Вестник РУДН. Серия: Инженерные исследования
RUDN Journal of Engineering Research

2024;25(3):203–215

ISSN 2312-8143 (Print); ISSN 2312-8151 (Online)
journals.rudn.ru/engineering-researches

DOI: 10.22363/2312-8143-2024-25-3-203-215
UDC 629.7.01
EDN: VOAIZM

Research article / Научная статья

Analysis of the Effects of Numerical Differentiation Methods on the Estimation of Longitudinal Stability and Control Derivatives of the Aircraft Mathematical Model

Oleg N. Korsun[a,b] ✉, **Moung Htang Om**[b], **Sekou Goro**[b]

[a] State Scientific Research Institute of Aviation Systems, *Moscow, Russia*
[b] Moscow Aviation Institute (National Research University), *Moscow, Russia*
✉ marmotto@rambler.ru

Article history

Received: May 20, 2024
Revised: July 13, 2024
Accepted: July 20, 2024

Conflicts of interest

The authors declare that there is no conflict of interest.

Authors' contribution

The authors have made an equal contribution to data collection and analysis.

Abstract. The estimation of the longitudinal stability and control derivatives of aircraft mathematical model was performed using the least square method, which requires the use of numerical differentiation. For the purpose of approximating the derivatives of pitch rate, the numerical differentiation methods such as: forward difference method, backward difference method, central difference method, combination of three finite difference methods "gradient" and Poplavsky method are applied. Based on the results that demonstrate the advantages and disadvantages of each of these methods, two approaches are proposed to ensure the improvement of the accuracy of the parameters estimation. The approach proposed in this paper combines the results obtained by separately using three finite difference methods to enhance of the accuracy of parameter estimation. This approach strengthens efficiency and compensates for weaknesses due to the nature and properties of finite difference methods.

Keywords: longitudinal control derivatives, parameters estimation, finite difference methods, Poplavsky method, combination of methods, combination of results

Funding

This work is partially supported by Russian Foundation for Fundamental Research, project 20-08-00449-a.

Acknowledgements

The authors express their deep appreciation to their teammates for passionate contributions that ensure the completion of this research work. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which improve the presentation of the research work.

For citation

Korsun ON, Om MH, Goro S. Analysis of the effects of numerical differentiation methods on the estimation of longitudinal stability and control derivatives of the aircraft mathematical model. *RUDN Journal of Engineering Research.* 2024;25(3):203–215. http://doi.org/10.22363/2312-8143-2024-25-3-203-215

Анализ влияния методов численного дифференцирования на оценку производных продольной устойчивости и управляемости математической модели летательного аппарата

О.Н. Корсун[a,b]✉, М.Х. Ом[b], С. Горо[b]

[a] Государственный научно-исследовательский институт авиационных систем, *Москва, Россия*
[b] Московский авиационный институт (Национальный исследовательский университет), *Москва, Россия*
✉ marmotto@rambler.ru

История статьи

Поступила в редакцию: 20 мая 2024 г.
Доработана: 13 июля 2024 г.
Принята к публикации: 20 июля 2024 г.

Заявление о конфликте интересов

Авторы заявляют об отсутствии конфликта интересов.

Вклад авторов

Авторы внесли равноценный вклад в сбор и анализ данных.

Аннотация. Оценка производных продольной устойчивости и управляемости математической модели летательного аппарата проводилось методом наименьших квадратов, применение которого требует использования численного дифференцирования. В целях аппроксимации производных угловой скорости тангажа применены методы численного дифференцирования, такие как метод левосторонней разности, метод правосторонней разности, метод двусторонней разности, комбинация трех методов конечных разностей «gradient» и метод Поплавского. На основании результатов, демонстрирующих преимущества и недостатки каждого из этих методов, разработано два подхода для обеспечения повышения точности оценивания коэффициентов. Предложенный в исследовании подход, путем комбинации результатов, полученных при раздельном использовании трех методов конечных разностей, обеспечивает повышение точности оценивания коэффициентов за счет увеличения эффективности и компенсации недостатков, обусловленных особенностями и свойствами методов конечных разностей.

Ключевые слова: производные продольной управляемости, оценка параметров, методы конечных разностей, метод Поплавского, комбинация методов, комбинация результатов

Финансирование

Исследование частично поддержано Российским фондом фундаментальных исследований, проект 20-08-00449-а.

