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Distribution of the peak age of information in a two-node transmission group modeled by a system with a group flow and a phase-type service time

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Abstract. This article continues the cycle of works by the authors devoted to the problem of the age of information (AoI), a metric used in information systems for monitoring and managing remote sources of information from the control center. The theoretical analysis of information transmission systems requires a quantitative assessment of the “freshness” of information delivered to the control center. The process of transferring information from peripheral sources to the center is usually modeled using queuing systems. In this paper, a queuing system with phase-type distributions is used to estimate the maximum value of the information age, called the peak age. This takes into account the special requirement of the transmission protocol, which consists in the fact that information enters the system in groups of random size. For this case, an expression is obtained for the Laplace–Stieltjes transformation of the stationary distribution function of the peak age of information and its average value. Based on the results of analytical modeling, a numerical study of the dependence of the average value of the peak age of information on the system load was carried out. The correctness of the expressions obtained was verified by comparing the analytical results with the results of simulation modeling.

Key words and phrases: information age, peak information age, queuing system, phase type distribution, group flow

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1. Introduction

The problem of timely delivery of information to the control and management center arises in various spheres of human activity: in energy systems, in the industrial Internet of things, in the field of autonomous transport, in video surveillance systems, etc. [1–3]. In 2011, to quantify the freshness of information received by the control and monitoring center, the Age of Information (AoI) metric was proposed, which is a function of the time between the generation of updates at the sending node and the delay in their delivery over the network to the control and monitoring center (recipient node) [4–13]. The most convenient device for studying the problem of information age is the device of queuing systems and networks. An overview of the works in which the analysis of the age of information is proposed to be carried out using this device can be found, for example, in [14]. It should be noted that

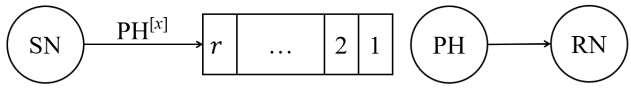

Figure 1. A two-node GT

most specialists limit themselves to simple models, for example, with an exponential distribution of time between the moments of generation of updates at the sending node and an exponential or deterministic distribution of the duration of update processing at the receiving node [15–17]. However, the simplest models of queuing systems allow us to obtain only a rough estimate of the age of information, since single-parameter distributions do not make it possible to take into account all the features of the protocols of modern dispatch control and data collection systems. In this paper, the process of transferring information from the sending node to the receiving node is modeled using a queuing system with phase-type distributions, the choice of phase parameters of which allows flexibly modeling complex dependencies that arise in modern data transmission systems.

2. Description of the model

Let's consider a group of information transmission (GT) consisting of a sender node (SN), a recipient node (RN) and a communication channel between them (Fig. 1).

Transmission from the SN to the RN is carried out by groups of packets of random length over a single communication channel. If the channel is busy, groups of packages line up in a queue with a limited number of waiting places. If there are no places in the queue, the group is lost and no longer has an impact on the information transfer process. A group is considered transferred if the last packet of this group is transmitted. By the peak age of the Z_n information transmitted by the n-th group, we will understand sum

$$Z_n = \hat{G}_{n+1} + W_{n+1}, \tag{1}$$

where $\hat{G}_{n+1}$ is the duration of the generation of the group following the n-th group, which ended with the first successful joining of the group to the transfer queue; W_{n+1} is the time of transfer of the $(n+1)$-st group from the SN to the RN.

We will model the transmission of information over the communication channel, using a single-line queuing system (QS) with a finite capacity accumulator r, $1 \leq r < \infty$.

Applications for the system are received in groups. The flow of groups of applications is recurrent with a distribution function $A(t)$, $t \geq 0$, of the phase interval [18]

$$A(t) = 1 - \boldsymbol{\alpha}^T \mathrm{e}^{\boldsymbol{\Lambda} t}\mathbf{1}, \quad \boldsymbol{\alpha}^T\mathbf{1} = 1, \tag{2}$$

with an irreducible PH representation $(\boldsymbol{\alpha}, \boldsymbol{\Lambda})$ of order l.

