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Modified Algorithm for Calculating the Parameters of Maneuvers of Coplanar Meeting of Spacecraft in a Near-Circular Orbit Using Low-Thrust Engines

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Conflicts of interest

The authors declare that there is no conflict of interest.

Abstract. A modified algorithm is presented for solving the problem of spacecraft rendezvous in a near-circular orbit. The study considers the calculation of maneuver parameters executed on several turns using a low-thrust propulsion system. It is assumed that the active spacecraft performs maneuvers within a predefined region around the target spacecraft, while the perturbative effects of Earth's gravitational field non-centrality and atmospheric drag are neglected. Well-established approximate mathematical models of spacecraft motion are employed to address the rendezvous problem. The methodology of determining the parameters of maneuvers is structured into three key stages: in the first and third stages, the parameters of impulsive transfer and low-thrust transfer are determined using analytical methods. In the second stage, maneuvers are allocated across the available turns to ensure a successful rendezvous by minimizing a selected control variable. The proposed approach is distinguished by its computational efficiency and robustness, making it suitable for onboard implementation in autonomous spacecraft navigation systems. As a case study, the paper analyzes the dependence of total characteristic velocity required for rendezvous on the magnitude of engine thrust and provides a comparative assessment of the total characteristic velocity for both impulsive and low-thrust maneuvering scenarios.

Keywords: spacecraft rendezvous, near-circular orbit, velocity impulse, maneuver parameters, approximate mathematical models, low-thrust propulsion

Authors' contribution

Baranov A.A. — research concept, validation, project supervision; *Olivio A.P.* — theory development, performing calculations, writing the text.

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Модифицированный алгоритм расчета параметров маневра копланарной встречи космических аппаратов на околокруговой орбите с использованием двигателей малой тяги

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Аннотация. Представлен модифицированный алгоритм решения задачи сближения космических аппаратов на околокруговой орбите. Рассмотрен расчет параметров маневра, выполняемого на нескольких витках с использованием двигательной установки малой тяги. Предполагается, что активный космический аппарат выполняет маневры в пределах заданной области вокруг целевого космического аппарата, при этом возмущающими эффектами нецентральности гравитационного поля Земли и атмосферного сопротивления пренебрегают. Для решения задачи сближения использованы хорошо зарекомендовавшие себя приближенные математические модели движения космического аппарата. Методология определения параметров маневров структурирована на три ключевых этапа: на первом и третьем этапах параметры импульсной передачи и передачи малой тяги определяются с использованием аналитических методов. На втором этапе маневры распределяются между доступными поворотами, чтобы обеспечить успешное сближение за счет минимизации выбранной управляющей переменной. Предлагаемый подход отличается своей вычислительной эффективностью и надежностью, что делает его пригодным для бортовой реализации в автономных навигационных системах космических аппаратов. В качестве примера в статье анализируется зависимость суммарной характеристической скорости, необходимой для сближения, от величины тяги двигателя и приводится сравнительная оценка суммарной характеристической скорости как для сценариев импульсного маневрирования, так и для маневрирования с малой тягой.

Ключевые слова: сближение космических аппаратов, околокруговая орбита, приближенные математические модели, двигательная установка малой тяги

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Все данные, модели и код, сгенерированные или использованные в ходе исследования, приведены в представленной статье.

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Introduction

The rendezvous of spacecraft (SC) in near-circular orbits is a highly intricate and technically demanding problem in astronautics. Its complexity arises from the interplay of nonlinear orbital dynamics, gravitational perturbations, and control constraints, all of which must be carefully managed to achieve mission success. The precise execution of spacecraft rendezvous is fundamental to a wide range of space operations, including satellite servicing, space station resupply, and autonomous docking maneuvers.

The choice of methodology for spacecraft rendezvous is strongly influenced by mission-specific objectives, which can vary significantly depending on operational requirements. These objectives dictate the selection of optimal control strategies, trajectory planning techniques, and guidance algorithms, all of which must balance fuel efficiency, time constraints, and navigational accuracy.

As advancements in space technology continue to push the boundaries of autonomous operations, the development of robust and efficient rendezvous strategies remains a critical area of research in astronautics. For instance, the trajectory optimization strategies used for SC rendezvous in near-circular orbits differ fundamentally from those applied in atmospheric observation missions. These disparities arise from variations in spacecraft modeling and the corresponding control system architecture.

In the realm of commercial and operational spaceflight, SC rendezvous problems exhibit notable similarities. While fundamental rendezvous algorithms have been developed and successfully implemented, continuous refinement is necessary to enhance their precision and efficiency. Typically, rendezvous operations involve two spacecraft: an active vehicle executing maneuvering procedures and a passive target following a free-flight trajectory. This paper aims to propose a modified algorithm for optimizing spacecraft rendezvous in near-circular orbits, ensuring maximum efficiency and accuracy while adhering to operational constraints.