Благодарности

Авторы выражают глубокую признательность своим коллегам по команде за посильный вклад, обеспечивший завершение исследовательской работы. Авторы также выражают признательность рецензентам за полезные комментарии и предложения, которые повысили качество исследовательской работы.

Для цитирования

Korsun O.N., Om M.H., Goro S. Analysis of the effects of numerical differentiation methods on the estimation of longitudinal stability and control derivatives of the aircraft mathematical model // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2024. Т. 25. № 3. С. 203–215. http://doi.org/10.22363/2312-8143-2024-25-3-203-215

Introduction

Estimating aircraft stability and control derivatives is relevant in aircraft engineering because these parameters provide crucial information regarding aircraft performance, stability, and control [1; 2]. Stability and control derivatives estimation also plays a significant role in flight testing, aircraft simulation, and control system design. It helps in predicting aircraft behavior during various flight conditions, such as takeoff, landing, and maneuvering. Methods for the estimation of these parameters are important in flight control system design, optimization of aircraft design, improvements in fuel efficiency, enhancement of flight safety, and help ensure the stability and maneuverability of aircraft, and these methods have been proposed in real time in some papers [3–5]. Moreover, aircraft stability and control derivative estimation is essential for simulation, aircraft performance analysis, and optimization, whereas it helps in the determination of the aircraft's maximum speed, range, payload capacity, and fuel consumption [6]. By understanding these parameters, aircraft operators can make informed decisions on flight planning, route selection, and operational efficiency. To estimate the dynamic stability derivatives, a computational fluid dynamics (CFD)-based force oscillation method was also applied by engineers [7]. Therefore, the accurate estimation of the aerodynamic parameters, aircraft stability, and control derivatives is relevant in the field of aircraft engineering. It is evident that measurement errors affect the accuracy of aerodynamic coefficient estimation [8]. There are various forms of input signals, i.e. the control organs' deflections, which also have a significant influence on the estimation accuracy [9–11]. It was observed that the application of appropriate methods for signal filtering can ensure an improvement in the accuracy of the estimation of the aerodynamic coefficients depending on the nature of the signals [12–14].

The main purpose of this study is to propose an approach for enhancing the accuracy of parameters estimation by analyzing the effects of numerical differentiation methods on the accuracy of estimation performed by the least-squares method without the application of any methods for filtering noisy signals. The derivative of the pitch rate is used as the output variable or the dependent variable in the least-squares method. Because the output vector represents the actual output values, its range and variations can influence parameter estimation accuracy.

1. Problem statement

In aircraft dynamics, the short-period mode of aircraft motion refers to the pitch motion, in which the aircraft rotates around its lateral axis. This motion includes changes in the pitch angle, pitch rate, and airspeed. In this short-period mode, the aircraft experiences rapid pitch angle fluctuations owing to interference such as turbulence or pilot actions. The stability and controllability characteristics of the aircraft in this mode are crucial for flight safety and efficiency. Therefore, to estimate the longitudinal stability and control derivatives of the aircraft mathematical model, the mathematical simulation of the short-period mode of the aircraft motion in a longitudinal channel is performed by applying the aircraft dynamic equations [15]. Because the problem under the current research focuses on the longitudinal stability and control derivatives, only the equations for the angle of attack and pitch rate are used. The elevator deflection was mathematically simulated, as shown in Figure 1. The necessary aerodynamic coefficients are determined to perform a mathematical simulation of the aircraft spatial motion, and they are estimated after the measurements.

In this study, the normally distributed random variables with a zero mean and different standard deviation values characterize the measurement noise.

The ordinary least-squares method (LSM) was used to estimate the aerodynamic coefficients. The derivative of the pitch rate was used to form the LSM output vector. It is very important to choose the correct numerical differentiation methods so that the accuracy of the estimation can be assured, whereas every numerical differentiation method has its own distinct effectiveness depending on the intensity of the measurement noise.

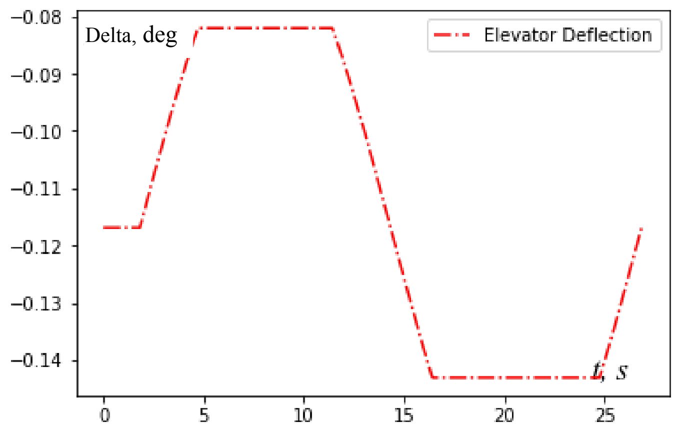


Figure 1. Angle of elevator deflection
Source: made by O.N. Korsun, M.H. Om, S. Goro

