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Dark matter hypothesis and new possibilities of the Skyrme–Faddeev chiral model

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Abstract. New possibilities of the 16-spinor realization of the Skyrme–Faddeev chiral model are discussed. Using gauge invariance principle, it is shown that there exist two independent ways for breaking the isotopic invariance symmetry. The first one concerns the interaction with the electromagnetic field (ordinary photons generated by the electric charge) and the second one includes the interaction with the new vector field (shadow/dark photons generated by the special neutrino charge). The neutrino oscillation phenomenon is explained.

Key words and phrases: chiral model, 16-spinor field, shadow/dark photons, neutrino charge

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1. Introduction. Structure of the $\mathrm{SO}(3)$ generators

The main idea behind this research concerns the problem of unifying the approaches suggested by Skyrme [1] and Faddeev [2] for interpreting baryons and leptons as topological solitons. To this aim, the 16-spinor Ψ realization of the Skyrme–Faddeev chiral models was considered some years ago [3]. Within the scope of this spinor realization there exist two kinds of the internal SO(3) generators:

$$\Lambda_i/2 = I_8 \otimes \sigma_i/2$$

and also

$$\lambda_i/2 = I_4 \otimes \sigma_i \otimes I_2/2,$$

where σ_i, $i = 1, 2, 3$, stands for the Pauli matrices and I_n denotes the unity matrix of the n-th order. The generators $\Lambda_i/2$ are used for constructing the S^2 manifold $(\bar{\Psi}\Lambda_i\Psi)^2 = \text{const}$ determining the Hopf invariant Q_{H}, which is interpreted as the lepton charge $\mathbb{L}$ according to Faddeev. As for the generators $\lambda_i/2$, they determine the isotopic space symmetry, with its localization implying the Yang–Mills axial vector field, which gives the main contribution to strong interactions.

2. Breaking the isotopic symmetry

However, the isotopic symmetry is broken by the electromagnetic interactions due to the extension of derivatives:

$$\partial_\mu \Psi \to \partial_\mu \Psi - \imath e_0 \Gamma_e A_\mu \Psi; \quad \mu = 0, 1, 2, 3,$$

where the electromagnetic coupling constant e_0 and the corresponding charge generator $\Gamma_e = P_3 \Lambda$ are introduced. Here the denotations are used: $P_3 = (1 - \lambda_3)/2$; $\Lambda = (1 - \Lambda_3)/2$. We are now in a position to mention the other possibility of the isotopic symmetry breaking due to extending the derivatives:

$$\partial_\mu \Psi \to \partial_\mu \Psi - \imath \tilde{e}_0 \Gamma_c C_\mu \Psi,$$

where the new massless vector field C_μ, the new coupling constant $\tilde{e}_0$ and the corresponding generator $\Gamma_c = N_3 \Lambda$, are introduced. Here the new isotopic projector $N_3 = (1 + \lambda_3)/2$ is used. It is worth while to underline that this new vector field should be generated by the special charge, which is similar to the electromagnetic one. However, only neutral leptons (neutrinos) should be endowed with this charge, so it can be called "the neutrino charge" [4, 5], and the corresponding vector field C_μ describes the unusual photons called "shadow" or "dark" ones. It should be also stressed that the universal vacuum state Ψ_0 there exists in this model, with the natural projector property being $\Lambda \Psi_0 = 0$, where the boundary condition at space infinity reads:

$$\Psi_0 = \lim_{|\vec{x}| \to \infty} \Psi. \tag{1}$$

3. Correspondence with quantum mechanics. Lepton part of the Lagrangian

The other problem to be solved in this model concerns the correspondence with quantum mechanics. According to Einstein [6, 7], particles should be represented as soliton-like configurations described by some regular solutions to field equations. Let us consider small excitations of the particle-soliton near the vacuum: $\Psi = \Psi_0 + \xi$, where $\xi \to 0$ as $|\vec{x}| \to \infty$. Correspondence with quantum mechanics means that the field ξ should satisfy the Klein–Gordon equation of the form

$$\left(\partial_\mu \partial^\mu + M^2\right) \xi = 0, \tag{2}$$

where M stands for the mass of the particle-soliton in natural units $\hbar = c = 1$. The latter condition implies the special structure of the Lagrangian density of the model in question [3]:

$$\mathcal{L} = \mathcal{L}_{\text{spin}} + \mathcal{L}_{\text{em}} + \mathcal{L}_c + \mathcal{L}_g,$$

where the following denotations are used:

$$\mathcal{L}_{\text{spin}} = \frac{1}{2\lambda^2} D + \frac{\varepsilon^2}{4} f_{\mu\nu} f^{\mu\nu} - V,$$

$$D = \overline{D_\mu \Psi} \gamma^\nu J_\nu D^\mu \Psi,$$

$$f_{\mu\nu} = (\bar{\Psi} \gamma^\alpha D_{[\mu} \Psi)(\overline{D_{\nu]} \Psi} \gamma_\alpha \Psi),$$

$$V = -\frac{2D^3}{\lambda^2 K^2 \ell_{\text{Pl}}^4 \varkappa_0^8} (J_\mu J^\mu - \varkappa_0^2)^2,$$

$$D_\mu \Psi = \partial_\mu \Psi - \left(\imath e_0 \Gamma_e A_\mu + \imath \tilde{e}_0 \Gamma_c C_\mu + \Gamma_\mu\right) \Psi.$$

Here the extended derivative contains the spinor affine connection Γ_μ. The Lagrangian includes the sigma-model part D with the projector $\gamma_0 J_\nu \gamma^\nu$ on the positive energy states. The first two terms in the Lagrangian $\mathcal{L}_{\text{spin}}$ imply the lower estimate of the energy through the corresponding topological charge (lepton or baryon one). Here the Dirac current reads $J_\mu = \bar{\Psi}\gamma_\mu \Psi$. At last, the Higgs potential V has the special structure based on the boundary condition:

$$\lim_{|\vec{x}|\to\infty} J_\mu J^\mu = \varkappa_0^2. \tag{3}$$

For providing the compatibility of the conditions (1), (3) and (4) the Higgs potential V includes the special gravitational invariant known as that of Kretschmann:

$$K = R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda},$$

that is the square of the Riemann curvature tensor. Finally, the Einstein gravitational term is included:

$$\mathcal{L}_g = -\frac{1}{2\varkappa} R,$$

where $\varkappa = 8\pi G/(c^4)$ and R, G stand for the scalar curvature and the Newton gravitational constant, respectively.

4. Quantization of the electric and the neutrino charges

It is worth while to underline that the discrete nature of the charges mentioned above can be provided through the special structure of the electromagnetic Lagrangian density and that of the dark/shadow photons part. Introducing intensity tensors for the electromagnetic and shadow fields respectively: $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$; $G_{\mu\nu} = \partial_{[\mu} C_{\nu]}$, let us write down the corresponding Lagrangian densities:

$$\mathcal{L}_{\text{em}} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \left[1 + \mu_0 \sin^2 \frac{\pi U}{2e}\right], \quad \mu_0 = \text{const};$$
$$\mathcal{L}_c = -\frac{1}{16\pi} G_{\mu\nu} G^{\mu\nu} \left[1 + \nu_0 \sin^2 \frac{\pi \tilde{U}}{2\tilde{e}}\right], \quad \nu_0 = \text{const};$$

where the special denotations are used:

$$U = (n_\mu A^\mu)^2 (-E_\nu E^\nu)^{-1/2}; \quad \tilde{U} = (n_\mu C^\mu)^2 (-G_\nu G^\nu)^{-1/2}.$$

Here the following quantities are introduced: $E_\nu = n^\mu F_{\mu\nu}$; $G_\nu = n^\mu G_{\mu\nu}$, where $n^\mu = J^\mu / J$ is the unit vector. It can be shown that at the space infinity the asymptotic behavior of the vector fields coincides with that of the Coulomb potential: $A_0 = q/r$; $C_0 = \tilde{q}/r$, where $r = |\vec{x}|$, with the corresponding charges taking integer values of the fundamental charges e and $\tilde{e}$.