The problem of spacecraft rendezvous in near-circular orbits using low-thrust propulsion is of critical importance in contemporary spaceflight. It plays a fundamental role in various applications, including coordinated spacecraft formations, satellite constellation deployment, active debris removal, and on-orbit servicing missions. Since the mid-20th century, electric propulsion systems have been extensively employed due to their high specific impulse, which significantly reduces propellant consumption rate for orbital maneuvers. However, the inherently low thrust of these systems results in prolonged maneuver durations, which must be meticulously accounted for in mission planning and control strategies.

Optimal low-thrust maneuvering has been extensively investigated in previous studies [1–12]. Of particular relevance are the contributions in [3–7], which address trajectory optimization under stringent constraints. Due to the mathematical complexity of low-thrust trajectory planning, numerical approaches based on Pontryagin's Maximum Principle and the continuation method have been traditionally employed. More recently, interior point methods [8] have gained prominence, demonstrating efficacy in solving largescale maneuvering problems.

Over the past two decades, spacecraft rendezvous has remained an active field of research [3–5; 9; 10]. Initial studies predominantly focused on high-thrust rendezvous strategies in near-circular orbits [11; 12], successfully addressing short-duration rendezvous (within three orbital revolutions) and classical mid-term rendezvous scenarios in coplanar circular orbits. Given the evolving landscape of space operations, continued advancements in rendezvous algorithms are essential to support emerging mission architectures and operational requirements.

Currently, the problem of multi-impulse spacecraft maneuvers remains one of the central challenges in astrodynamics, requiring the development of increasingly efficient and reliable computational methods. Due to the inherent complexity of this problem, contemporary approaches typically adopt a multi-stage resolution framework that combines analytical and numerical techniques.

The primary difficulty in formulating and solving these problems arises from the need to model space trajectories under multiple dynamic and operational constraints. To address these challenges, various algorithms have been proposed to decompose the solution into structured steps, as demonstrated in studies [13–21].

Analytical methods, as presented in [13–17], are widely used to solve orbital maneuvering and orbital plane rotation problems independently. Although this approach may lead to an increase in the total characteristic velocity required for maneuvers, it is advantageous due to its simplicity and operational reliability.

Additionally, numerical methods have been employed to determine optimal solutions in highly complex multi-impulse scenarios, taking into account specific constraints, as detailed in [18; 19]. The authors of [20; 21] developed efficient algo-rithms for maneuver parameter calculations, which are widely used due to their accuracy and applicability.

An alternative based on solving Lambert's problem was presented in [21]. In this approach, the parameters corresponding to a two-impulse trajectory are initially determined, followed by an analysis of the behavior of the hodograph of the base vector associated with the solution. If necessary, additional velocity impulses are introduced to refine the trajectory and ensure an optimized solution.

Finally, the studies [4; 22] propose hybrid numerical-analytical methods for solving multi-impulse rendezvous problems, aiming to effectively address contemporary practical challenges. These approaches integrate the principles established in previous studies [13–21], providing a more comprehensive solution adapted to the demands of modern astrodynamics.

This paper presents a modified algorithm with enhanced capabilities for addressing the rendezvous problem of two SC in near-circular orbits under low-thrust propulsion. The proposed modification of this algorithm aims to overcome the limitations of existing methods, which often fail to obtain viable solutions in low-thrust regimes, as demonstrated in study [9]. With the implemented improvements, it becomes possible to successfully execute maneuvers even under minimal thrust conditions, thereby expanding the algorithm's applicability.

In this study, the SC rendezvous problem is analyzed both from the perspective of impulsive maneuvers and considering the continuous operation of low-thrust propulsion systems. In contrast to previous studies [3–6; 9], which employed different strategies for solving the rendezvous problem in coplanar orbits, the proposed algorithm offers a more comprehensive approach, being applicable both in ground-based control centers and onboard satellites, thus enabling greater operational autonomy.

Various specialized mathematical models are used to describe the relative motion of spacecraft in near-circular orbits. One of the most widely employed is the Hill-Clohessy-Wiltshire (HCW) model [23; 24], which assumes that the separation between SC is small compared to the orbital radius. However, in this study, we adopt an alternative linearized formulation derived in [25], which provides greater accuracy and applicability for maneuver planning in low-thrust regimes.

With the increasing number of SC and the growing demand for real-time problem-solving, there is a significant shift toward onboard computation of maneuver parameters. This necessitates the development of computationally efficient and highly reliable algorithms. The proposed method meets these requirements, ensuring computational robustness and enhancing the feasibility of autonomous execution of orbital maneuvers.

