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Development of a Time Domain Identification Algorithm with a Spectral Objective Function

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The authors declare that there is no conflict of interest.

Abstract. A reliable method has been developed for identifying aerodynamic coefficients and systematic errors in the aircraft measuring system, using the advantages of frequency domain analysis. The parameter identification problem is formulated using maximum likelihood estimation method. The models of object and observation are formulated in time domain and the objective function is defined in frequency domain that is able to decouple the aircraft's response at different frequencies, effectively mitigating the impact of noise and potential non-linearities inherent in time-domain data. This transformation from time domain to frequency domain also facilitates the identification of delays in measurement system, which are often difficult to estimate accurately in the time domain. A modified Newton's method is employed to efficiently minimize the objective function in frequency domain, yielding optimal estimates for the lateral aerodynamic derivatives and delays. The effectiveness of this approach is validated through examples of identifying the parameters of a flight vehicle motion model, demonstrating its capability to accurately characterize lateral aircraft dynamics. This method provides a valuable tool for enhancing flight control system design and analysis by enabling more precise modeling of aircraft behavior.

Keywords: parameter identification, time-spectral algorithm, frequency domain, aerodynamic coefficients, on-board measurement errors

Authors' contribution:
Korsun O.N. — research concept, analysis of the data obtained; *Om M.H.* — collection and processing of materials, writing the text.

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Разработка алгоритма идентификации во временной области со спектральной целевой функцией

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Заявление о конфликте интересов

Авторы заявляют об отсутствии конфликта интересов.

Аннотация. Разработан надежный метод определения аэродинамических коэффициентов и систематических ошибок в измерительной системе самолета, в котором используются преимущества анализа в частотной области. Задача определения параметров формулируется в рамках метода максимума правдоподобия. Модели объекта и наблюдения задаются во временной области, а функционал определяется в частотной области, что позволяет разделить динамические характеристики самолета на разных частотах, эффективно уменьшая влияние шума и потенциальных нелинейностей, присущих данным во временной области. Этот переход из временной области в частотную также облегчает определение задержек в измерительной системе, которые часто сложно точно оценить во временной области. Для минимизации целевой функции в частотной области применяется модифицированный метод Ньютона, что позволяет получить оптимальные оценки боковых аэродинамических коэффициентов и запаздываний. Эффективность данного подхода подтверждается примерами идентификации параметров модели движения летательного аппарата, демонстрируя его способность точно охарактеризовать боковую динамику самолета. Этот метод может стать эффективным инструментом для оптимизации проектирования и анализа систем управления полетом. Он дает возможность с высокой точностью моделировать поведение летательного аппарата.

Ключевые слова: идентификация параметров, спектрально-временной алгоритм, частотная область, аэродинамические коэффициенты, погрешности бортовых измерений

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Introduction

Algorithms for the parameter identification of dynamic systems are traditionally divided into frequency- and time-domain methods, and each group of methods has its own advantages and disadvantages [1; 2]. Therefore, time-domain algorithms provide a simpler account of the nonlinearities and nonstationarities of an object, whereas frequency-domain algorithms allow for the selection of the most effective band of operating frequencies for a given task. Time and spectral domain parameter identification methods form an effective instrument for aircraft flight tests [3]. An example of spectral parameter identification was presented in [4]. In previous studies [5–8], the use of inputs in the frequency domain to augment the quality of identification estimates was discussed. Other fields where time- and frequency-domain identification play an important role are on-board measurement systematic error identification [9–11], aircraft thrust and drag force estimation [12; 13], satellite orbit parameter determination [14; 15], and analyses and improvement of piloting processes [16–19].

In recent years, there has been a trend towards the development of mixed time-frequency methods that aim to combine the advantages of both approaches.

As an example, one can cite the spectral-temporal identification method [20], in which the calculation of residuals between the experimentally measured and model-predicted signal values is performed in the time domain, whereas the minimized function is formulated in the frequency domain, specifically in the complex variable domain of the Laplace transform. The main limitation of this method is the requirement of linearity in the object and observation models. The proposed algorithm is free from this constraint and is based on a well-known method for identifying nonlinear nonstationary dynamic systems using the maximum likelihood approach [21]. Furthermore, a significant advantage of the proposed algorithm is its use of concepts familiar to engineering practice, such as frequency domains and spectral densities.