Each group receives a random number of applications η with a given probability in advance:

$$\alpha_k = \mathrm{P}\{\eta = k\}, \quad k = \overline{1, r+1}.$$

At the same time

$$\sum_{k=1}^{r+1} \alpha_k = 1.$$

If, upon receipt of an application in the system, the number of applications in a group exceeds the number of available places in it, then the available places are filled, and the remainder of the applications from the new group is lost. A group is considered accepted into the system if at least one application from its membership is accepted. In the opposite case, the group is lost and the generation of the next group immediately begins.

Applications are served one at a time. Their service times do not depend on the aggregate and do not depend on the duration of generation and the number of applications in the system and have a common PH of $B(t)$ phase type:

$$B(t) = 1 - \boldsymbol{\beta}^T \mathrm{e}^{\boldsymbol{M}t}\mathbf{1}, \quad t \geq 0, \quad \boldsymbol{\beta}^T\mathbf{1} = 1, \tag{3}$$

with an irreducible PH representation $(\boldsymbol{\beta}, \boldsymbol{M})$ of the order of m.

We will assume that groups of applications are served in the order they are received, and applications within the group are served in a random order. In other words, an arbitrary application may end up in the right place in a group of k applications with the same probability of $1/k$.

The QS in question will be encoded in Kendall's notation as $PH^{[x]}/PH/1/r$.

3. Mathematical model

This system was considered in [19]. In this work, the functioning of the system was described by a homogeneous Markov process (MP) $\{X(t),\ t \geq 0\}$ over a set of states

$$\mathscr{X} = \bigcup_{n=0}^{r+1} \mathscr{X}_n,$$

where $\mathscr{X}_0 = \{(i,0),\ i = \overline{1,l}\}$, $\mathscr{X}_n = \{(i,n,j),\ i = \overline{1,l},\ j = \overline{1,m}\}$, $n = \overline{1,r+1}$.

Here and further, the expression $X(t) = (i,0)$ means that at time t the system is empty, and the generation process is going through phase i; $X(t) = (i,n,j)$ means that at time t there are n applications in the system, the generation process is going through phase i, and the maintenance process is phase j.

Since all states (MP) $\{X(t),\ t \geq 0\}$ communicate with each other, it is ergodic.

The limiting probabilities

$$p_{i0} = \lim_{t\to\infty} \mathrm{P}\{X(t) = (i,0)\},$$

$$p_{inj} = \lim_{t\to\infty} \mathrm{P}\{X(t) = (i,n,j)\},$$

exist, are strictly positive, do not depend on the initial distribution, coincide with stationary and they are the only solution to the system of equilibrium equations (SEE) with a normalization condition.

If you want to get acquainted with the conclusion of the SEE, we recommend that you refer to the materials [19]. We will limit ourselves to reproducing the main result of this work related to the development of a recurrent matrix algorithm for calculating stationary probabilities of system states. To do this, we introduce vectors:

$$\boldsymbol{p}_0^T = (p_{10}, p_{20}, \dots, p_{l0}),$$

$$\boldsymbol{p}_n^T = (p_{1n1}, \dots, p_{1nm}, p_{2n1}, \dots, p_{2nm}, \dots, p_{lnm})$$

and let's introduce the notation:

$$\boldsymbol{\lambda} = -\boldsymbol{\Lambda}\mathbf{1},$$

$$\boldsymbol{\mu} = -\boldsymbol{M}\mathbf{1},$$

$$A_k = \begin{cases} 1, \; k = 0, \\ 1 - \sum_{s=1}^{k} a_s, \end{cases}$$

where $1 - \sum_{s=1}^{k} a_s$ is the probability that the group size will be greater than k, $k = \overline{1, r+1}$.