1. Mathematical Formulation of the Rendezvous Problem

The maneuver planning for a spacecraft transferring between two closely spaced near-circular orbits is analyzed within the framework of unperturbed Keplerian motion. The problem is approached using an approximate impulsive model, where the trajectory is discretized into N velocity impulses applied over a predefined time horizon. By employing a linearized approximation, the conditions governing the transition from an initial orbit to a target coplanar orbit can be expressed as follows [26; 27]:

$$\Delta V_{r1}\sin\alpha_1 + 2\Delta V_{t1}\cos\alpha_1 + \ldots + \\ + \Delta V_{rN}\sin\alpha_N + 2\Delta V_{tN}\cos\alpha_N = \Delta e_x; \quad (1)$$

$$-\Delta V_{ri}\cos\alpha_1 + 2\Delta V_{ti}\sin\alpha_1 + \ldots - \\ - \Delta V_{rN}\cos\alpha_N + 2\Delta V_{tN}\sin\alpha_N = \Delta e_y; \quad (2)$$

$$\Delta V_{t1} + \Delta V_{t2} + \Delta V_{t3} + \ldots + \\ + \ldots + \Delta V_{tN} = \Delta a / 2; \quad (3)$$

$$2\Delta V_{r1}\left(1 - \cos\alpha_1\right) + \Delta V_{t1}\left(-3\alpha_1 + 4\sin\alpha_1\right) + \ldots + \\ + 2\Delta V_{rN}\left(1 - \cos\alpha_N\right) + \\ + \Delta V_{tN}\left(-3\alpha_N + 4\sin\alpha_N\right) = \Delta t, \quad (4)$$

where $\Delta a = \left(a_f - a_0\right)/r_0$, $\Delta t = n_0\left(t_f - t_0\right)$, $\Delta V_{ti} = \Delta V_{ti}^* / V_0$, $\Delta V_{ri} = \Delta V_{ri}^* / V_0$.

Here, a_f and a_0 represent semi-major axes of the orbits. The initial and final time is given by t_f and t_0. The reference circular orbit, characterized by a radius r_0 such that $r_0 = a_f$, imposes the constraints V_0 and n_0, which respectively represent the orbital velocity and angular velocity of the spacecraft's motion. The maneuvering strategy consists of N discrete velocity impulses, each applied at an angle φ_i, -measured from the line connecting the spacecraft to the target point in the direction of motion. The *i*-th velocity correction is decomposed into its transverse ΔV_{ti}^* and radial ΔV_{ri}^* components, each playing a critical role in shaping the transfer trajectory.

The maneuver optimization problem is defined as minimizing the total characteristic velocity ΔV associated with the executed maneuvers:

$$\Delta V = \min\sum_{i=1}^{N}\Delta V_i = \min\sum_{i=1}^{N}\sqrt{\Delta V_{ri}^2 + \Delta V_{ti}^2}$$

under restrictions (1)–(6).

2. Algorithm for Solving the Rendezvous Problem

The rendezvous problem is solved based on the resolution of the orbital transfer problem. To achieve this, an algorithm presented in papers [9; 27] is employed, where the authors assume that the correction of the eccentricity vector and the impulse application angles can be performed by applying velocity impulses at optimal points along the trajectory. The determination of these points is formalized by the following expressions:

$$\mathrm{tg}\alpha_e = \frac{\Delta e_y}{\Delta e_x}, \quad \alpha_1 = \alpha_e, \quad \alpha_2 = \alpha_1 + \pi,$$

where α_e is the angle defining the optimal direction for correcting deviations in the eccentricity vector.

The optimization conditions result in three distinct categories of solutions, as presented in [4; 27]. The optimal impulse magnitudes can be obtained from the analysis of the first three equations of system (1)–(4), following the approach described below:

$$\Delta V_{1t} = \left(\Delta a + \Delta e\right)/4; \quad (5)$$

$$\Delta V_{2t} = \left(\Delta a - \Delta e\right)/4. \quad (6)$$

Once the optimal impulse magnitudes are determined, they will be used as initial approximations to solve the rendezvous problem. Subsequently, the velocity impulses ΔV_{t1} and ΔV_{t2} are distributed over the N available orbital revolutions designated for maneuver execution [9; 27]:

$$\Delta V_{1t} = \Delta V_{1t1} + \Delta V_{1t2} + \ldots + \Delta V_{1tN}; \quad (7)$$

$$\Delta V_{2t} = \Delta V_{2t1} + \Delta V_{2t2} + \ldots + \Delta V_{2tN}. \quad (8)$$

The next goal is to determine the distribution of velocity impulses over the turns in a manner that satisfies equation (4). We will adopt a significant simplification, assuming that the variation of velocity impulses over the turns occurs linearly to make the analysis more tractable, that is, allowing the approximation to reduce the complexity of the meeting problem:

$$\Delta V_{1ti} = \Delta V_{1t1} + \left(i-1\right)\left(\Delta V_{1tN} - \Delta V_{1t1}\right)/\left(N-1\right); \quad (9)$$

$$\Delta V_{2ti} = \Delta V_{2t1} + \left(i-1\right)\left(\Delta V_{2tN} - \Delta V_{2t1}\right)/\left(N-1\right). \quad (10)$$