1. Time-Frequency Identification

The identification algorithm is obtained in which the models of the object and observations are formulated in the time domain, whereas the minimized functional is defined in the frequency domain. It is assumed that the new algorithm, while retaining the main properties of the original time-domain method, will acquire new beneficial qualities, primarily because of its ability to select bands of operating frequencies.

The nonlinear and nonstationary model of the object and observations are defined as follows:

$$y'(t) = f\left(y(t), a, t\right), \tag{1}$$

$$z(t_i) = h\left(y(t_i, a, t_i) + \upsilon(t_i)\right). \tag{2}$$

where, $y(t)$, $z(t_i)$ is the state vector and observation vector with dimensions n and r respectively; a is the vector of the parameters that shall be identified with dimension p; $\upsilon(t_i)$ is the vector random process of the white noise type of dimension r, having a normal distribution and a correlation matrix $R(t_i)$.

The control signal $u(t)$, is considered to be a known function of time and is accounted for by the time dependence on the right-hand side of (1). Observation equation (2) is defined for discrete time instants t_i, $i = 1, 2, \ldots, N$, which corresponds to digital registration and processing.

Traditionally, the minimized functional in the time domain is formulated as follows:

$$J(a) = (1/N)\sum_{i=1}^{N}\left(z(t_i) - \hat{z}(t_i, a)\right)^{\mathrm{T}} \times$$
$$\times R^{-1}(t_i)\left(z(t_i) - \hat{z}(t_i, a)\right), \tag{3}$$

where N is the number of observations; $\hat{z}(t_i, a)$ is the prediction of the observation vector, which is determined by the numerical integration of equations (1), (2) at $\upsilon(t_i) = 0$ for given initial conditions and at a fixed value of the parameter vector a.

Considering that the operation $(1/N)\sum_{i=1}^{N}(.)$ corresponds to the estimate of the mean and is denoted by $\varepsilon(t_i, a)$ the residual between the observation and prediction:

$$\varepsilon(t_i, a) = z(t_i) - \hat{z}(t_i, a) \quad (4)$$

the functional (3) may be written in the form:

$$J(a) = \hat{M}\left[\varepsilon^{\mathrm{T}}(t_i, a) R^{-1}(t_i) \varepsilon(t_i, a)\right], \quad (5)$$

where $\hat{M}[.]$ is the estimate of the mean for N number of observations.

It is necessary to introduce several notations immediately before proceeding to the formulation of the functional in the frequency domain.

For a scalar signal $x(t)$, presented by N number of measurements of $x(t_i), i = 1, 2, ..., N$ conducted at regular intervals $\Delta t = t_{i+1} - t_i$ with the registration frequency $f_{\text{reg}} = 1/\Delta t$, let us denote $F(x(t))$ the result of the discrete Fourier transform [10], calculated for discrete values of frequencies $f(x)$, $k = 1, 2, ..., N/2 + 1$ within the frequency band 0 … 0.5 f_{reg}.

Let us denote $F_k(x(t))$, the component of $F(x(t))$, corresponding to the frequency f_k. In the case of a vector signal

$$x(t) = \left[x_1(t) x_2(t), ..., x_r(t)\right]^{\mathrm{T}},$$

the discrete transformation is applied separately to each component, and $F_k(x(t))$ represents a vector of the same dimension.

$$F_k(x(t)) = \left[F_k(x_1(t))\, F_k(x_2(t)), ..., F_k(x_r(t))\right]^{\mathrm{T}}.$$

For the scalar signal $x(t)$ described above, at each discrete frequency value f_k the estimate of the spectral density of the power flux $\hat{S}_k(x(t))$ is denoted as [10]:

$$\hat{S}_k(x(t)) = \hat{M}\left[F_k^*(x(t)) F_k(x(t))\right]. \quad (6)$$

Methods for numerically determining the estimate of mean in for (6) for N number of measured values of $x(t_i)$ are presented, for instance, in [22].

