□	+	7 □□□□□□□	=	**10** (**10** · *5*) **:** 2 **= 25 =** $\mathbf{5^2}$,
3.	**5** □□□□□	+	5 □□□□□	=	**10** ***hence***
4.	**7** □□□□□□□	+	3 □□□	=	**10** (**200** · *100*) **:** 2 **= 10000 =** $\mathbf{100^2}$
5.	**9** □□□□□□□□□	+	1 □	=	**10**



Here, three squares identical to the square at the bottom left are added next to it, followed by five more squares, then seven, nine, and so on, until finally 199 squares are added.

a) Calculate the area of the square described.

b) Try to form and calculate a similar sum to the one described in this task.

7. Put together three problems on arithmetic (or geometric) progressions. The tasks should be different and designed as a test, or formulate all kinds of tasks that can be included in a test on ‘progressions.’

Part II (test tasks in 2 variants)

Part III

Try to identify the specific aspects of the following Tasks 1−3 and propose a method for assessing their completion results. We suggest that you select and complete one of the following tasks:

1. Create a series of questions about a given sequence $\{a_n\}$, $n = 1, 2, \ldots$, in such a way that by answering these questions, you obtain complete information about the sequence (its form, terms, properties, historical facts related to this sequence). How many questions are needed?

2. For an upcoming TV commercial in a math series, try to create a presentation script for a specific sequence. First, think about what sequence you could successfully present to a non-math audience and what features of this sequence could be emphasized.

3. Try to devise a metaphor for the word ‘progression’ and illustrate it in color.

An analysis of the integration of logical and intuitive experience in the process of learning mathematics leads to the following conclusions:

1. The dominance of one type of experience over the other hinders the understanding of mathematical material.

2. The balance between logical and intuitive experience should be carefully considered when planning a lesson.

3. The integration of the students’ logical and intuitive experiences improves the quality of their knowledge.

The relationship between logic and intuition has been studied by Y. Ponomarev [2]. He developed a detailed model of mathematical activity, which, in its most complete version, is used only in major scientific discoveries.

In this model, the following phases are distinguished:

1. Collecting certain facts and patterns through observation, calculation, and measurement.
2. Forming hypotheses based on this information (intuitive).
3. Proving or disproving these hypotheses through logical thinking.
4. Systematizing the proven facts and creating a theory.
5. Practical application.

Moving through these stages by integrating students' logical and intuitive experiences will improve their understanding of the mathematical material.

According to the concept of Y. Ponomarev, there are two types of creative tasks:

● Tasks that are solved with the help of conscious techniques (this process can be controlled);

● Tasks that are solved using unconscious techniques (this process cannot be algorithmized but can be partially controlled by creating conditions that promote intuitive insights).

To ensure understanding of the mathematical material, it is important that both types of tasks occur in class and that the teacher creates conditions that encourage students' intuition (e.g., involving them in forming hypotheses and justifying or refuting them). However, it is important to remember that accurate intuition can only develop on a solid foundation of students' knowledge (definitions, theorems, and the basic concepts of their proofs).

Integrating a student's logical and intuitive experiences contributes to the following:

● Understanding of the subject matter: the student becomes a full subject of the teaching activity, and knowledge is experienced, appropriated, and personalized.

● Student motivation: the intuitively proposed hypothesis stimulates students to search for its justification, i.e., a logical proof. Students master the informational component of mathematical knowledge (knowledge, skills, abilities) and the awareness of their application and use. At the same time, they acquire methods of general scientific knowledge (heuristic and logical), which should also be the subject of conscious discussion.

Creative activity, including mathematical activity, involves a process of the search for new results:

1. Intuitive processes are crucial to generating and developing new ideas. These processes are based on ‘plausible’ conclusions drawn from concrete cases through comparison, induction, analogy, and generalization. Among these processes, intuition stands out as it emerges from the repetition of the brain’s logical thinking and eventually becomes part of cognitive skills. These related skills are interconnected; they reinforce each other and merge into intuition. Mastery of cognitive skills in conjunction with ‘plausible thinking’ and both general and specific methods of scientific inquiry creates an optimal environment for developing intuition in education.
2. Logical processes are based on deductive reasoning, and arguments are based on logical laws and forms.

Therefore, creative mathematical activities should be based on integrating students’ logical and intuitive experiences in their dialectical unity.