Therefore, substituting the values of the velocity impulses determined using expressions (9) and (10) into equations (7) and (8), we will obtain the following equations:

$$\Delta V_{1tN} = \frac{2\Delta V_{1t}}{N} - \Delta V_{1t1}; \quad (11)$$

$$\Delta V_{2tN} = \frac{2\Delta V_{2t}}{N} - \Delta V_{2t1}. \quad (12)$$

Consequently, substituting the obtained values $\Delta V_{1tN}, \Delta V_{2tN}$ into equations (9) and (10), we obtain:

$$\Delta V_{1ti} = \left[2(i-1)\Delta V_1 + N(N+1-2i)\Delta V_{1t1}\right] / N(N-1); \quad (13)$$

$$\Delta V_{2ti} = \left[2(i-1)\Delta V_2 + N(N+1-2i)\Delta V_{2t1}\right] / N(N-1). \quad (14)$$

Thus, we found the values of all velocity impulses expressed only through ΔV_{1t1} and ΔV_{2t1}. Substituting them into equation (3), we obtain a linear equation with two unknowns ΔV_{1t1}, ΔV_{2t1}. The coefficients of the velocity impulses are known, since their angles of application are known:

$$\alpha_{1i} = \alpha_e + 2\pi(N_i - N); \quad (15)$$

$$\alpha_{2i} = \alpha_e + \pi + 2\pi(N_i - N). \quad (16)$$

By iterating the value of the variable ΔV_{1t1}, within the specified interval, we determine the corresponding value of the variable ΔV_{2t1} for each case based on equation (3).

Next, the values of all velocity impulses are calculated based on equations (13) and (14). The sum of the magnitudes of these impulses defines the total characteristic velocity for each obtained solution. The solution corresponding to the lowest total characteristic velocity is considered the optimal rendezvous trajectory. If the total characteristic velocity of the selected solution matches that of the transfer problem, it can be inferred that the trajectory with the minimum achievable characteristic velocity has been determined.

In the next step, the duration of each identified maneuver is estimated using the following formula:

$$\Delta\varphi_i = \frac{\omega_c}{\omega}\Delta V_i, \quad (17)$$

where ω_c is the represents the centripetal acceleration of the reference circular orbit, ω is the acceleration generated by the propulsion system, m denotes the mass of the active SC, T is the thrust of its engine.

If $\Delta\varphi_i \leq 20\circ$ (the duration of the largest velocity impulse), the solution can be considered approximately equivalent to an impulsive-based approach, and the problem is considered resolved. However, when the maneuver duration becomes significant, the solution shifts to one that involves low thrust.

3. Solving the Problem With "Low Thrust"

The velocity impulses applied for each turn result, in a certain way, in the change in eccentricity and semi-major axis, so to take these changes into account we will use the following expressions:

$$\Delta e_i = 2\Delta V_{1ti} - 2\Delta V_{2ti}; \quad (18)$$

$$\Delta a_i = 2\Delta V_{1ti} + 2\Delta V_{2ti}. \quad (19)$$

Therefore, we calculate the necessary duration of low-impulse maneuvers that will result in the same change in these elements [25]:

$$\Delta\varphi_1 = \frac{\omega_c \Delta a}{4\omega n} + 2\arcsin\left[\frac{\omega_c \Delta e}{8\omega n \cos\left(\frac{\omega_c \Delta a}{8\omega n}\right)}\right];$$

$$\Delta\varphi_2 = \frac{\omega_c \Delta a}{4\omega n} - 2\arcsin\left[\frac{\omega_c \Delta e}{8\omega n \cos\left(\frac{\omega_c \Delta a}{8\omega n}\right)}\right]. \quad (20)$$

Thus, the duration of each maneuver is determined iteratively, turn by turn, ensuring the successful resolution of the low-thrust problem. If the arcsine argument exceeds unity, no feasible solution exists under the given thrust constraints and spacecraft mass for the specified number of orbital turns.

The computed low-thrust solution exhibits a similar evolution of the semi-major axis and eccentricity vector compared to the corresponding impulsive transfer. Equation (4) is satisfied with high accuracy, as the midpoints of the extended-duration maneuvers coincide with the instants at which velocity impulses are applied in the impulsive case. This alignment ensures a comparable modification of the orbit's major axis and guarantees arrival at the designated rendezvous point within the required timeframe.

However, the rendezvous problem has been solved using linearized equations of motion, which neglect perturbative effects such as the non-centrality of the gravitational field, atmospheric drag, and other external influences. Consequently, the accuracy in satisfying the terminal conditions defined in system (1)–(6) remains insufficient. To enhance precision, an iterative correction scheme may be necessary [18; 19].