It is now possible to proceed to the formulation of the functional and derivation of the algorithm.

The following assumption is made: the object model and observation model are defined by the equations (1) and (2). The functional in the frequency domain can be defined as follows:

$$J_f(a) = \hat{M}\left[\sum_{k=1}^{L} F_k^*(\varepsilon(t,a)) G_k F_k(\varepsilon(t,a))\right], \quad (7)$$

where $\varepsilon(t,a)$ is the vector of residual with dimension r between observation and prediction, calculated by formula (4); G_k is the diagonal matrix of real dimensional weight coefficients with dimension $r \times r$.

Substituting into (7)

$$G_k = \operatorname{diag}\left[g_{k1}\;\; g_{k2}\;, ..., g_{kr}\right],$$

equation (6, 8) can be obtained in the form:

$$J_f(a) = \sum_{k=1}^{L}\sum_{j=1}^{r} g_{kj} \hat{S}_k(\varepsilon_j(t,a)). \quad (8)$$

From (8), considering (6), it follows that functional (7) with the specified choice of the weight matrix G_k is real-valued, although it contains complex components $F_k(\varepsilon(t,a))$.

A recurrent algorithm for finding an estimate of the vector of parameters $\hat{a}$ minimizing the functional (7) is obtained as a modification of Newton's method [21]:

$$\hat{a}_{k+1} = \hat{a}_k - \left(D(\hat{a}_k)\right)^{-1} \frac{dJ_f(\hat{a}_k)}{d\hat{a}_k}, \quad (9)$$

where, $\dfrac{dJ_f(\hat{a}_k)}{d\hat{a}_k}$ is a vector with the dimension p of the first derivatives of the functional with respect to the vector of parameters; $D(\hat{a}_k)$ is a matrix with dimension $p \times p$, approximately equal to the matrix of the second derivatives of the functional with respect to the vector of parameters.

We will examine the matrix $D(\hat{a}_k)$ in more detail during the derivation of the algorithm. It is important to note that the distinction between the recurrent algorithm (9) and classical Newton method lies in the way it is computed. To implement (9), it is necessary to determine the first- and second-order derivatives of the functional (7) with respect to the vector parameters that should be identified. In the differentiation process, we utilize the linearity property of the expectation operators, differentiation, and Fourier transforms, along with the following formula:

$$\frac{d\left(x^*(a)Gx(a)\right)}{da} = 2\mathrm{Re}\left[\frac{dx^*(a)}{da}Gx(a)\right], \qquad (10)$$

where G is a valid diagonal matrix; $x(a)$ is a complex vector that is a function of a valid vector argument a.

The equation (10) is obtained by differentiating the left side, where the elements of the vector $x(a)$ and matrix G can be written explicitly. The derivative of the quadratic form is $x^*(a)Gx(a)$.

It is assumed that $x(a)$ is a complex vector of dimension n, which is a function of the valid vector argument a of dimension p;

G is a diagonal real-dimensional matrix of the weight coefficients of the dimension $n \times n$, $G = \mathrm{diag}(g_1 \;\; g_2, \; \ldots, \; g_n)$.

Let us demonstrate, that the derivative of quadratic form by vector a

$$\frac{d(x^*(a)\, G\, x(a))}{da} = 2\,\mathrm{Re}\left[\frac{dx^*(a)}{da}_{(p\times n)} G_{(n\times n)}\; x(a)_{(n\times 1)}\right].$$

Writing out the elements of the vectors explicitly,

$$x^*(a)\, G\, x(a) = g_1\, x_1^*(a)\, x_1(a) + \\ + g_2\, x_2^*(a)\, x_2(a) + \ldots + g_n\, x_n^*(a)\, x_n(a).$$