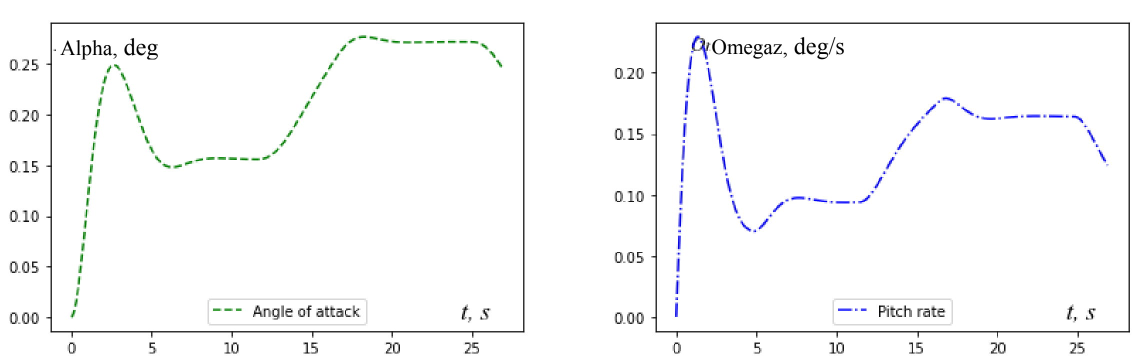
For the simulation, it was necessary to form the object model and determine the output signals. To form the object model, the angle of attack and pitch rate of the aircraft were simulated using mathematical equations. In this study, integration was performed by applying simpler Euler methods for the angle of attack and pitch rate. Usually, it is convenient and easy to perform simulation and identification in a discrete form; for this reason, the simulation of all the necessary signals is performed in a discrete form. The mathematical formulas in discrete form for the angle of attack (1) and pitch rate (2) are as follows:

$$\alpha\left(t_{(i+1)}\right)=\alpha\left(t_i\right)+\Delta t\left[-Y^{\alpha}\alpha\left(t_i\right)+\omega_z\left(t_i\right)+\left(-Y^{\delta}\delta\left(t_i\right)\right)\right]; \quad (1)$$

$$\omega_z\left(t_{(i+1)}\right)=\omega_z\left(t_i\right)+\Delta t\left[M_z^{\alpha}\,\alpha\left(t_i\right)+M_z^{\omega_z}\omega_z\left(t_i\right)+M_z^{\delta}\delta\left(t_i\right)\right], \quad (2)$$

where $\alpha\left(t_{(i+1)}\right)$ is the angle of attack for time instant $\left(t_{(i+1)}\right)$ (rad), $\omega_z\left(t_{(i+1)}\right)$ is the pitch rate for time instant $\left(t_{(i+1)}\right)$ (rad/s), $\alpha\left(t_i\right)$ is the angle of attack for time instant $\left(t_i\right)$, $\omega_z\left(t_i\right)$ is the pitch rate for time instant $\left(t_i\right)$, and Y^{α}, Y^{φ}, M_z^{α}, $M_z^{\omega_z}$, M_z^{δ} are the aerodynamic parameters.

After the simulation of the short-period mode of the aircraft motion in longitudinal motion was performed, it was necessary to proceed with the measurement of the signals (Figure 2) used in the process of parameter estimation.



a *b*

Figure 2. Measurement of signals: *a* is the simulated angle of attack; *b* is the pitch rate
Source: made by O.N. Korsun, M.H. Om, S. Goro

To imitate the measurement mathematically, the normally distributed random variables with zero mean and different standard deviation values were subjected to the measurement noise for every signal. The values of the standard deviations of the measurements are listed in Table 1. It is assumed that the measurements took place under various intensities of measurement noise (Figure 3); therefore, the effectiveness of the estimation will be analyzed based on the numerical methods applied to approximate the derivatives of the pitch rate under the influence of various levels of noise intensity. In Table 1, Std represents the values of the standard deviations in degrees subjected to measurement noise.