5. Mirror symmetry and the intersection problem of lepton and baryon states

Let us now recall the Skyrme's idea [1] to determine the S^3 manifold by the O(4) invariant condition

$$(\bar{\Psi}\Psi)^2 + (\imath \bar{\Psi}\gamma_5 \vec{\lambda} \Psi)^2 = \text{const} \tag{4}$$

and identify the baryon charge $\mathbb{B}$ with the winding number $\deg(S^3 \to S^3)$. However, the main question arises: how to exclude the intersection of baryon and lepton sectors? The answer is given by the special mirror symmetry that should be attributed to the states in the lepton sector:

$$\Psi_{\mathrm{L}} = \gamma_{(0)}\Psi_{\mathrm{L}}. \tag{5}$$

The similar mirror symmetry but in the isotopic space should be attributed to the states in the baryon sector:

$$\Psi_{\mathrm{B}} = \gamma_{(0)}\gamma_5\gamma_{(2)}\lambda_2\Psi^*_{\mathrm{B}}. \tag{6}$$

Here the following structure of the plane Dirac γ-matrices are used:

$$\gamma_{(0)} = I_2 \otimes \sigma_1 \otimes I_4; \quad \gamma_{(k)} = -\imath\sigma_k \otimes \sigma_2 \otimes I_4,$$

where $k = 1, 2, 3$, and $\gamma_5 = I_2 \otimes \sigma_3 \otimes I_4$. The physical origin of the symmetry (6) reduces to the fact of charge independence of strong interactions. To prove the impossibility of intersecting lepton and baryon states let us introduce the following representation of the 16-spinor [8]:

$$\Psi = \oplus_{j=1}^{2}\left(\varphi_j \oplus \chi_j \oplus \xi_j \oplus \zeta_j\right), \tag{7}$$

where φ_j, χ_j, ξ_j, ζ_j stand for some 2-spinors. Applying the symmetry (6) to (7), one finds $\varphi_j = \chi_j$, $\xi_j = \zeta_j$ for the Weyl representation of γ-matrices. For this effective 8-spinor one obtains $(\bar\Psi\vec\Lambda\Psi) \neq 0$, but $(\bar\Psi\gamma_5\vec\lambda\Psi) = 0$. Therefore, in view of (5) one gets $\mathbb{B} = 0$, $\mathbb{L} \neq 0$.

On the contrary, applying the symmetry (6) to (7), one finds for the baryon sector $\xi_j = \imath\sigma_2\varphi_j^*$, $\zeta_j = \imath\sigma_2\chi_j^*$. For this effective 8-spinor one gets $(\bar\Psi\Lambda_2\Psi) = 0$, but $(\bar\Psi\Psi) \neq 0$, $(\bar\Psi\gamma_5\vec\lambda\Psi) \neq 0$. Thus, as a consequence, $\mathbb{B} \neq 0$, $\mathbb{L} = 0$. Taking these facts into account, one concludes about impossibility of intersecting lepton and baryon states.

Finally, one can deduce the structure of the vacuum state Ψ_0. To this end, due to the universal character of the vacuum state, let us apply to Ψ_0 both (6) and (6) symmetries, with the result being:

$$\Psi_0 = \mathrm{col}\left\{\begin{bmatrix} a_0 \\ 0 \end{bmatrix}, \begin{bmatrix} a_0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -a_0^* \end{bmatrix}, \begin{bmatrix} 0 \\ -a_0^* \end{bmatrix}\right\}.$$

Here $4|a_0|^2 = \varkappa_0$ is the new fundamental constant, characterizing the vacuum state.

6. Axially symmetric states

First one remarks that due to space reflection symmetry of the lepton sector one gets $\varphi_j = \chi_j$ and also $\xi_j = \zeta_j$. To study the angular structure of the spinor field let us use the principle of symmetric criticality [9] and consider a class of axially symmetric states invariant under the special group of combined space and isotopic rotations:

$$G = \mathrm{diag}\left[\mathrm{SO}(2)_S \otimes \mathrm{SO}(2)_I\right], \tag{8}$$

with the corresponding generators reading: $\bar J_3 = -\imath\partial_\phi + \sigma_3/2$; $T_3 = \lambda_3\Lambda_3/2$, respectively. Solving the invariance equations:

$$\bar J_3\Psi_1 = \frac{\lambda_3}{2}\Psi_1, \quad \bar J_3\Psi_2 = -\frac{\lambda_3}{2}\Psi_2,$$

one can find the dependence of the fields on the azimuth angle ϕ:

$$\varphi_1 = \begin{bmatrix} f_1 \\ g_1 \exp[\imath\phi] \end{bmatrix}; \quad \zeta_1 = \begin{bmatrix} u_1 \exp[-\imath\phi] \\ v_1 \end{bmatrix};$$
$$\varphi_2 = \begin{bmatrix} g_2 \exp[-\imath\phi] \\ f_2 \end{bmatrix}; \quad \zeta_2 = \begin{bmatrix} u_2 \\ v_2 \exp[\imath\phi] \end{bmatrix}.$$