Furthermore, the previously proposed algorithm proved inadequate for rendezvous maneuvers involving spacecraft equipped with low-thrust engines of very small thrust magnitude [9]. As the thrust level decreases, the duration of certain critical maneuvers extends beyond the required correction time for the eccentricity vector. To mitigate this issue, the duration of these maneuvers is constrained to the upper bound at which the eccentricity correction remains maximized (180° change), while increasing the number of intermediate maneuvers. If these adjustments do not sufficiently impact the arrival time at the rendezvous point, an additional velocity impulse can be introduced at a specific orbital position. To compensate for the residual trajectory deviation at the rendezvous point in the absence of impulsive corrections, we subtract the effect of pre-defined discrete impulses (typically 4, 5, or 6, depending on their influence on the final arrival time). This approach ensures a more precise alignment with the target conditions while maintaining the feasibility of the low-thrust transfer strategy.

4. Algorithm for Solving the Meeting Problem When Fixing Velocity Impulses

The formulated rendezvous problem uses an algorithm that consists of the following stages:

1. For long-term maneuvers, we calculate how fixed (*N* impulses) maneuvers on the outer turns change the eccentricity and major semi-axis;
2. Then we solve the transfer problem for the remaining misses in eccentricity and major semi-axis;
3. We calculate the change in arrival time due to the influence of N impulses;
4. Then we distribute two new calculated velocity impulses between the remaining turns to correct the time miss remaining after the fixed impulses;
5. We determine the change in eccentricity and major semi-axis on each turn;
6. We take into account the duration of maneuvers;
7. If the new internal impulses are also greater than the permissible value, then the procedure is repeated, new fixed velocity impulses appear;
8. We calculate the total costs.

5. Examples of Solving the Coplanar Rendezvous Problem When Recording Velocity Impulses

Let us analyze the motion of a SC relative to a reference point O, which follows a near-circular orbit of radius 6871 km around the Earth, under the assumption of an unperturbed gravitational field. The Earth's gravitational parameter is taken as $3.9860044{\cdot}1014\ m^3/s^2$. The objective is to resolve the problem of a flight, where the spacecraft performs *N* velocity impulses within a fixed time interval to transition from an initial orbit to a target location in phase space. The initial conditions are: $r_0 = (10, 100, -5)$ km and $v_f = (0, 0, 0)$ m/s with the goal of reaching the origin of the reference frame, i.e., $r_f = (0, 0, 0)$ km and $v_f = (0, 0, 0)$ m/s in the tenth turn N=10. The spacecraft has an initial mass of 1000 kg, and its propulsion system operates with a specific impulse of 220 s, corresponding to an effective exhaust velocity of 2157.463 m/s. The thrust magnitude varies within the range 0.19 to 0.362 *N*.

Table 1 presents the parameters of the coplanar transition maneuvers, where the first velocity impulse is braking, and the second is accelerating. This occurs because the orbits intersect.

When addressing the rendezvous problem, the velocity impulses were not only distributed across the turns, as shown in Figure 1, but also optimized with respect to a single parameter, ensuring compliance with the time constraint.

Solution to the problem with low thrust:

In certain instances, the previous solution algorithm is unavailable because the argument of the arcsine falls outside the range (–1; 1). Additionally, as thrust increases, the maneuver duration decreases, and the total velocity costs for the low-thrust solution with thrust entrainment align with those of the impulse-based solution. Table 2 shows the correction of eccentricity and semi-major axis using velocity impulses for each turn.

After correcting the orbital elements, Table 3 shows the calculated results for the problem with low thrust ($T = 0.362$N) for $N = 10$.

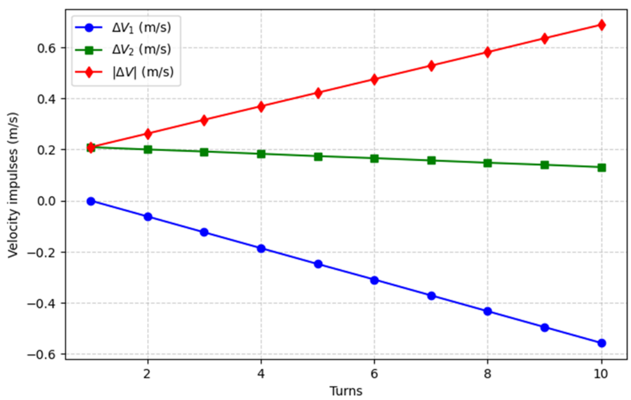

Figure 1. Distribution of the two-impulse optimal maneuvers by turns

Source: made by A.A. Baranov, A.P. Olivio

Table 1

Results of the calculation the parameters of coplanar transition maneuvers

ΔV_1 m/s	$\lvert\Delta V\rvert$ m/s	$\lvert\Delta V\rvert$ m/s	α_e°	α_1°	α_2°
–2.785	1.7	4.485	6.4	186.4	366.4