Table 1

The values of standard deviations subjected to measurement noise

Signal, deg	Std, deg	Std, deg	Std, deg	Std, deg	Std, deg
Angle of attack	0.02	0.08	0.2	0.4	0.8
Pitch rate	0.02	0.08	0.2	0.4	0.8
Elevator deflection	0.02	0.08	0.2	0.4	0.8

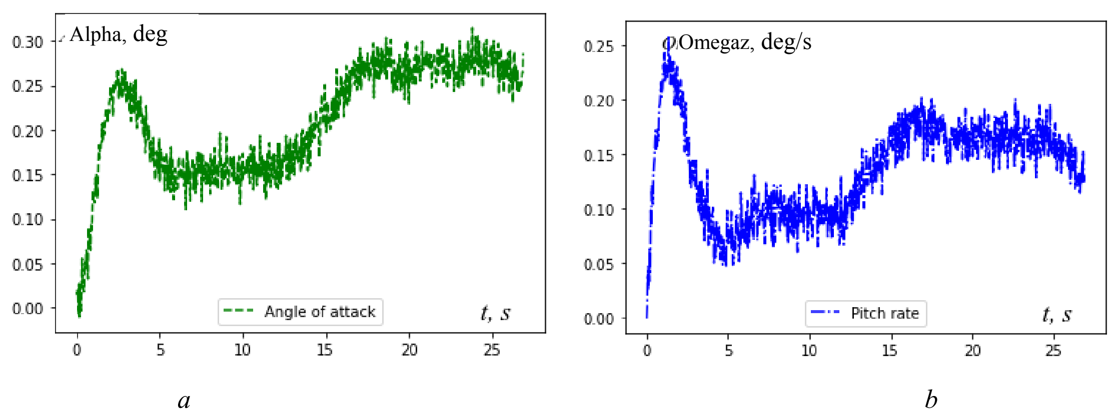


Figure 3. Measurement of signals under the influence of measurement noise:
a — Angle of attack; *b* — Pitch rate
S o u r c e: made by O.N. Korsun, M.H. Om, S. Goro

2. Estimation of stability and control derivatives

There are many distinct methods for parametric system identification, and each method has its advantages and disadvantages depending on the system itself [16; 17]. These parametric identification methods are also applicable for detecting dynamic errors in on-board measurements of aircraft based on flight data [18].

The least-squares method (1) was used in this study to estimate the aerodynamic coefficients. Evidently, the least-squares method is more effective for linear systems. The mathematical model of an object can be described as:

$$y_{(t)} = \hat{a}^{\mathrm{T}} x_{(t)}, \tag{3}$$

where $y_{(t)}$ is the vector of the output signal, $x_{(t)}$ is the vector of the regressors, and $\hat{a}$ is the vector of unknown parameters.

The observation model for N number of observations can be expressed as

$$z_{(t)} = y_{(t)} + \varepsilon_{(t)}, \tag{4}$$

where $\varepsilon_{(t)}$ denotes the measurement noise.

Therefore, for each of the N discrete time points at which the measurements are available, it can be described in matrix notation as:

$$Y = X\hat{a} + \varepsilon, \tag{5}$$

where $Y = [y_1 y_2 y_3 \ldots\ldots y_N]^{\mathrm{T}}$, $\varepsilon = [\varepsilon_1 \varepsilon_2 \varepsilon_3 \ldots\ldots \varepsilon_N]^{\mathrm{T}}$ are $N \times 1$ size vectors and X is the $N \times p$ $N \times p$ matrix of the independent variables:

$$X = \begin{bmatrix} 1 & x_{1t_{(1)}} & x_{2t_{(1)}} & x_{3t_{(1)}} \\ 1 & x_{1t_{(2)}} & x_{2t_{(2)}} & x_{3t_{(2)}} \\ & \ldots & \ldots & \ldots \\ 1 & x_{1t_{(N)}} & x_{2t_{(N)}} & x_{3t_{(N)}} \end{bmatrix}.$$

Unlike the maximum likelihood, in the least-squares method (6), the parameters to be estimated must arise in the expressions for the means of the observations. When the parameters appear linearly in these expressions, the least-squares estimation problem can be solved in closed form, and it is relatively straightforward to derive the statistical properties for the resulting parameter estimates. The least-squares method is given by

$$\hat{a} = \left(X^{\mathrm{T}} X\right)^{-1} X^{\mathrm{T}} Y, \qquad (6)$$

where $\hat{a}$ is the vector of unknown parameters to be estimated, X is the matrix of the object model, and Y is the vector of the output signal.