$\boldsymbol{\Lambda} \oplus \boldsymbol{M} = \boldsymbol{\Lambda} \otimes \boldsymbol{I} + \boldsymbol{I} \otimes \boldsymbol{M}$ is the Kronecker sum $\boldsymbol{\Lambda}$ and $\boldsymbol{M}$, the symbol $\otimes$ denotes the tensor product of matrices; $\lambda = -(\boldsymbol{\alpha}^T \boldsymbol{\Lambda}^{-1} \mathbf{1})^{-1}$ is the intensity of receipt of groups of applications; $\mu = -(\boldsymbol{\beta}^T \boldsymbol{m}^{-1} \mathbf{1})^{-1}$ is the intensity of service of applications.

$$\widetilde{\boldsymbol{\Lambda}} = -(\boldsymbol{\Lambda} \oplus \boldsymbol{M}) + \mathbf{1}\boldsymbol{\alpha}^T \otimes \boldsymbol{M},$$

$$\widetilde{\boldsymbol{M}} = -(\boldsymbol{\Lambda} \oplus \boldsymbol{M}) + \boldsymbol{\Lambda} \otimes \mathbf{1}\boldsymbol{\beta}^T,$$

$$\widetilde{\widetilde{\boldsymbol{M}}} = \boldsymbol{\Lambda} \oplus \boldsymbol{M} + \lambda \boldsymbol{\alpha}^T \otimes \boldsymbol{I},$$

$$\boldsymbol{F}_1 = -(\boldsymbol{\Lambda} \otimes \boldsymbol{\beta}^T),$$

$$\boldsymbol{F}_2 = \lambda \boldsymbol{\alpha}^T \otimes \boldsymbol{\beta}^T,$$

$$\boldsymbol{F}_3 = \widetilde{\boldsymbol{\Lambda}} - (\lambda \boldsymbol{\alpha}^T \otimes \boldsymbol{I}),$$

$$\boldsymbol{F}_4 = \lambda \boldsymbol{\alpha}^T \otimes \boldsymbol{I},$$

$$\boldsymbol{F}_5 = \lambda \boldsymbol{\alpha}^T \otimes \mathbf{1}\boldsymbol{\beta}^T,$$

$$\boldsymbol{W}_1 = \boldsymbol{F}_1 \widetilde{\boldsymbol{M}}^{-1},$$

$$\boldsymbol{W}_n = \left[A_{n-1}\boldsymbol{F}_2 + \sum_{k=1}^{n-1} \boldsymbol{W}_k (a_{n-k}\boldsymbol{F}_4 + A_{n-k}\boldsymbol{F}_5) + \boldsymbol{W}_{n-1}\boldsymbol{F}_3 \right] \widetilde{\boldsymbol{M}}^{-1},$$

$$\boldsymbol{W}_{r+1} = -\left(a_{r+1}\boldsymbol{F}_2 + \sum_{k=1}^{r} A_{r-k} \boldsymbol{W}_k \boldsymbol{F}_4 \right) \widetilde{\widetilde{\boldsymbol{M}}}^{-1},$$

$$\boldsymbol{v} = \mathbf{1} + \sum_{k=1}^{r+1} \boldsymbol{W}_k (\mathbf{1} \otimes \mathbf{1}),$$

$$\boldsymbol{Z} = \left[-\boldsymbol{W}_r \boldsymbol{F}_3 + \boldsymbol{W}_{r+1} (\widetilde{\boldsymbol{M}} + \widetilde{\widetilde{\boldsymbol{M}}}) \right] (\boldsymbol{I} \otimes \mathbf{1}).$$

The following theorem is proved in [19]:

Theorem 1. *The stationary probability distribution* $\{\boldsymbol{p}_n, n = \overline{0, r+1}\}$ *for QS* $PH^{[x]}/PH/1/r$ *is determined by the expressions:*

$$\boldsymbol{p}_n^T = \boldsymbol{p}_0^T \boldsymbol{W}_n, \quad n = \overline{1, r+1}, \tag{4}$$

where $\boldsymbol{p}_0$ *is the only solution to the system of equations:*

$$\boldsymbol{p}_0^T \boldsymbol{Z} = \mathbf{0}^T, \tag{5}$$

$$\boldsymbol{p}_0^T \boldsymbol{v} = 1. \tag{6}$$

4. Laplace–Stiltjes transformation of the peak age of information

First, we will find the distribution of the time spent by the group of applications in the system under consideration. To do this, we construct a Markov chain (CM) $\{X_k^- = X(t_k - 0),\ k \geq 1\}$ nested at the moments $t_k - 0$ of the arrival of groups of appearances, over the set of states

$$\mathscr{X}_A^- = \{(0), (n, j),\ n = \overline{1, r+1},\ j = \overline{1, m}\}.$$

We denote by q_0 and q_{nj} the stationary probabilities of the states of the CM (0) and (n, j) and introduce the vectors:

$$\boldsymbol{q}_n^T = (q_{n1}, ..., q_{nm}).$$

From [20] we get:

$$q_0 = \frac{1}{\lambda} \boldsymbol{p}_0^T \boldsymbol{\lambda}, \tag{7}$$

$$\boldsymbol{q}_n^T = \frac{1}{\lambda} \boldsymbol{p}_n^T (\boldsymbol{\lambda} \otimes \boldsymbol{I}), \quad n = \overline{1, r+1}. \tag{8}$$

Letq_{xk} be $x \in \mathscr{X}_A^-$ there is a stationary probability that at the moment $t - 0$ before the group arrived, the system was in state x, and a group of k applications entered the system.

Next, we introduce the vectors:

$$\boldsymbol{q}_{nk}^T = (q_{(n,1),k}, ..., q_{(n,m),k}).$$

It is obvious that

$$q_{0k} = q_0 a_k, \quad k = \overline{1, r+1}, \tag{9}$$

$$\boldsymbol{q}_{nk}^T = \boldsymbol{q}_n^T a_k, \quad k = \overline{1, r+1}, \tag{10}$$

$$\boldsymbol{q}_{n,r+1-n}^T = \boldsymbol{q}_n^T A_{r-n}, \quad n = \overline{1, r}. \tag{11}$$

Let us denote by $W(t|(0), k)$, $k = \overline{1, r+1}$, the conditional distribution function (CDF) of the time spent in the system by a group of k applications, provided that at the time $t - 0$ the group was received in the system 0 applications.

Through $W(t|(n, j), k)$, $k = \overline{1, r+1-n}$ is the CDF of the time spent in the system by a group of k applications, provided that at the time $t - 0$ of the group's receipt, there were n applications in the system, and the application was serviced on the device phase j, $j = \overline{1, m}$.

The Laplace–Stieltjes transform (LST) CDF $W(t|(0), k)$ and $W(t|(n, j), k)$ are denoted by $w_0(s|k)$ and $w_n(s|j, k)$ and we introduce vectors

$$\boldsymbol{w}_n^T(s|k) = (w_n(s|1, k), ..., w_n(s|m, k))\,.$$

Note that if at the moment $t - 0$ of the group's receipt the system was in the state (0), then the first application from the group immediately goes to service, and the entire group of size k is serviced after the last service is completed (k-th) applications from this group. Therefore

$$w_0(s|k) = \beta^k(s),$$

where $\beta(s)$ is the LST of the CDF $B(t)$, is determined by the formula

$$\beta(s) = \boldsymbol{\beta}^T (s\boldsymbol{I} - \boldsymbol{M})^{-1} \boldsymbol{\mu}. \tag{12}$$

If the incoming group finds the system in the state (n, j), $n = \overline{1, r}$, then the service of the first application from the group will begin after the service of the application on the device is completed, $n-1$ application from queues and all k applications of this group will be served

Thus, we have:

$$\boldsymbol{w}_n(s|k) = (s\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{\mu}(\beta(s))^{n-1+k}. \tag{13}$$