Source: made by A.A. Baranov, A.P. Olivio

Table 2

Results of the correction of eccentricity and semi-major axis by turns

N	$\Delta e_i \left(10^{-4}\right)$	$\Delta a_{0i} \left(10^{-4}\right)$
1	–0.5492	0.5485
2	–0.6888	0.3633
3	–0.8285	0.1781
4	–0.9682	–0.00711
5	–1.108	–0.1923
6	–1.248	–0.3775
7	–1.387	–0.5627
8	–1.527	–0.7479
9	–1.667	–0.9331
10	–1.806	–1.118

Source: made by A.A. Baranov, A.P. Olivio

Table 3

Results of calculation of the problem with low thrust "$T = 0.362$ N" for $N = 10$

N	ΔV_{1i} m/s	ΔV_{2i} m/s	$\lvert\Delta V\rvert$ m/s	$\Delta\varphi_{1i}^\circ$	$\Delta\varphi_{2i}^\circ$	$\lvert\Delta\varphi\rvert^\circ$
1	–0.002	0.211	0.213	–0.342	36.991	37.333
2	–0.064	0.202	0.266	–11.166	35.44	46.606
3	–0.126	0.193	0.319	–22.044	33.944	55.988
4	–0.188	0.186	0.374	–33.029	32.554	65.583
5	–0.252	0.179	0.431	–44.173	31.324	75.497
6	–0.317	0.173	0.49	–55.54	30.317	85.857
7	–0.383	0.169	0.552	–67.2	29.602	96.802
8	–0.452	0.167	0.619	–79.237	29.265	108.502
9	–0.523	0.168	0.691	–91.758	29.412	121.17
10	–0.598	0.172	0.77	–104.902	30.181	135.083
$\sum$	–2.903	1.818	4.721	–509.392	319.029	828.421

Source: made by A.A. Baranov, A.P. Olivio

It can be stated that with such a thrust the problem is solved optimally and the introduction of fixed impulses is not required.

Further, in order to move to an algorithm with fixed maneuvers, it is necessary to solve the problem with such a low thrust that impulses appear greater than the permissible value. However, we are interested in solutions with a thrust of 0.24, 0.22, 0.21, 0.2 and 0.19 *N*, and thrusts less than 0.19 *N* are not interesting.

Figure 2 presents the results of calculating the problem with low thrusts for T = 0.24, 0.22, 0.21, 0.2, 0.19 *N*.

It can be seen that there is no solution, since the first maneuver is larger than permissible.

We can observe the evolution of the velocity impulses and durations of the maneuvers at the different thrust levels. With the increase in thrust, the velocity impulses and durations of the maneuvers in the last turns will not exist (this is visualized in the Figures of levels 2 to 5). In these same figures we can observe the reduction of the values of N, in reality there was no reduction, but rather the non-existence of the values of the velocity impulses and duration of the maneuvers in the last turns.

Next, we take one of the solutions presented in Figure 2 and transform it into a solution with fixed impulses. We fix the impulses on the last turn. The duration of the first impulse on the turn is –180 degrees, and the duration of the second impulse is 72 degrees. It is necessary to calculate and present in the table the influence of these impulses on the difference in the orbital elements (OE), compare the values of the orbital elements before and after the maneuvers are performed. After that, the standard algorithm for the first 9 turns presented in [9] can be applied.

Table 4 presents the orbital elements (eccentricity, semi-major axis, and flight time) before and after applying these fixed impulses. It is important to note that, to calculate the parameters for coplanar transfer maneuvers, new orbital elements must be calculated from the initial orbital elements. Here and everywhere, deviations (Δa and Δt) will be presented in dimensionless variables. Afterward, we need to multiply by r_0 and λ_0 to convert them back to original units. These updated values will be defined in Table 5, and so on.

Table 4

Difference of orbital elements

Orbital elements (OE)	OE_1	OE_2	Difference of OE
$\Delta e_i\left(10^{-4}\right)$	–1.806	–1.655	–0.151
$\Delta a_{0i}\left(10^{-4}\right)$	–1.118	–0.9823	–0.1357
$\Delta t_i\left(10^{-3}\right)$	–12	–1.51	–0.1049

N o t e : **OE_1** = Orbital elements before impulse fixation and **OE_2** = Orbital elements after impulse fixation.

S o u r c e: made by A.A. Baranov, A.P. Olivio

Table 5

Results of calculation of parameters of coplanar transfer maneuvers

ΔV_1 m/s	ΔV_2 m/s	$\lvert\Delta V\rvert$ m/s
–2.283	1.572	3.855

S o u r c e: made by A.A. Baranov, A.P. Olivio

Subsequently, the changes in eccentricity and the semi-major axis for each turn are determined to satisfy the spacecraft's flight time condition, with the results presented in Table 6.

