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Liquid radial flows with a vortex through porous media

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Abstract. The filtration process is studied for a popular class of filters with radial cartridges that proved their high effectiveness in purification of water. The mass balance equation for radial flows in porous media is obtained by using the lattice approximation method, the transverse diffusion process being taken into account. The Euler dynamical equations are modified by including the Darcy force proportional to the velocity of the filtration flow. The system of equations is written for the stationary axially symmetric radial flow and solved by the perturbation method, if the vertical velocity is supposed to be small.

Key words and phrases: filtration, porous medium, Darcy force

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1. Introduction. The mass balance equation in porous media

The hydrodynamics of liquid flow in a porous medium modeling the grain filling in filters is studied [1–11]. The main concept behind this research appears to be the necessity to modify the fundamental equations of hydrodynamics to meet the requirements of mass and momentum balance under specific conditions of liquid flows through porous media. As can be shown later, bearing on the lattice approximation, the structure of the fluid current and the transverse diffusion coefficient D are derived, the latter proving to be proportional to the diameter d of the grains as constituents of the medium [12–16].

Our study concerns radial filtration process, where the purification proves to be more effective than that for cylindrical geometry. First, let us apply the lattice approximation to the mass balance equation and use the cylindrical coordinates ρ, ϕ, z, with ϕ being the azimuth angle. Let us number the lattice vertices by the indices i, j (transverse to the flow) and k (along the flow), the corresponding cylindrical coordinates being ϕ, z and ρ, respectively. Let us denote the local radial stream of the fluid by

$$G_{ijk} = \Delta S_k \, u_{ijk}, \tag{1}$$

where u_{ijk} stands for the radial velocity of the flow and ΔS_k is the area of the gap between the grains. It means that

$$\Delta S_k = \rho_k \Delta\phi \Delta z \, S_k, \tag{2}$$

where S_k denotes the porosity of the medium, with the fluid density being taken unity. Therefore, the local mass conservation law reads

$$G_{ijk} = r_{k-1}G_{ijk-1} + p_{k-1}(G_{i-1jk-1} + G_{i+1jk-1}) + q_{k-1}(G_{ij-1k-1} + G_{ij+1k-1}), \tag{3}$$

where the branching coefficients r, p, q are introduced. Thus, the mass conservation equation reads

$$\sum_{ij} G_{ijk} = \sum_{ij} G_{ijk-1},$$

and implies the constraint on the branching coefficients:

$$r_k + 2(p_k + q_k) = 1. \tag{4}$$

In view of (1), (2) and the constraint (4) one can represent the equation (3) in the form:

$$r_k\rho_k\, S_k\, u_{\rho ij}^{(k)} - r_{k-1}\rho_{k-1}\, S_{k-1}\, u_{\rho ij}^{(k-1)} = p_{k-1}\rho_{k-1}S_{k-1}\left(u_{\rho i-1j}^{(k-1)} + u_{\rho i+1j}^{(k-1)}\right) + \\ + q_{k-1}\rho_{k-1}S_{k-1}\left(u_{\rho ij-1}^{(k-1)} + u_{\rho ij+1}^{(k-1)}\right) - 2\rho_k\, S_k(p_k + q_k)\, u_{\rho ij}^{(k)}.$$

Identifying now the lattice spacing with the diameter d of the grain, one can prove through the latter relation that in the continuous limit the following differential equation is valid:

$$\partial_z\big[\rho S(u - D_z\,\partial_z\, w)\big] + \partial_\rho(r\, S\rho w) + \partial_\phi(S\, v) - \partial_\phi^2[D_\phi S\, w/\rho] = 0, \tag{5}$$

where the transverse diffusion coefficients are introduced: $D_z = q\,d$, $D_\phi = p\,d$ and the following denotations for the components of the fluid velocity are used: $u_\rho = w$, $u_\phi = v$, $u_z = u$. However, in virtue of the axial symmetry of the flow it is necessary to put $p_k = 0$. Therefore, the equation (5) takes the form of the stationary mass conservation law:

$$div\,\mathbf{j} = 0, \tag{6}$$

with the components of the current $\mathbf{j}$ reading:

$$j_\rho = r(\rho)S(\rho)w, \quad j_z = S(\rho)\big[u - D(\rho)\,\partial_z\, w\big], \tag{7}$$

where the local transverse diffusion coefficient is introduced:

$$D(\rho) = q(\rho)d(\rho). \tag{8}$$

It is worth while to stress that the effect of the transverse diffusion in porous media is widely discussed in literature [13, 14].