And finally, it should be taken into account that if a group finds the system in one of the states $(r+1, j)$, then the entire group is lost. Consequently, in accordance with (8), the probability of loss of the group π is determined through the probabilities of the states $(r+1, j)$ of the CM nested at the moments $t-0$ of the application groups as follows:

$$\pi = \frac{1}{\lambda}\boldsymbol{p}_{r+1}^T(\boldsymbol{\lambda} \otimes \mathbf{1}). \tag{14}$$

Summing up our reasoning and applying the formula of complete certainty, we come to the following result:

Theorem 2. *LST of the residence time of a group of applications accepted in the QS $PH^{[x]}/PH/1/r$ is defined by the expression:*

$$w(s) = \frac{1}{1-\pi}\left[\sum_{k=1}^{r+1} a_k q_0 \beta^k(s) + \sum_{n=1}^{r}\left[\sum_{k=1}^{r+1-n} a_k \boldsymbol{q}_n^T(s\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{\mu}(\beta(s))^{n-1+k} + \right.\right.$$
$$\left.\left. + A_{r+1-n}\boldsymbol{q}_n^T(s\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{\mu}(\beta(s))^r\right]\right]. \tag{15}$$

As already noted above, the peak age of the Z_n of the n-th group of turnouts is equal to the sum of two terms $\hat{G}_{n+1}$ – the duration of generation of groups following the nth and completed the first successful connection to the queue and W_{n+1} is the time spent by the $(n+1)$-th group in the system. It is obvious that

$$\hat{G}_{n+1} = \frac{1}{1-\pi}G_{n+1},$$

where G_{n+1} is the duration of generation of the $(n+1)$-th group with CDF $A(t)$ and LST

$$\alpha(s) = \boldsymbol{\alpha}^T(s\boldsymbol{I} - \boldsymbol{\Lambda})^{-1}\boldsymbol{\lambda}. \tag{16}$$

Considering the independence of $\hat{G}_{n+1}$ and W_{n+1}, their independence from n and the fact that the LST of $\frac{1}{1-\pi}G$ is equal to $\alpha\left(\frac{s}{1-\pi}\right)$, we come to the following result.

Theorem 3. *The LST of the peak age of information transmitted by a group of applications in the $PH^{[x]}/PH/1/r$ system is determined by the expression:*

$$z(s) = \alpha\left(\frac{s}{1-\pi}\right)w(s), \tag{17}$$

where $w(s)$ and $\alpha\left(\frac{s}{1-\pi}\right)$ calculated according to (15)–(16)

5. LST of the peak age information for QS $M^{[x]}/M/1/r$

As an example, let us consider a special case – QS with a group Poisson flow of applications of intensity λ and exponential service of intensity μ.

In this case, the parameters of the initial system will be determined by the expressions:

$$\boldsymbol{\Lambda} = (-\lambda), \quad \boldsymbol{\lambda} = (\lambda), \quad \boldsymbol{\alpha} = (1), \quad \alpha(s) = \frac{\lambda}{\lambda + s}, \tag{18}$$

$$\boldsymbol{M} = (-\mu), \quad \boldsymbol{\mu} = (\mu), \quad \boldsymbol{\beta} = (1), \quad \beta(s) = \frac{\mu}{\mu + s}, \tag{19}$$

and the algorithm of Theorem 1 is expressed in the form of a recurrent formula:

$$p = \rho \sum_{k=0}^{n-1} p_k A_{n-k-1}, \quad n = \overline{1, r+1}, \tag{20}$$

where $\rho = \lambda/\mu$ and p_n are the stationary probabilities that there is n applications. The probability p_0 is determined from the normalization condition.