In order to proceed, it is necessary to determine the derivative of the summand of the form included in the resulting expression, $g_i\, x_i^*(a)\, x_i(a)$:

$$g_i \frac{d(x_i^*(a)\, x_i(a))}{da} = g_i\left(\frac{dx_i^*(a)}{da}_{(p\times 1)} x_i(a) + x_i^*(a)\,\frac{dx_i(a)}{da}_{(p\times 1)}\right) =$$

$$= g_i\left(\frac{dx_i^*(a)}{da}_{(p\times 1)} x_i(a)_{(1\times 1)} + \left(\frac{dx_i^*(a)}{da}\, x_i(a)\right)^*\right) =$$

$$= 2\, g_i\, \mathrm{Re}\left[\frac{dx_i^*(a)}{da}\, x_i(a)\right]_{(p\times 1)}.$$

Let us find the desired derivative using the following result:

$$\frac{d(x^*(a)\, G\, x(a))}{da} = \frac{d}{da}\left[\sum_{i=1}^{n} g_i\, x_i^*(a)\, x_i(a)\right] =$$

$$= 2\,\mathrm{Re}\left[g_1 \frac{dx_1^*(a)}{da} x_1(a) + g_2 \frac{dx_2^*(a)}{da} x_2(a) + \ldots + g_n \frac{dx_n^*(a)}{da} x_n(a)\right] =$$

$$= 2\,\mathrm{Re}\left[\begin{bmatrix}\frac{dx_1^*(a)}{da} & \frac{dx_2^*(a)}{da} & \cdots & \frac{dx_n^*(a)}{da}\end{bmatrix} \begin{bmatrix} g_1 & 0 & \ldots & 0 \\ 0 & g_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & g_n \end{bmatrix} \begin{bmatrix} x_1(a) \\ x_2(a) \\ \ldots \\ x_n(a) \end{bmatrix}\right] =$$

$$= 2\,\mathrm{Re}\left[\frac{dx^*(a)}{da}_{(p\times n)} G_{(n\times n)}\; x(a)_{(n\times 1)}\right].$$

Thus, the equation (10) is proven.

It is necessary now to find the first derivative of functional (7):

$$\frac{dJ_f(a)}{da} = \hat{M}\left[\sum_{k=1}^{L} 2\mathrm{Re}\left\{F_k^*\left(\frac{d\varepsilon(t,a)}{da}\right) G_k F_k(\varepsilon(t,a))\right\}\right] =$$

$$= -2\hat{M}\left[\sum_{k=1}^{L} \mathrm{Re}\left\{F_k^*\left(\frac{d\hat{z}(t,a)}{da}\right) G_k F_k(\varepsilon(t,a))\right\}\right]. \qquad (11)$$

Note that in the second equation (11), the expression (4) is used for the residual $\varepsilon(t,a)$.

In the special case, for $G_k = E$ the derivative (11) takes the form:

$$\frac{dJ_f(a)}{da} = -2\mathrm{Re}\left\{\sum_{k=1}^{L} \hat{S}_k\left(\frac{d\hat{z}(t,a)}{da}, \varepsilon(t,a)\right)\right\}, \qquad (12)$$

where, $\hat{S}_k\left(\frac{d\hat{z}(t,a)}{da}, \varepsilon(t,a)\right) = \hat{M}\left[F_k^*\left(\frac{d\hat{z}(t,a)}{da}\right) F_k(\varepsilon(t,a)\right],$

there is an estimate of the cross-spectral density of signals $d\hat{z}(t,a)/da$ and $\varepsilon(t,a)$ is calculated from N values of these signals recorded at discrete time points t_i, $i=1,2,...,N$.

More generally, after substituting $G_k = \text{diag}\,[g_{k1}\ \ g_{k2},\ldots,g_{kr}]$ into (11), equation (13) will be obtained:

$$\frac{dJ_f(a)}{da} = -2\text{Re}\left\{\sum_{k=1}^{L}\begin{bmatrix}\sum_{i=1}^{r}\hat{S}_k(i1,i)g_{ki}\\ \sum_{i=1}^{r}\hat{S}_k(i2,i)g_{ki}\\ \ldots\\ \sum_{i=1}^{r}\hat{S}_k(ip,i)g_{ki}\end{bmatrix}\right\}, \quad (13)$$

where, $\hat{S}_k(ij,i)$ is the estimate of the cross-spectral density of the signal corresponding to the i-th row of the j-th column of the matrix $d\hat{z}(t,a)/da$ (i.e., the signal of the derivative of the i-th element of the vector of observation prediction $\hat{z}(t,a)$ with respect to the j-th element of the vector of parameters a) and the i-th element of the vector of the residual $\varepsilon(t,a)$.