Only three moment coefficients involved in the angle of attack and pitch rate were estimated, whereas the present study focused on analyzing the effect of numerical differentiation methods on the estimation. The estimation of three moment coefficients using LSM requires the derivative of the pitch rate $\dot{\omega}_z$ for the formation of the output signal vector Y. The object model matrix X is formed by the angle of attack α, pitch rate ω_z and elevator deflection δ. Then, the object model matrix, output signal vector and vector of unknown parameters take the following form:

$$X = \begin{bmatrix} 1 & \alpha_{t_{(1)}} & \omega_{zt_{(1)}} & \delta_{t_{(1)}} \\ 1 & \alpha_{t_{(2)}} & \omega_{zt_{(2)}} & \delta_{t_{(2)}} \\ & \ldots & \ldots & \ldots \\ 1 & \alpha_{t_{(N)}} & \omega_{zt_{(N)}} & \delta_{t_{(N)}} \end{bmatrix},$$

$$Y = \begin{bmatrix} \dfrac{d\omega_{z(1)}}{dt} \\ \dfrac{d\omega_{z(2)}}{dt} \\ \cdot \\ \cdot \\ \dfrac{d\,\omega_{z(N)}}{dt} \end{bmatrix}, \quad \hat{a} = \begin{bmatrix} M_z^{\alpha} & M_z^{\omega_z} & M_z^{\delta} \end{bmatrix}^{\mathrm{T}}.$$

3. Numerical differentiation methods for the approximation of derivatives

The aerodynamic moments acting on the aircraft in flight are proportional to the derivatives of the corresponding angular velocities of the aircraft, which are usually obtained from the onboard measurements. Therefore, the numerical differentiation methods that ensure the accurate approximation of the derivatives are crucial in aircraft engineering [19].

Generally, differential equations can be solved analytically; however, significant effort and effective mathematical theory are often required, and the closed form of the solution may be too confusing to be useful. When an analytical solution to a differential equation is not available, it is too difficult to deduce, or it takes on a sophisticated form that is unhelpful to apply, an approximate solution can be considered. There are two approaches to this purpose. The first approach is a semi-analytical methods that consider the use of series, that is, integral equations, perturbation methods, or asymptotic methods for obtaining an approximate solution expressed in terms of simpler functions. The second one is numerical solutions. Discrete numeric values can represent solutions with a certain precision. Currently, these numerical arrays (and their associated tables or graphs) are obtained using computers to provide effective solutions to many problems that were previously impossible to obtain [20].

In this study, to approximate the derivative of the pitch rate, forward difference, backward difference, and central difference methods were applied. The finite difference methods — forward difference, backward difference, and central difference —

are used for the numerical approximation of the derivatives of a certain function. The forward difference method approximates the derivative of a function at a particular point by considering the function values at the very point and a nearby point ahead of it. This can be expressed mathematically as follows:

$$f'(x) = \frac{\left(f_{(x+h)} - f_{(x)}\right)}{h}. \qquad (7)$$

The backward difference method considers the function values at that point and the nearby point behind it to approximate the derivatives of a function at a particular point. This can be expressed mathematically as follows:

$$f'(x) = \frac{\left(f_{(x)} - f_{(x-h)}\right)}{h}. \qquad (8)$$

The central difference method takes the function values at two nearby points, one ahead and one behind it to approximate the derivative of a function at a particular point. This can be expressed mathematically as follows:

$$f'(x) = \frac{\left(f_{(x+h)} - f_{(x-h)}\right)}{2h}, \qquad (9)$$

where $f'_{(x)}$ is the derivative of a function, $f_{(x)}$ is the function that is differentiated, and h is the step size.

Moreover, the combination of these three methods "gradient" that is implemented in Python is also utilized for the approximation of the derivative of pitch rate. This combination method "gradient" uses the forward/backward difference methods for approximating the boundary points (first and last points) of the signal and the central difference method is used to calculate the derivatives of interior points of the signal.

Poplavsky method was also applied in this study to approximate the derivatives [19].

The approximation polynomials of k degrees were used to estimate the first derivative. This is expressed as follows:

$$S = \sum_{j=-m}^{m} b_j y(t_i), \qquad (10)$$

$$b_j = \frac{5\left[5\left(3m^4 + 6m^3 - 3m + 1\right)j - 7\left(3m^2 + 3m - 1\right)j^3\right]}{h\left(m^2 - 1\right)m\left(m + 2\right)\left(4m^2 - 1\right)\left(2m + 3\right)}, \qquad (11)$$

where h is the sampling interval, and m is the sliding interval size.