It should be also underlined that the fields satisfy the vacuum boundary condition at the space infinity $\Psi \underset{r\to\infty}{\to} \Psi_0$, if the following nontrivial boundary conditions read:

$$f_1 \underset{r\to\infty}{\to} a_0, \tag{9}$$
$$v_1 \underset{r\to\infty}{\to} -a_0^*. \tag{10}$$

7. Structure of the lepton charge

First of all one should remark that all calculations in the lepton sector appear to be drastically simplified through using the toroidal coordinates $x \geq 0$, $\xi \in [-\pi, \pi]$. Their connection to the cylindrical ones reads:

$$\rho = a\frac{\sinh x}{\cosh x - \cos\xi}, \quad z = a\frac{\sin\xi}{\cosh x - \cos\xi},$$

where a stands for the length parameter. The explanation for this effect can be seen if one identifies, following Faddeev, the lepton charge $\mathbb{L}$ with the Hopf invariant $Q_{\mathrm{H}} = \mathbb{L}$. To define the structure of Q_{H}, let us introduce the unit 3-vector $\vec{n} = \vec{V}/|\vec{V}|$, where $\vec{V} = (\Psi^+\vec{\Lambda}\Psi)$, the manifold S^2 being determined by the condition $\vec{n}^2 = 1$. In view of the boundary conditions (9) and (10) one can put: $u_1 = g_1 = f_2 = v_2 = 0$ and also:

$$f_1 = sa_0, \quad v_1 = -sa_0^*, \quad s^* = s;$$
$$g_2 = a_0 v \exp[\imath\mu], \; u_2 = u, \; v^* = v, \; u^* = u, \; \mu^* = \mu.$$

As a result, one finds that the quantity $V_1 + \imath V_2 = 2\Psi_1^+\Psi_2$ reduces to the following one:

$$V_1 + \imath V_2 = \varkappa_0 s v \exp\left[\imath(\mu - \phi)\right]. \tag{11}$$

On the other hand, there exists the following correspondence between the 3-vector n^a, $a = 1, 2, 3$, and the special 4-vector a^μ:

$$\partial_\mu a_\nu - \partial_\nu a_\mu = 2\epsilon_{abc}\partial_\mu n^a \partial_\nu n^b n^c. \tag{12}$$

On the basis of the relation (12) one can construct the identically conserved topological current:

$$j^\mu_{\mathrm{H}} = -(128\pi^2)^{-1}\epsilon^{\mu\nu\sigma\tau}\left(\partial_\nu a_\sigma - \partial_\sigma a_\nu\right) a_\tau, \tag{13}$$

the conservation equation $\partial_\mu j^\mu_{\mathrm{H}} = 0$ being implied by the S^2-condition $\vec{n}^2 = 1$. Identifying the conserved charge $\int dV j^0_{\mathrm{H}}$, where dV is the element of the 3-volume, with the Hopf topological invariant, one finds the well-known Whitehead formula for the degree of knottedness or link invariant [10, 11]:

$$Q_{\mathrm{H}} = -(8\pi)^{-2}\int dV([\nabla\vec{a}]\vec{a}). \tag{14}$$

In our case of axially symmetric configurations the expression (14) can be reduced to the standard winding number $\deg(S^3 \to S^3)$ through applying the Hopf mapping $S^3 \to S^2$ and the definitions (12), (13), (14). To this end, let us introduce the auxiliary 2-spinor:

$$\chi = \mathrm{col}\Big[\cos\tilde{A} + \imath\sin\tilde{A}\cos\tilde{B}, \sin\tilde{A}\sin\tilde{B}\exp(\imath\tilde{C})\Big] \tag{15}$$

and calculate the following 3-vectors:

$$\vec{n} = \chi^{+}\vec{\sigma}\chi, \quad \vec{a} = -\imath\chi^{+}\nabla\chi, \quad [\nabla\vec{a}] = -2\imath[\nabla\chi^{+}\nabla\chi].$$