As can be observed, and in this case, finding the first derivative is reduced to calculating the cross-spectral densities.

Then, it is necessary to approximate the second by derivation differentiating (11) using the vector of parameters a:

$$\frac{d^2J_f(a)}{da^2} = \frac{d}{da}\left[\frac{dJ_f(a)}{da}\right] =$$

$$= -2\text{Re}\left\{\hat{M}\left[\sum_{k=1}^{L}F_k^*\left(\frac{d^2\hat{z}(t,a)}{da^2}\right)\right]G_kF_k(\varepsilon(t,a))\right\} +$$

$$+2\text{Re}\left\{\hat{M}\left[\sum_{k=1}^{L}F_k^*\left(\frac{d\hat{z}(t,a)}{da}\right)G_kF_k\left(\frac{d\hat{z}(t,a)}{da}\right)\right]\right\} \approx$$

$$\approx 2\text{Re}\left\{\hat{M}\left[\sum_{k=1}^{L}F_k^*\left(\frac{d\hat{z}(t,a)}{da}G_kF_k\left(\frac{d\hat{z}(t,a)}{da}\right)\right)\right]\right\}. \quad (14)$$

In equation (14), the term containing the prediction of the second derivatives with respect to the parameter vector has been omitted. This reduces the computational burden and eliminates errors associated with calculating the second derivatives. The main idea is that near the extremum, the omitted term is small because it is proportional to the residual $\varepsilon(t,a)$, and far from the extremum, its influence on the convergence of the algorithm is not decisive.

When $G_k = E$, from equation (14), the following equation (15) will be obtained:

$$\frac{d^2J_f(a)}{da^2} \approx 2\text{Re}\left\{\sum_{k=1}^{L}\hat{S}_k\left(\frac{d\hat{z}^{\text{T}}(t,a)}{da},\frac{d\hat{z}(t,a)}{da}\right)\right\}, \quad (15)$$

where, under the sign of the sum, there is an estimate of the cross-spectral density of the signals $d\hat{z}(t,a)/da$.

By substituting $G_k = diag\ [g_{k1}, g_{k2}, \ldots, g_{kr}]$ into (14), equation (16) will be obtained as follows:

$$\frac{d^2J_f(a)}{da^2} \approx 2\text{Re}\left\{\sum_{k=1}^{L}\begin{bmatrix}\sum_{i=1}^{r}\hat{S}_k(i1,i1)g_{ki} & \sum_{i=1}^{r}\hat{S}_k(i1,i2)g_{ki} & \ldots & \sum_{i=1}^{r}\hat{S}_k(i1,ip)g_{ki}\\ \sum_{i=1}^{r}\hat{S}_k(i2,i1)g_{ki} & \sum_{i=1}^{r}\hat{S}_k(i2,i2)g_{ki} & \ldots & \sum_{i=1}^{r}\hat{S}_k(i2,ip)g_{ki}\\ \ldots & \ldots & \ldots & \ldots\\ \sum_{i=1}^{r}\hat{S}_k(ip,i1)g_{ki} & \sum_{i=1}^{r}\hat{S}_k(ip,i2)g_{ki} & \ldots & \sum_{i=1}^{r}\hat{S}_k(ip,ip)g_{ki}\end{bmatrix}\right\}. \quad (16)$$

The term $\hat{S}_k(ij,in)$ represents an estimate of the cross-spectral density between the derivative of the i-th element of the prediction vector with respect to the j-th element of the parameter vector, and the derivative of the i-th element of the prediction vector with respect to the n-th element of the parameter vector.