First, the estimation of aerodynamic parameters using LSM was performed under noise-free conditions to clarify how the method functions without noise. Table 2 presents the results. The estimation was repeatedly executed 15 times and the average of the relative errors of the estimates was calculated.

Table 2

Relative errors of estimates given by numerical differentiation methods under noise-free condition

Method	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
Forward Difference	1.90588286e-12	8.38218384e-12	3.89965837e-12
Backward Difference	56.54558069	167.07475111	90.06760292
Central Difference	28.27279035	83.53737555	45.03380146
Combination of Methods	1.89378435	1.36827312	0.96303547
Combination of Results	1.89414061	1.36895486	0.96301928
Poplavsky Method_1	8.40614113	28.35047731	14.42287996
Poplavsky Method_2	1.89026931	1.33060582	0.97035183

After obtaining the results calculated by the forward difference method, backward difference method, central difference method, and Poplavsky method, two approaches are proposed considering the efficiency and weaknesses of these methods. The first one is Poplavsky methods-2, where the sample points are cut off along the left and right edges of the dataset in accordance with the size of the sliding window used in the Poplavsky method. The second approach considered a combination of the results obtained by separately using three finite difference methods.

As shown in Table 2, the forward difference method demonstrated superior performance compared to the other methods under noise-free conditions. Combination of methods "gradient", Combination of results and Poplavsky method also demonstrate their effectiveness in providing satisfactory estimates. The backward difference method is generally less accurate, and when it is used for the output vector, which is crucial in estimating parameters, it provides less favorable estimates. The results provided by the central difference method are less favorable because this method does not accurately approximate the derivatives at the boun-dary points. The results show that Poplavsky method also provides gratifying accuracy in the estimation of the parameters under noise-free conditions.

4. Results and discussion

For better accuracy and reliability of the estimates of the stability and control derivatives, the estimation was performed 15 times. For each execution of the program, whereas the normally distributed random variables with zero mean and several values of standard deviation characterized the measurement noise, the relative errors of the estimates were calculated. Subsequently, the average values of the relative errors of the estimates according to the repeated execution of the estimation are obtained. The results obtained by the five methods and two approaches are presented in Tables 3–9 and their graphical presentation is shown in Figures (4–6).

Table 3

Relative errors of estimates given by forward difference method

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	0.15205137	0.86688247	0.29560156
0.08	0.66371885	11.94143302	3.2670063
0.2	3.55255766	69.70950238	18.22990503
0.4	19.66459307	258.66536699	63.01107033
0.8	116.90595768	724.5247589	132.21844664

Table 4

Relative errors of estimates given by backward difference method

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	56.46582798	167.64983046	90.18434052
0.08	57.33883809	181.78023716	95.01045211
0.2	58.34679468	250.40740951	116.16826948
0.4	54.42187212	459.89022414	175.76298856
0.8	18.73253971	960.14423628	275.98508977

Table 5

Relative errors of estimates given by central difference method

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	28.30462418	83.64873361	45.08821755
0.08	28.66114439	83.85506646	45.40073055
0.2	30.98583624	89.82494974	48.79572234
0.4	37.10499183	104.01791904	57.41742689
0.8	48.81781812	117.835604	69.74591019

Table 6

Relative errors of estimates given by combination of three methods "gradient"

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	1.7906732	1.53301677	0.83952767
0.08	1.37399282	3.27782994	1.37076629
0.2	3.40201398	15.52870946	6.96162448
0.4	13.13329098	39.18691594	20.89832966
0.8	36.63975232	85.35310955	51.29733977

Table 7

Relative errors of estimates given by combination of results given by finite difference methods

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	1.84662819	1.36845403	0.93535781
0.08	1.44177607	3.93460101	1.52677994
0.2	2.89059324	14.78572012	6.41968791
0.4	11.46850026	35.22754859	18.59589663
0.8	35.05152409	87.77718717	51.02816458

Table 8

Relative errors of estimates given by Poplavsky method_1

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	8.44056384	28.78966508	14.56942007
0.08	8.18407016	29.35411495	14.57706342
0.2	11.86598703	39.18062433	19.97106469
0.4	22.38493718	65.62501345	35.51506445
0.8	41.11094292	101.28620295	59.36737503

Table 9

Relative errors of estimates given by Poplavsky method_2

Std, deg	Relative error M_z^{α}, %	Relative error $M_z^{\omega_z}$, %	Relative error M_z^{δ}, %
0.02	1.76466215	1.89400133	0.71172912
0.08	1.73815893	7.24275427	3.36221295
0.2	12.51837471	27.97205869	17.2391036
0.4	49.55022288	65.28447612	54.77609201
0.8	79.72952628	90.19302059	83.77492038

In the presented Tables 3–9, the notation "Std" stands for the values of the standard deviations that characterized the measurement noise for the simulation of the measurement of the aircraft performance signals using the mathematical equations of flight dynamics. The relative errors in the estimates of the pitch moment coefficient with respect to the angle of attack, which is a static longitudinal stability derivative, are shown in Figure 4.