2. The hydrodynamics of the radial flow in porous media: Darcy's law

To find the profiles of the velocity $\mathbf{u}$ and the pressure P, it is necessary to solve the Euler equation, with the force density $\mathbf{f}$ including the gravity acceleration $\mathbf{g}$ and the Darcy force $\mathbf{f}_D = -k_D\mathbf{u}$. In the first approximation, the Darcy coefficient k_D appears to be constant: $k_D = k_0 = const$, but in general it should be some function of the velocity and the pressure [7, 15–21]. In particular, recently some deviations from the standard Darcy's law appear to be evident [22, 23]. Let us now add to the mass balance equation also the stationary Euler equation:

$$(\mathbf{u}\nabla)\mathbf{u} + \nabla P = \mathbf{g} - k_0\mathbf{u}. \tag{9}$$

Let us now rewrite the equations (6) and (9) in cylindrical coordinates:

$$\partial_\rho(r\rho S\, w) + \partial_z\big[\rho S(u - D\,\partial_z\, w)\big] = 0,$$
$$(w\,\partial_\rho + u\,\partial_z)u + \partial_z\, P + g + k_0\, u = 0,$$
$$(w\,\partial_\rho + u\,\partial_z)w - \frac{v^2}{\rho} + \partial_\rho\, P + k_0\, w = 0,$$
$$(w\,\partial_\rho + u\,\partial_z)v + \frac{v\, w}{\rho} + k_0\, v = 0.$$

3. Perturbation method

Let us suppose that the radial part of our filter has the external diameter $2b$, the internal one $2a$ and several plates (layers or cartridges) of the height $2l \ll a$. Therefore, $a \le \rho \le b$, $-l \le z \le l \ll a$ and the boundary condition for the fluid vertical velocity reads:

$$u(\rho, z = \pm l) = 0.$$

In view of the condition $l \ll a$, the vertical velocity u is supposed to be small: $u \ll v,\ w$. Therefore, in the first approximation one can put in equations (2), (2), (2), (2) $u = 0$, $\partial_z\, w = \partial_z\, v = 0$ and obtain the following structure of the velocities $w_0(\rho)$, $v_0(\rho)$ and the pressure $P = -gz + p_0(\rho)$:

$$v_0(\rho) = \frac{C_1\, a}{\rho}\ \exp\!\left(-\int\limits_a^\rho \frac{d\rho}{w_0}\right),$$

$$p_0(\rho) = C_2 - \frac{w_0^2}{2} + \int\limits_a^\rho \left(\frac{v_0^2}{\rho} - k_0\, w_0\right) d\rho,$$

where $w_0(\rho) = C_0(r\rho S)^{-1}$ and C_0, C_1, C_2 denote some constants.

In the second approximation, in view of the boundary condition (3), one can put:

$$u = \alpha z(l^2 - z^2), \quad w = w_0 + \beta z^2, \quad v = v_0 + \gamma z^2;$$

where the functions $\alpha(\rho)$, $\beta(\rho)$, $\gamma(\rho)$ take the form:

$$\alpha(\rho) = \frac{2C_3\, D}{r\rho S\, l^2}\ \exp\!\left(\int\limits_a^\rho \frac{6D}{r\, l^2}\, d\rho\right);$$

$$\beta(\rho) = \frac{C_3}{r\rho S}\ \exp\!\left(\int\limits_a^\rho \frac{6D}{r\, l^2}\, d\rho\right);$$

$$\gamma(\rho) = C(\rho)\frac{a}{\rho}\ \exp\!\left(-\int\limits_a^\rho \frac{k_0 + 4\beta D}{w_0}\, d\rho\right).$$

Finally, inserting functions (3), (3), (3) into the equations (2) and (2), one can obtain the modified pressure:

$$P = -g\, z + p_0 - (k_0\, \alpha + w_0\, \partial_\rho\, \alpha)\, z^2(l^2 - z^2/2)/2 -$$
$$- z^2\left[w_0\beta - \int\limits_a^\rho \left(\frac{2v_0\gamma}{\rho} - k_0\, \beta\right) d\rho\right],$$

where

$$C(\rho) = C_4 + \int_a^\rho \frac{k_0 v_0 \beta \rho}{w_0^2\, a} \exp\left(\int_a^\rho \frac{k_0 + 4\beta D}{w_0}\, d\rho\right)$$

and C_3, C_4 denote some constants.

4. Conclusion

Several important effects were revealed in our study of radial flows with a vortex through porous media. First, the unusual structure of the mass balance equation (2), having the form of the transverse diffusion law, was found. In this equation the transverse diffusion coefficient $D(\rho)$ appears to be proportional to the diameter $d(\rho)$ of the grain filling modeling a porous medium.

Second, the simplest Darcy's force with constant Darcy coefficient $k_D = k_0$ was used, the important dependence of the vortex velocity (3) on k_0 being established. This fact supports the necessity of generalizing the Darcy's law, in accordance with the effects mentioned in [22, 23].

Third, the important influence of the vortex velocity $v(\rho, z)$ on the purification efficiency becomes evident from the structure of the pressure (3) and (3). It is worth while to stress the connection of this effect with the fluidization process discussed in [24, 25].

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Радиальные потоки жидкости с вихрем через пористые среды

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Аннотация. Изучается процесс фильтрации для популярного класса фильтров с радиальными картриджами, доказавших свою высокую эффективность при очистке воды. Уравнение баланса массы для радиальных потоков в пористых средах получено с использованием метода решёточного приближения с учётом процесса поперечной диффузии. Динамические уравнения Эйлера модифицированы путём включения силы Дарси, пропорциональной скорости фильтрационного потока. Система уравнений записана для стационарного осесимметричного радиального потока и решена методом возмущений, если вертикальная скорость предполагается малой.

Ключевые слова: фильтрация, пористая среда, сила Дарси