Thus, the final algorithm is as follows: the model of the object and observations is defined by equations (1) and (2), and the parameter estimates are computed using the recurrent formula (9), in which the first derivative of the functional is determined based on the form of the matrix G_k using formulas (12) or (13), whereas the approximate matrix of the second derivatives $D(\hat{a}_k)$ is given by formulas (15) or (16). Prior parameter estimates must be provided to initiate the algorithm. The computations based on formula (9) are concluded when the magnitude of the vector $\hat{a}_{k+1} - \hat{a}_k$ becomes less than a certain small value, for example, 1 to 2% of the magnitude of the vector $\hat{a}_k$. The derivative of the forecast with respect to the parameters $d\hat{z}(t,a)/da$ is determined numerically by sequentially assigning small increments to each of the elements of the vector a.

The numerical estimation of the spectral and cross-spectral densities of the power flow is performed using one of the known methods. A number of practically effective algorithms are provided, for example, in [22]. In particular, this work used the Fast Fourier Transform, the Goodman-Otnes-Enokson spectral window, and frequency averaging, although other options are also possible.

The choice of operating frequency bands is based on the conditions of a specific problem by setting the coefficients of the matrix G_k or by excluding individual frequency components from the summation over $k = 1, 2, ..., L$. The latter method must be used when using simpler formulae correspond to $G_k = E$.

2. Identification of Lateral Aerodynamic Coefficients of an Aircraft Using Time-Frequency Identification Algorithm

This study considers the functionality of the proposed algorithm by means of examples of identifying the parameters of an aircraft motion model.

Example 1. Let us consider the lateral motion equations of an aircraft, which we will extract from the complete system of spatial motion equations:

$$\frac{d\beta}{dt} = a_z\cos\beta - (a_x\sin\beta - \omega_y)\cos\alpha + (a_y\sin\beta + \omega_x)\sin\alpha;$$

$$\frac{d\omega_y}{dt} = \frac{J_z - J_x}{J_y}\omega_x\omega_z + q\frac{Sl}{J_y}m_y + (K_{\text{дв}}\omega_z + (P_{\text{прав}} - P_{\text{лев}})z_{\text{дв}})/J_y;$$

$$\frac{d\omega_x}{dt} = \frac{J_y - J_z}{J_x}\omega_y\omega_z + q\frac{Sl}{J_x}m_x;$$

$$\frac{d\gamma}{dt} = \omega_x - tg\,\upsilon\,(\omega_y\cos y - \omega_z\sin y), \quad (17)$$

where

$$a_x = qS(-c_x + c_p)/m - g\sin\upsilon;$$

$$a_y = qSc_y/m - g\cos\upsilon\cos\gamma;$$

$$a_z = qSc_z/m - g\cos\upsilon\sin\gamma.$$

The aerodynamic force and moment coefficients are generally defined using the following mathematical expressions:

$$c_z = c_z^{\beta}\beta + c_z^{\delta н}\delta_н;$$

$$m_x = m_x^{\beta}\beta + m_x^{\omega x}\omega_x + m_x^{\omega y}\omega_y + m_x^{\delta э}\delta_э + m_x^{\delta н}\delta_н;$$

$$m_y = m_y^{\beta}\beta + m_y^{\omega x}\omega_x + m_y^{\omega y}\omega_y + m_y^{\delta э}\delta_э + m_y^{\delta н}\delta_н. \quad (18)$$

The normal, lateral, and longitudinal overloads were calculated using the following formulae:

$$n_y = a_y/g + \cos\upsilon\cos\gamma;$$

$$n_z = a_z/g + \cos\upsilon\sin\gamma;$$

$$n_x = a_x/g + \sin\upsilon. \quad (19)$$

This notation corresponds mainly to the generally accepted notation in flight dynamics. When performing lateral motion identification, the longitudinal motion parameters (angles of attack α, pitch angle J and pitch rate ω_z) are replaced with the measured values, and the parameter a_y is calculated from (19) using the measured values of the normal overload n_y. The changes in flight speed V and parameter a_x were considered similarly.