The relative errors in the estimates of the pitch moment coefficient with respect to the pitch rate, which is a dynamic longitudinal stability derivative, are shown in Figure 5.

The relative errors in the estimates of the pitch moment coefficient with respect to the elevator deflection, which is the longitudinal control derivative, are shown in Figure 6.

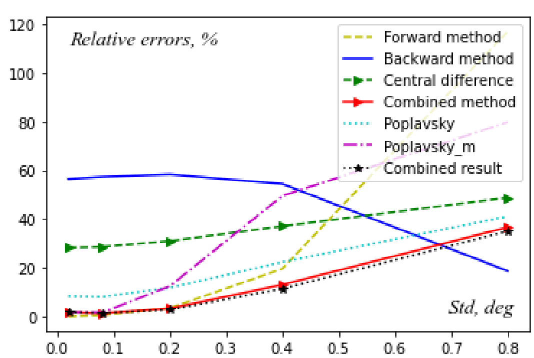


Figure 4. Relative errors of the estimates of pitch moment coefficient with respect to angle of attack M_z^{α}
Source: made by O.N. Korsun, M.H. Om, S. Goro

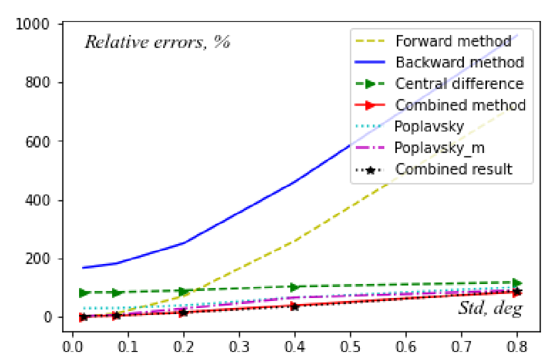


Figure 5. Relative errors of the estimates of pitch moment coefficient with respect to pitch rate $M_z^{\omega_z}$
Source: made by O.N. Korsun, M.H. Om, S. Goro

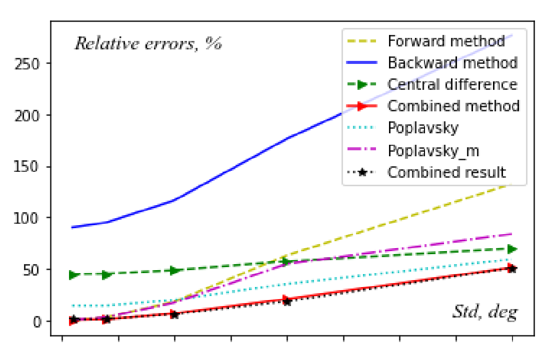


Figure 6. Relative errors of the estimates of pitch moment coefficient with respect to elevator deflection M_z^{δ}
Source: made by O.N. Korsun, M.H. Om, S. Goro

In Figures 4–6, the *X* axes of each figure represent the values of standard deviations in degrees that characterize the measurement noise, while the *Y* axes represent the relative errors of the estimates of the longitudinal stability and control derivatives calculated by the least-squares method, which requires the derivatives of the pitch rate.

According to the results presented in Tables 3–9 and Figures 4–6, it can be observed that the backward difference method is less effective for estimating coefficients. The forward difference method ensures a good and satisfactory result when the noise intensity is relatively low; however, its ability deteriorates with an increase in the noise intensity. The central difference method provides moderate and stable accuracy of the estimation at every level of noise intensity, but it provides a less accurate approximation of the derivatives at the boundary points, resulting in a significant deterioration in the accuracy of the estimation. Poplavsky method also provides a good accuracy of the estimates, and it can be observed that this method is robust and stable despite changes in the noise intensity. The combination of three finite difference methods "gradient" ensures gratifying and stable accuracy for every level of noise intensity.