Table 6

Results of the correction of eccentricity and semi-major axis by turns

N	$\Delta e_i\left(10^{-4}\right)$	$\Delta a_{0i}\left(10^{-4}\right)$
1	–0.499	0.4983
2	–0.6554	0.3219
3	–0.8119	0.1454
4	–0.9683	–0.0311
5	–1.125	–0.2074
6	–1.281	–0.3838
7	–1.438	–0.5603
8	–1.594	–0.7368
9	–1.75	–0.9132
10	–1.655	–0.9821

S o u r c e: made by A.A. Baranov, A.P. Olivio

Figure 3 shows the parameters of the optimal solution to the meeting problem for N = 10.

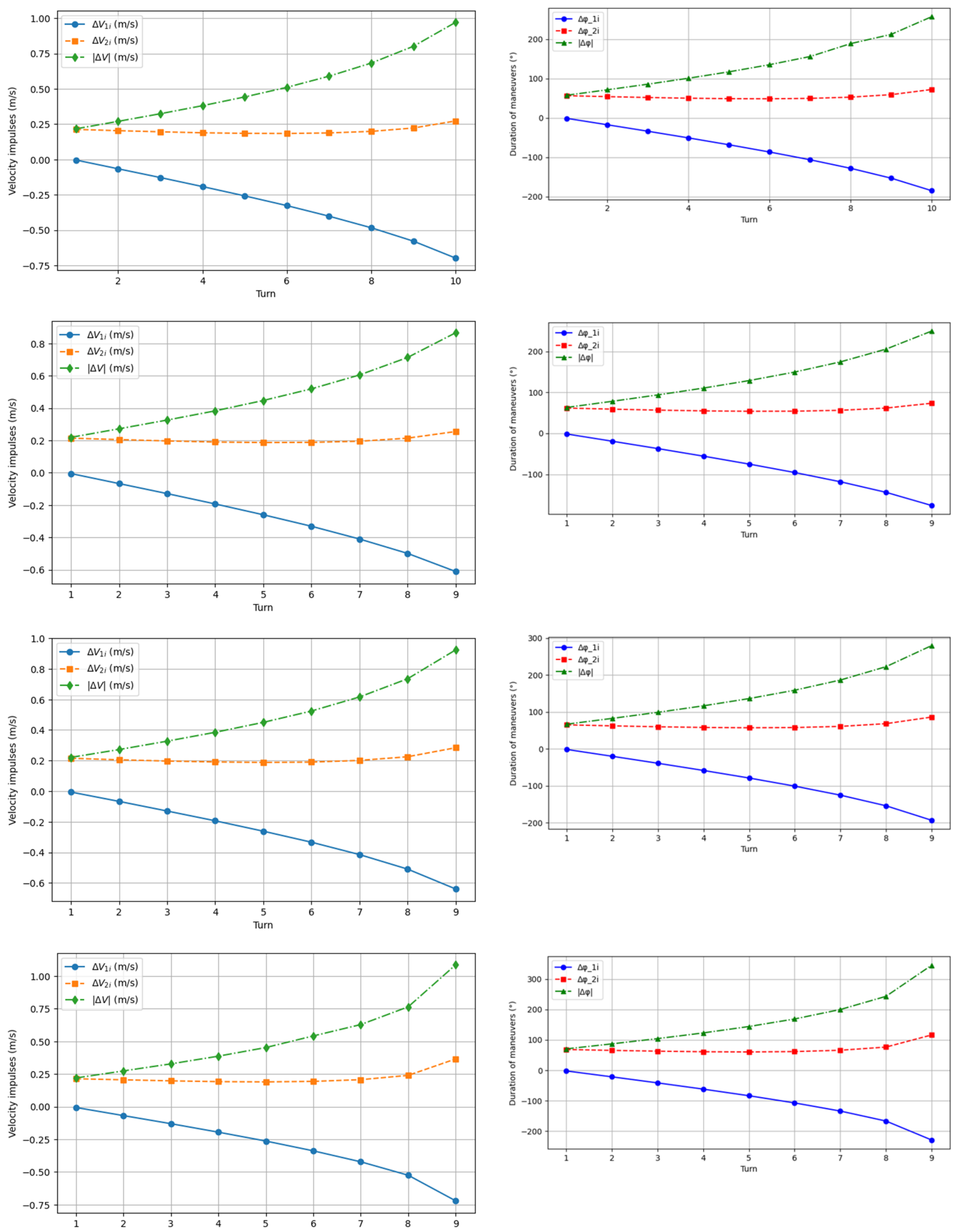

Figure 2. Results of calculation of the problem with low thrust for $N = 10$
Source: made by A.A. Baranov, A.P. Olivio