In order to obtain the initial data for identification, a straight and level flight was simulated. Relative to this straight and level flight, a series of aileron inputs to the left and right, lasting 1–2.5 s was performed. The duration of the identification section was 32 s, with a registration frequency of 8 Hz. For identification, the values of the angular velocities ω_x, ω_y and lateral load factor n_z, as well as the measurements of the input signal - aileron deflections $\delta_э$, were used. The rudder was not deflected during the identification section, i.e. $\delta_н = 0$ was assumed. The amplitude of the aileron deflections was chosen so that the maximum deviations of the signals were ± 8 deg/s for ω_x ± 1.7 deg/s for ω_y ± 0.08 units of overload for n_z. The measurement noises were modeled as discrete Gaussian random sequences of the white noise type with standard deviations of 0.08 deg/s for ω_x, 0.02 deg/s for ω_y, 0.02 units of overload for n_z, 0.02 deg/s for $\delta_э$.

The entire volume of the simulated data was used for identification, that is, a section with a duration of 32 s. The coefficients of the aerodynamic forces and moments (18) in various combinations were selected as the parameters to be identified.

At the initial stage, the same input data were processed using the proposed algorithm and the traditional Maximum Likelihood Estimation (MLE) algorithm, with identical object and observation models and for the same set of parameters to be identified. In the frequency functional, summation was performed over the entire frequency band. In this case, the estimates obtained by both algorithms were nearly identical, with estimation errors not exceeding 2–3%. The results indicate that when models (1) and (2) correspond to the object and measurement system with respect to the parameters to be identified, respectively, and the disturbance represents the aforementioned broadband noise that is uncorrelated with the useful signal, the use of the frequency functional does not provide advantages over the time method, and both algorithms are equivalent. Therefore, further research should focus on cases of disturbances that are correlated with the object's signals.

At the initial stage, it was also established that components with frequencies above 0.8 Hz do not have a significant impact on the parameter estimates, as they fall outside the object's pass band and correspond to noise. Therefore, in subsequent implementations of the algorithm with the frequency functional, these components were excluded from the summation process.

In the main stage, the identification of the roll channel parameters $m_x^{\delta э}$, $m_x^{\omega x}$ was considered as the first example, with the signal of angular velocity ω_x containing additive sinusoidal noise. The results are presented in Table 1.

Table 1

Relative errors of lateral aerodynamic moment coefficients identification

Frequency band for the calculation of functional, Hz	Relative errors of the estimates of parameters, %		Noise parameters		Standard criterion Cr, %
	$m_x^{\delta э}$	$m_x^{\omega x}$	Amplitude, Degree	Frequency, Hz	
0...0.8	9.6	7.1	3.0	0,5	16.3
0...0.33	3.2	1.7	3.0	0.5	0.42
0...0.24	0.1	0.12	3.0	0.5	0.001
0...0.8	6.1	14.0	3.0	0.2	17.7
0.24...0.8	1.4	9.7	3.0	0.2	3.9
0.33...0.8	0.25	4.9	3.0	0.2	0.67
0...0.8	8.7	15.1	4.0	0.1	24.2
0.33...0.8	1.4	3.8	4.0	0.1	0.61
0.42...0.8	1.39	0.2	4.0	0.1	0.32

Source: made by O.N. Korsun and M.H. Om

The relative errors in the parameter estimation are presented in Columns 2 and 3 of the Table 1. The table indicates that the presence of noise leads to an increase in the estimation errors of the parameters when calculating the functional across the entire frequency band of the object (0–0.8 Hz). The errors decrease significantly when calculating the functional within a band that does not contain noise.

To select frequency bands effectively, a criterion that allows for the comparison of different options is required. In this study, the following normalized criterion was used, with the values presented in the last column of the Table:

$$Cr = \frac{\sum_{k=1}^{L}(\hat{S}_k(\varepsilon_1(t,a)) + \hat{S}_k(\varepsilon_2(t,a)) + ... + \hat{S}_k(\varepsilon_r(t,a)))\cdot 100\% \cdot}{\sum_{k=1}^{L}(\hat{S}_k(\hat{z}_1(t,a)) + \hat{S}_k(\hat{z}_2(t,a)) + ... + \hat{S}_k(\hat{z}_r(t,a)))} \quad (20)$$

In formula (20), the numerator contains estimates of the spectral densities of the residuals for each component r of the measurement vector, whereas the denominator includes estimates of the spectral densities of the components of the prediction vector. The summation in (20) is performed over the frequency components belonging to the frequency band used in the functional calculation. Criterion (20) is proportional to the ratio of the powers of the residual and the output signal within the frequency band utilized for identification.