As a result, one gets from (15) the desired form of Q_{H}:

$$Q_{\mathrm{H}} = \frac{1}{2\pi^2}\int dV \sin^2\tilde{A}\sin\tilde{B}\big(\nabla\tilde{C}[\nabla\tilde{A}\nabla\tilde{B}]\big). \tag{16}$$

However, the integral (16) can be calculated exactly for the axially symmetric states via the substitution:

$$\sin\tilde{A}\sin\tilde{B} = \sin(\tilde{\sigma}/2), \quad \tan\tilde{A}\cos\tilde{B} = \tan\varphi.$$

Thus, the integral (16) takes the form:

$$Q_{\mathrm{H}} = \frac{1}{8\pi^2}\int dV\big([\nabla(\varphi\nabla n_3)]\nabla\tilde{C}\big), \tag{17}$$

where $n_3 = \cos\tilde{\sigma}$. Comparing the phases in (11) and in $n_1 + \imath n_2 = 2\chi_1^{+}\chi_2 \sim \exp[\imath(\tilde{C} - \varphi)]$, one finds $\tilde{C} = -\phi$ and $\varphi = -\mu$. First, one deduces from (17) that

$$Q_{\mathrm{H}} = \frac{1}{8\pi^2}\int dV\big([\nabla(\mu\nabla n_3)]\nabla\phi\big).$$

Using the Stokes theorem and performing the ϕ-integration, one gets the contour integral:

$$Q_{\mathrm{H}} = -\frac{1}{4\pi}\oint\limits_{L}\mu dn_3. \tag{18}$$

It is worth while to stress that the contour L in (18) contains the z-axis, the large asymptotic circumference, where $\sin\tilde{\sigma} = 0$, and also surrounds the interval $0 \le \rho \le a$, $z = 0$. Therefore, one concludes that the integral (18) reduces to the jump $[\mu]$ of the function μ on that interval, the latter one connecting the north and the south poles of the sphere S^2, i. e. the points $n_3 = \pm 1$, respectively. Finally, one obtains the value of the lepton number $\mathbb{L} = n$ characterizing our particle-soliton, since

$$\frac{1}{4\pi}\int_{-1}^{+1}[\mu]dn_3 = n.$$

Here the evident property of the angular variable ξ was taken into account:

$$\lim_{z\to+0}\xi = \pi, \quad \lim_{z\to-0}\xi = -\pi.$$

Therefore, the following relations hold: $\mu = n\xi$, $[\mu] = n[\xi] = 2\pi n$, with n being some integer number.

Let us now recall the final angular structure of the spinor field Ψ:

$$\begin{aligned} \varphi_1 &= \begin{bmatrix} sa_0 \\ 0 \end{bmatrix}, & \varphi_2 &= \begin{bmatrix} va_0\exp[\imath(n\xi - \phi)] \\ 0 \end{bmatrix}, \\ \zeta_1 &= \begin{bmatrix} 0 \\ -sa_0^{*} \end{bmatrix}, & \zeta_2 &= \begin{bmatrix} u \\ 0 \end{bmatrix}; \end{aligned} \tag{19}$$

with s, v, u being some real functions of the radial toroidal coordinate x. Inserting (19) into the current $J_\mu = \bar{\Psi}\gamma_\mu\Psi$, one gets $J_\mu J^\mu = J^2$, where $J = 2\left[|a_0|^2(2s^2 + v^2) + u^2\right]$. This structure of the current J_μ suggests the following simplifying substitution:

$$s|a_0| = \frac{\sqrt{J}}{2}\sin A\cos B, \quad u = \left(\frac{J}{2}\right)^{1/2}\cos A, \quad v|a_0| = \left(\frac{J}{2}\right)^{1/2}\sin A\sin B.$$

8. Neutrino dark charge and spin. Neutrino oscillations

Let us first recall the invariant definition of the neutrino charge (or shadow/dark charge):

$$\tilde{U} = \frac{C_0^2}{|\dot{C}_0|} = \tilde{e} \equiv \tilde{e}_0(\hbar c).$$