In accordance with (7),(8) and (14), we obtain:

$$q_n = p_n, \quad n = \overline{0, r+1},$$

$$\pi = p_{r+1}. \tag{21}$$

And finally, based on (15) and (17), taking into account (18)–(21), we come to the following result:

Theorem 4. *LST of the peak age of information transmitted the group of applications in the $M^{[x]}/M/1/r$ system is defined by the expression:*

$$z(s) = \alpha\left(\frac{s}{1 - p_{r+1}}\right) w(s),$$

where $\alpha(s) = \dfrac{\lambda}{\lambda + s}$;

$$w(s) = \frac{1}{1 - p_{r+1}} \left[\sum_{k=0}^{r+1} a_k p_0 \left[\frac{\mu}{\mu + s}\right]^k + \sum_{n=1}^{r} \sum_{k=1}^{r+1} a_k p_n \times \right.$$
$$\left. \times \left[u(r + 2 - n - k) \left[\frac{\mu}{\mu + s}\right]^{n+k} + u(k - (r + 1 - n)) \left[\frac{\mu}{\mu + s}\right]^{r+1} \right] \right]. \tag{22}$$

$$u(x) = \begin{cases} 1, & x > 0, \\ 0, & x \le 0, \end{cases} \qquad \textit{is the Heaviside function.}$$

Corollary 1. *The average value MZ of the peak age of information transmitted by a group of applications in the $M^{[x]}/M/1/r$ system is determined by the expression:*

$$MZ = \frac{1}{1 - p_{r+1}} \left[\frac{1}{\lambda} + \sum_{k=1}^{r+1} p_0 a_k \frac{k}{\mu} + \sum_{n=1}^{r} p_n \sum_{k=1}^{r+1} a_k \left[u(r + 2 - n - k)\frac{n + k}{\mu} + \right.\right.$$
$$\left.\left. + u(k - (r + 1 - n))\frac{r + 1}{\mu} \right] \right]. \tag{23}$$

The proof is based on the application of the formula:

$$MZ = -z'(0).$$

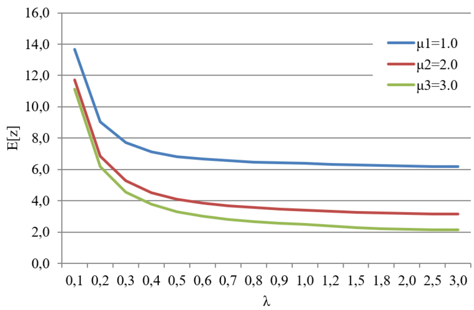

Figure 2. Dependence of the average peak age of information on the intensity of the incoming flow λ at fixed values of μ

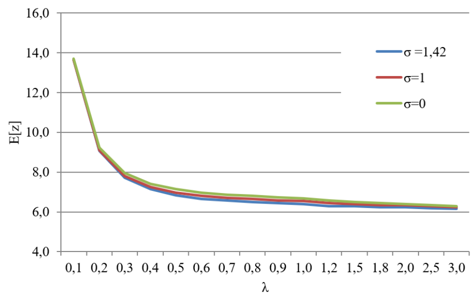

Figure 3. The dependence of the average peak age of information on the intensity of the incoming flow λ for different distributions of the number of applications in the group

6. The numerical results

We conducted a numerical study of the peak age of information for QS $M^{[x]}/M/1/r$ the results of which are shown in fig. 2 and fig. 3.

Figure 2 shows the dependence of the average peak increase on the intensity of the flow λ for three different values of the service intensity μ: $\mu = 1.0$, $\mu = 2.0$, $\mu = 3.0.0$ and storage capacity $r = 4$. At the same time, the distribution of the number of applications in the group was set by the formula:

$$\mathrm{P}\{\eta = k\} = \frac{1}{5}, \quad k = \overline{1,5}. \tag{24}$$

Note that the variant with $\mu = 1.0$ was calculated in two ways: analytically and imitatively. As can be seen from Figure 2, with the growth of λ, there is a decrease in the peak age. Moreover, the larger the μ, the lower the average age, which is quite expected, because with large λ and μ, the transfer of information from the SN and RN occurs more often and faster.