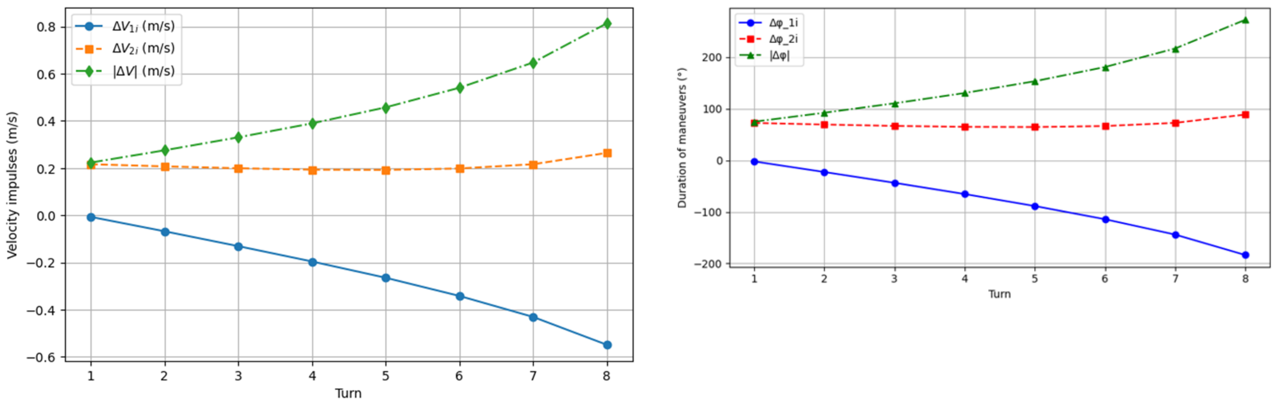

Figure 2. Results of calculation of the problem with low thrust for *N* = 10 *(continuation)*

Source: made by A.A. Baranov, A.P. Olivio

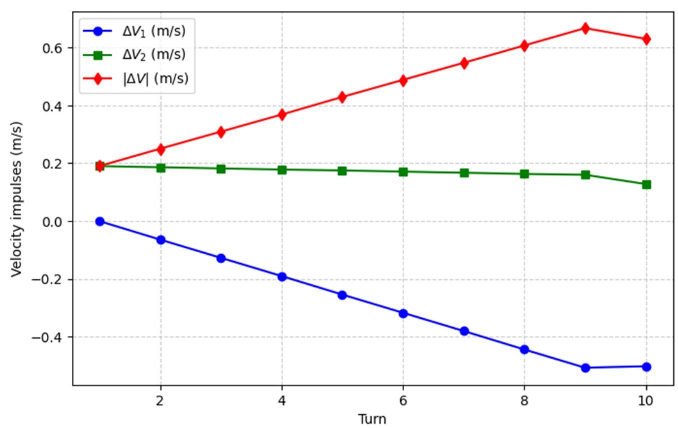

Figure 3. Distribution of the two-impulse optimal maneuvers by turns for the rendezvous problem

Source: made by A.A. Baranov, A.P. Olivio

Table 7

Results of calculation of the problem with low thrust "*T* = 0.22 *N*" for *N* = 10

N	ΔV_1 m/s	ΔV_2 m/s	$\lvert\Delta V\rvert$ m/s	$\Delta\varphi_{1i}^{\circ}$	$\Delta\varphi_{2i}^{\circ}$	$\lvert\Delta\varphi\rvert^{\circ}$
1	–0.004	0.194	0.198	–1.133	55.898	57.031
2	–0.067	0.19	0.257	–19.414	54.786	74.2
3	–0.132	0.187	0.319	–37.966	53.945	91.911
4	–0.198	0.186	0.384	–57.068	53.654	110.722
5	–0.267	0.188	0.455	–77.076	54.269	131.345
6	–0.341	0.195	0.536	–98.493	56.293	154.786
7	–0.423	0.21	0.633	–122.144	60.551	182.695
8	–0.519	0.238	0.757	–149.693	68.707	218.4
9	–0.645	0.297	0.942	–186.245	85.866	272.111
10	–0.623	0.249	0.872	–180	72	252
Σ	–3.219	2.134	5.353	–929.232	615.97	1545.202

Source: made by A.A. Baranov, A.P. Olivio

As shown in Figure 2, no solution was found for the tenth turn, as the maneuver durations exceeded the permissible limit. However, after applying the new algorithm, the results presented in Table 7 show that a solution for the tenth turn is now available. This indicates that the algorithm was successful.

Conclusion

The paper proposes a modified algorithm for calculating the parameters of a multi-turn rendezvous. The main advantage of the proposed algorithm is its simplicity and reliability, which allows it to be used not only in ground control centers, but also on board a spacecraft. At the same time, this modification of the algorithm for calculating the parameters of a multi-turn rendezvous allows us to obtain a solution to the problem at low thrust. The examples given in the article confirm the operability of this modified algorithm and the high quality of the resulting solution. Furthermore, the algorithm's ability to adapt to varying mission conditions, such as changes in thrust or trajectory, demonstrates its versatility and potential for broader applications in future space missions. This enhancement could significantly contribute to improving mission efficiency and accuracy, particularly for long-duration spaceflights requiring precise maneuvering.

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