Solving this equation, one finds $C_0 = \tilde{e}(1+t)^{-1}$; $t = -\log\tanh(x/2)$, that corresponds to the closed string approximation $x \to \infty$, or $t \to 0$. On the other hand, one can use the charge conservation law due to the Nœther's theorem:

$$\tilde{U} = \int \frac{\partial \mathcal{L}}{\partial C^0}dV = \tilde{e}. \tag{20}$$

However, there exists the definition of the spin z-projection:

$$S_3 = \int 2\Re\left[\frac{\partial \mathcal{L}}{\partial(D^0\Psi)}\bar{J}_3\Psi\right]dV,$$

which is equivalent, in view of the symmetry group (8), to the relation:

$$S_3 = \frac{1}{2c}\int \frac{\partial \mathcal{L}}{\partial(\tilde{e}_0 C^0)}dV. \tag{21}$$

Therefore, unifying (20) and (21), one gets

$$S_3 = \tilde{U}(2c\tilde{e}_0)^{-1} = \hbar/2.$$

Fixing the neutrino charge $Q_\nu = 1$ in the units of $\tilde{e}$, it is possible to attack the very old neutrino oscillations problem. According to observations [12–20], in the flux of ν_τ some time later there can be found a mixture of ν_μ, ν_e, $\bar{\nu}_\mu$, $\bar{\nu}_e$. The same concerns the flux of ν_μ. First of all, one should take into account the conservation law of the lepton number $\mathbb{L}$ due to its topological origin. Let us recall the properties of the lepton family:

$$\mathbb{L} = 1 : (e^-, \nu_e); \quad \mathbb{L} = 2 : (\mu^-, \nu_\mu); \quad \mathbb{L} = 3 : (\tau^-, \nu_\tau).$$

Taking into account the neutron decay: $n \to p + e^- + \bar{\nu}_e$, one should attribute to the neutron the neutrino charge $Q_\nu = -1$. The conservation laws of $\mathbb{L}$ and Q_ν imply the following reaction and its inversion process (in a medium):

$$\nu_\tau \to 2\nu_\mu + \bar{\nu}_e, \quad 2\nu_\mu + p \to \nu_e + \nu_\tau + p,$$

that explains the oscillations of ν_τ.

Now let us consider the flux of high energy ν_μ in a medium: Then the following reactions hold:

$$\nu_\mu + n \to p + \mu^-, \quad \mu^- \to e^- + \bar{\nu}_e + \nu'_\mu,$$

that explains the oscillations of ν_μ.

9. Results and Discussions

Some aspects of the dark matter problem are discussed in this paper. First, one remarks two possibilities of breaking the isotopic invariance, which imply the existence of two kinds of electric charges and corresponding electromagnetic fields and photons: ordinary and dark/shadow ones. The realization of this program specifies the adequate structure of the basic field model, the Skyrme–Faddeev chiral model. The topological solitons in this model correspond to the two classes of particles: leptons and baryons, endowed with the lepton $\mathbb{L}$ and baryon $\mathbb{B}$ charges, respectively. The intersection problem of these classes can be solved via introducing the Brioschi 16-spinors Ψ as fundamental unitary fields advocated by Einstein. The hypothesis of the alternative (dark) electric charge, called the neutrino charge Q_ν, permits one to attack and to solve the neutrino oscillation problem.

10. Conclusion

Basing on the dark matter hypothesis, the neutrino oscillation problem is attacked. The solution of this problem is given due to the conservation laws of the lepton and of the neutrino charges.

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Гипотеза о тёмной материи и новые возможности киральной модели Скирма–Фаддеева

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Аннотация. Обсуждаются новые возможности 16-спинорной реализации киральной модели Скирма–Фаддеева. Используя принцип калибровочной инвариантности, показывается, что есть два независимых способа нарушения изотопической симметрии. Первый способ состоит в том, чтобы включить взаимодействие с электромагнитным полем (обыкновенными фотонами, порождаемыми электрическим зарядом), а второй опирается на взаимодействие с новым векторным полем (теневыми/тёмными фотонами, порождаемыми специальным нейтринным зарядом). Объясняется явление нейтринных осцилляций.

Ключевые слова: киральная модель, 16-спинорное поле, теневые/тёмные фотоны, нейтринный заряд