Table 1 shows that, as the accuracy of the estimates increases, the value of the criterion decreases. In the identification of the roll channel parameters, it was assumed that the other parameters of lateral motion (18) were known exactly. Now, let us assume that they are known with errors, and assess the possibility of isolated identification of the roll channel parameters.

We introduced 30% errors in the parameters $m_x^{\beta}, m_x^{\omega_y}, m_y^{\omega_y}, m_y^{\omega_x}, c_z^{\beta}$ and identified the roll channel.

The results presented in Table 2 show that the transition to the relatively high-frequency band significantly reduced the estimation errors.

3. Identification of the Errors of the Measurement System from the Flight Experiment Data

Example 2. For the conditions in Example 1, consider another task. Suppose it is necessary to identify the errors of the measurement system from the flight experiment data, namely, the delays $\Delta T\omega_x, \Delta T\omega_y$ relative to the start time of the information frame for the angular velocity signals ω_x and ω_y. During the generation of the initial data, these delays were assumed to be equal to 1/2 of the sampling interval, that is, $\Delta T\omega_x = \Delta T\omega_y = 1/16$ s. The errors in specifying the aircraft model parameters were considered as noise. The results are presented in Table 3.

Table 2

Relative errors of lateral aerodynamic moment coefficients identification

Frequency band for the calculation of functional, Hz	Relative errors of the estimates of parameters, %		Type of error	Standard criterion Cr, %
	$m_x^{\delta э}$	$m_x^{\omega x}$		
0...0.8 0.33...0.8 0.42...0.8	7.5 3.4 3.3	16.7 4.0 2.7	Errors 30% for parameters $m_x^{\beta}, m_x^{\omega_y}, m_y^{\omega_y}, m_y^{\omega_x}, c_z^{\beta}$	4.65 0.24 0.025

S o u r c e: made by O.N. Korsun and M.H. Om

Table 3

Relative errors of identification estimates

Frequency band for the calculation of functional, Hz	Relative errors of the estimates of parameters, %		Type of noise	Standard criterion Cr, %
	$\Delta T\omega_x$	$\Delta T\omega_y$		
0...0.8	0.5	1.5	Without noise	0.012
0...0.8 0.33...0.8	2.5 4.2	50.2 0.5	Errors 30% for m_x^{β}	1.05 0.036
0...0.8 0.33...0.8 0.51...0.8	9.2 8.6 4.8	106.1 23 9.7	Errors 30% for $m_x^{\beta}, m_x^{\omega_y}, m_y^{\omega_y}$	0.88 0.054 0.035

S o u r c e: made by O.N. Korsun and M.H. Om

As we can see, in this case, the transition to the relatively high-frequency band also significantly reduces the influence of noise, and the comparison of options is facilitated by criterion (20). The advantages of the algorithm include the ability to select the operational frequency band, which substantially diminishes the impact of noise related to the useful signal, typically caused by inaccuracies or incompleteness of the model, neglect of coupled system models, and other problems mentioned in [23].

Conclusion

This study proposes an algorithm for the identification of dynamic systems, in which the nonlinear models of the object and observations are defined in the time domain, while the minimized functional is defined in the frequency domain. The algorithm offers a significant advantage by allowing for the selection of the operational frequency band. This capability greatly minimizes the interference from noise that can affect useful signals, which often arises from various factors such as model inaccuracies, incomplete data, and the neglect of coupled system interactions. By effectively targeting specific frequency ranges, the algorithm enhances signal clarity and reliability, making it a valuable tool in complex modeling scenarios. In contrast, traditional algorithms often lead to significant biases in parameter estimates due to such types of errors and noises. The effectiveness of the algorithm has been validated through examples of identifying the parameters of a flight vehicle motion model.

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