How can we teach students to conclude? How can we incorporate their logical and intuitive experiences? D. Pólya recommends structuring this process as follows: students first observe how the teacher argues and learn to argue ‘plausibly’ by imitating the teacher.

Here, you will find a system for understanding the teaching material that is based on the integration of students’ logical and intuitive experiences:

1. Choose a simple topic: choose a topic that is simple enough to avoid overly complex explanations.
2. Students should know how to research both with a teacher and independently.
3. Be aware that the chain of reasoning leading to a hypothesis can be lengthy and may not fit into the time frame of a lesson. However, it should be clear and understandable to the students.
4. The goal of the lesson should not only be to learn new material but also to teach methods of learning and scientific inquiry (both heuristic and deductive). This will help students develop the ability to ask questions and formulate hypotheses.
5. Prioritize logical reasoning: the logical component should play the primary role in this process, as understanding the logical structure of definitions, theorem statements, and the nature of mathematical proofs guides heuristic exploration. Hypotheses established intuitively through ‘plausible’ reasoning should be proven or disproven deductively.

6. Clarify the basis of reasoning: students need to clearly understand the basis for their conclusions, whether they are arrived at by plausible reasoning based on induction or by correct reasoning based on deduction.

7. Students should be trained to present complete arguments and avoid claims that are ‘almost’ proven.

The teacher must guide students through the process of proving theorems, as this promotes a conscious understanding of the different methods of scientific inquiry (both general and specific). This can be done both during the proof process and afterward, when the teacher draws the students’ attention to the proof method, its characteristics, and its nature.

Based on the research conducted and the analysis of the psychological, educational, scientific-methodological, and pedagogical literature on this subject, we conclude that integrating students’ logical and intuitive experiences facilitates understanding complex learning content. This integration helps students to make connections between formal concepts, their meanings, and content, which allows them to develop a holistic view, internalize new knowledge, and make it personal.

Overcoming difficulties in mastering complex or difficult educational material occurs at the intersection of understanding, where students link formal definitions of concepts to their logical, physical, geometric, and other meanings, as well as to their deep content – from underlying ideas and historical context to their place within mathematical theory and students’ personal associations. This integration of logical and intuitive experiences is crucial. In addition, the variability of information representation (using different forms and types) can help to expand and enrich internal representations of the new knowledge. It is also important to develop a holistic understanding of the subject matter that results from making formal-logical structural and functional connections.

Overcoming difficulties in learning complex educational material occurs when students combine logical and intuitive experiences. This helps them to grasp not only formal definitions but also the logical, physical, and geometric meanings of concepts and develop a deeper understanding – from the history of the idea to its role in theory to their personal associations with the material. In addition, using different forms of presenting information (including different types and formats) can expand and enrich the inner picture of new knowledge. It is also important to develop a holistic view of the material, which is built by making formal, logical, structural, and functional connections [13, 14].

At present, it's important to re-evaluate both the current experience of developing rational thinking in math education and the process of building intuition in mathematics and to create conditions for their integration.

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ИНТЕГРАЦИЯ ЛОГИЧЕСКОГО И ИНТУИТИВНОГО ОПЫТА УЧЕНИКА КАК УСЛОВИЕ ПОНИМАНИЯ МАТЕМАТИКИ

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Аннотация. В современных психолого-педагогических работах исследуется дуальность человеческого мышления, когда две противоположные системы – эвристическая и аналитическая – взаимодействуют в диалектическом единстве. Отсюда и два соответствующих способа познания мира: логический и интуитивный. Акцент на формирование только логического опыта ученика, в конечном итоге, сводится к формированию умения решать определенный, достаточно узкий класс типовых задач, а также не способствует гармоничному всестороннему развитию личности. Поэтому в настоящее время большое внимание уделяется вопросу «понимающего» освоения учебных дисциплин и преодолению формализма в процессе обучения посредством активизации учебно-познавательной деятельности и применения методических способов интеграции логического и интуитивного опыта ученика, способствующих достижению более глубокого уровня понимания учебного материала. Данное исследование посвящено анализу особенностей интеграции логического и интуитивного познавательных стилей мышления и их использования в процессе обучения математике.

Ключевые слова: *интеллект, познавательные стили мышления, интенциональный опыт, понимающее обучение математике*

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