Figure 3 reflects the dependence of the average peak age of information on the intensity of the flow λ for $\mu = 1.0$ and $r = 4$ for different variations of the distribution of the number of applications in the group. So in option 1, the distribution of the number of applications in the group is given by the formula (24). In option 2:

$$\mathrm{P}\{\eta = k\} = \begin{cases} \frac{1}{2}, & k = 2,4; \\ 0, & k = 1,3,5. \end{cases}$$

In option 3, the number of applications in the group is constant and equal to 3. Obviously, for all three options, the average number of applications in the group is the same and equal to 3, but the standard deviation is different: $\sigma_1 = \sqrt{2}$; $\sigma_2 = 1$; $\sigma_3 = 0$. Despite the fairly close values of the average peak age for all three variants, there is a well-defined dependence: the greater the standard deviation, the lower the value of the average peak age. What is the reason for this pattern, it is not yet clear to us. Perhaps this is due to the loss of applications due to a limited storage device. In any case, this issue remains a topic for discussion.

7. Conclusion

As a result of the conducted research, we managed to obtain an expression for the Laplace–Stieltjes transformation of the stationary distribution function of the peak age of information transmitted from a peripheral source to the control center, modeling the transmission process using a queuing system with a group flow of applications and with phase-type distributions. This study allows us to obtain fairly accurate estimates of the age of information for real technical systems, due to the versatility of phase-type distributions.

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References

1. Sultan, A. *Ultra Reliable and Low Latency Communications. 3GPP* 2023.
2. Kamoun, F. & Kleinrock, L. Analysis of Shared Finite Storage in a Computer Network Node Environment Under General Traffic Conditions. *IEEE Transactions on Communications* **28,** 992–1003. doi:10.1109/TCOM.1980.1094756 (1980).
3. Baskett, F., Chandy, K. M., Muntz, R. R. & Palacios, F. G. Open, Closed, and Mixed Networks of Queues with Different Classes of Customers. *Journal of the ACM* **22,** 248–260. doi:10.1145/321879.321887 (1975).
4. Bedewy, A. M., Sun, Y. & Shroff, N. B. *Age-optimal information updates in multihop networks* in *2017 IEEE International Symposium on Information Theory (ISIT)* (2017), 576–580. doi:10.1109/ISIT.2017.8006593.
5. Bojan, T. M., Kumar, U. R. & Bojan, V. M. *An internet of things based intelligent transportation system* in *2014 IEEE International Conference on Vehicular Electronics and Safety* (2014), 174–179. doi:10.1109/ICVES.2014.7063743.
6. Hu, C. & Dong, Y. Age of information of two-way data exchanging systems with power-splitting. *Journal of Communications and Networks* **21,** 295–306. doi:10.1109/JCN.2019.000037 (2019).
7. Costa, M., Codreanu, M. & Ephremides, A. *Age of information with packet management* in *IEEE International Symposium on Information Theory (ISIT)* (2014), 1583–1587.
8. Chiariotti, F., Vikhrova, O., Soret, B. & Popovski, P. Peak Age of Information Distribution for Edge Computing With Wireless Links. *IEEE Transactions on Communications* **69,** 3176–3191. doi:10.1109/TCOMM.2021.3053038 (2021).
9. Kadota, I., Uysal-Biyikoglu, E., Singh, R. & Modiano, E. *Minimizing the Age of Information in broadcast wireless networks* in *2016 54th Annual Allerton Conference on Communication, Control, and Computing (Allerton)* (2016), 844–851. doi:10.1109/ALLERTON.2016.7852321.
10. Kosta, A., Pappas, N., Ephremides, A. & Angelakis, V. *Non-linear Age of Information in a Discrete Time Queue: Stationary Distribution and Average Performance Analysis* in *ICC 2020 — 2020 IEEE International Conference on Communications (ICC)* (2020), 1–6. doi:10.1109/ICC40277.2020.9148775.

11. Talak, R., Karaman, S. & Modiano, E. Improving Age of Information in Wireless Networks With Perfect Channel State Information. *IEEE/ACM Transactions on Networking* **28,** 1765–1778. doi:10.1109/TNET.2020.2996237 (2020).
12. Tripathi, V., Talak, R. & Modiano, E. *Age of information for discrete time queues* 2019.
13. Wijerathna Basnayaka, C. M., Jayakody, D. N. K., Ponnimbaduge Perera, T. D. & Vidal Ribeiro, M. *Age of Information in an URLLC-enabled Decode-and-Forward Wireless Communication System* in *2021 IEEE 93rd Vehicular Technology Conference (VTC2021-Spring)* (2021), 1–6. doi:10.1109/VTC2021-Spring51267.2021.9449007.
14. Zhbankova, E., Khakimov, A., Markova, E. & Gaidamaka, Y. The Age of Information in Wireless Cellular Systems: Gaps, Open Problems, and Research Challenges. *Sensors* **23.** doi:10.3390/s23198238 (2023).
15. Kaul, S., Yates, R. & Gruteser, M. *Real-time status: How often should one update?* in *2012 Proceedings IEEE INFOCOM* (2012), 2731–2735. doi:10.1109/INFCOM.2012.6195689.
16. Costa, M., Codreanu, M. & Ephremides, A. On the Age of Information in Status Update Systems With Packet Management. *IEEE Transactions on Information Theory* **62,** 1897–1910. doi:10.1109/TIT.2016.2533395 (2016).
17. Kaul, S. K., Yates, R. D. & Gruteser, M. *Status updates through queues* in *2012 46th Annual Conference on Information Sciences and Systems (CISS)* (2012), 1–6. doi:10.1109/CISS.2012.6310931.
18. Bocharov, P. P. & Pechinkin, A. V. *Queueing Theory [Teoriya massovogo obsluzhivaniya]* in Russian. 529 pp. (Izd-vo RUDN, Moscow, 1995).
19. Bocharov, P. P. & Yakutina, S. V. Stationary queue distribution in a finite capacity service system with group flow and phase-type service time [Stacionarnoe raspredelenie ocheredi v sisteme obslugivaniya konechnoy emkosty s gruppovim potokom i vremenem obslugivaniya fazovogo tipa]. *Avtomatika i Telemekhanika.* in Russian, 106–119 (1994).
20. Naumov, V. A. *O predelnih veroyatnostyah polumarkovskogo processa [On the limiting probabilities of a semi-Markov process]* in Russian. 35–39 (Universitet drugbi narodov, Moscow, 1975).

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Распределение пикового возраста информации в двухузловой группе передачи, моделируемой системой обслуживания с групповым потоком и обслуживанием фазового типа

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Аннотация. Данная статья продолжает цикл работ авторов, посвященных проблеме возраста информации (AoI) – метрики, используемой в информационных системах для мониторинга и управления удаленными источниками информации со стороны центра управления. Теоретический анализ систем передачи информации требует количественной оценки «свежести» информации, доставляемой в центр управления. Процесс передачи информации от периферийных источников к центру обычно моделируется с помощью систем массового обслуживания. В данной работе для оценки максимального значения возраста информации, называемого пиковым возрастом, используется система массового обслуживания с распределениями фазового типа. При этом учитывается специальное требование протокола передачи, состоящее в том, что информация в систему поступает группами случайного размера. Для данного случая получено выражение для преобразования Лапласа–Стилтьеса стационарной функции распределения пикового возраста информации и его среднего значения. По результатам аналитического моделирования проведено численное исследование зависимости среднего значения пикового возраста информации от загрузки системы. Корректность полученных выражений проверена путем сравнения аналитических результатов с результатами имитационного моделирования.

Ключевые слова: возраст информации, пиковый возраст информации, система массового обслуживания, распределение фазового типа, групповой поток