The forward difference method is efficient for approximating derivatives at the left boundary, and the backward difference method is effective at the right boundary. The central difference method provides better results than the other two methods, but it is observed that this method is much more suitable for finding the derivatives of the interior points. Based on these advantages and disadvantages, a combination of three finite-difference methods was proposed. The three combined methods were observed to be more efficient in handling different levels of noise intensity throughout the entire dataset, as they approximated the derivatives at the boundary points using the forward and backward difference methods, which are generally effective at the boundary points. It also potentially allows for a more accurate approximation of the derivative considering the specific characteristics of noise in different locations.

Nevertheless, in the combination of the three methods, the central difference method still struggles at the second and second last points of the processing time because these two points become the first and last points at the left and right boundaries, where the method has weakness.

The combination of the results obtained by separately approximating the derivatives using three finite-difference methods compensates for this weakness and ensures the enhancement of the accuracy of parameter estimation. Poplavsky method also struggles at the left and right boundaries, but after cutting off the edges according to the size of the sliding window, it becomes more efficient under a low intensity of noise. However, the efficiency of this approach deteriorates with an increase in the intensity of the measurement noise.

Conclusion

This study comprehensively analyzed the effects of numerical differentiation methods: the forward difference method, backward difference method, central difference method, combination of finite difference methods, and Poplavsky method on the estimation of the longitudinal stability and control derivatives of the mathematical model of the motion of the aircraft. Moreover, two proposed approaches take into account the combination of the results separately obtained by three finite difference methods and Poplavsky method-2, where the unnecessary edges of the dataset at the boundaries are cut off.

The numerical differentiation methods are used to approximate the derivatives of the pitch rate that is necessary for forming the output vector in the least-squares method. It is important to note that the choice of method depends on specific tasks and requirements. All of these numerical differentiation methods may have advantages depending on the task. The combination of the results separately obtained by the three finite difference methods can be especially useful in practical applications where noise is present, for example, in scientific experiments, data analysis, or signal processing. This method also has the ability to reduce the effects of noise and provides reliable estimates of derivatives, even at high noise levels.

Based on the results obtained in this work, it can also be noted that every single point of the dataset is crucial for the parameter estimation, as the first and last points, that is, boundary points, significantly affect the accuracy of estimation. This advantage of the proposed approach enhances the accuracy and reliability of numerical differentiation methods in various scientific research fields.

References / Список литературы

1. Anderson JD. *Introduction to Flight*. 8th edition. USA: McGraw-Hill Publ.; 2016.

2. Lutskiĭv V, Galashev E. *Aerodynamics, stability and controllability of supersonic aircraft*. Moscow: Nauka Publ.; 1998. (In Russ.)

3. Vepa R. *Flight Dynamics, Simulation, and Control, For Rigid and Flexible Aircraft.* Boca Raton: CRC Press; 2014. https://doi.org/10.1201/b17346

4. Korsun ON, Poplavsky BK. Technology for identifycation of aerodynamic coefficients of aircraft based on flight test data. *Modeling of aviation systems*. Moscow: GosNIIAS Publ.; 2011. P. 444–451. (In Russ.) EDN: RROKCT

Корсун О.Н., Поплавский Б.К. Технология идентификации аэродинамических коэффициентов летательных аппаратов по данным летных испытаний // Моделирование авиационных систем. М.: Изд-во ГосНИИАС, 2011. С. 444–451. EDN: RROKCT

5. Korsun ON, Om MH, Latt KZ, Stulovskii AV. Real-time aerodynamic parameter identification for the purpose of aircraft intelligent technical state monitoring. *Procedia Computer Science.* 2017;103:67–74. https://doi.org/10.1016/j.procs.2017.01.014

6. Anton N, Botez RM, Popescu D. New Methods and Code for Aircraft Stability Derivatives Calculations from its Geometrical Data. *Canadian Aeronautical Society Institute CASI Aircraft Design and Development Symposium*. Kanata, Ont., Canada; 2009:6046.

7. Juliawan N, Chung HS, Lee JW, Kim S. Estimation and Separation of Longitudinal Dynamic Stability Derivatives with Forced Oscillation Method Using Computational Fluid Dynamics. *Aerospace.* 2021;8(11):354. https://doi.org/10.3390/aerospace8110354

8. Om MH, Latt KZ, Karapetyan TS. Estimation of aerodynamic parameters in conditions of measurement. *ITM Web of Conferences*. 2017;10:01007. https://doi.org/10.1051/itmconf/20171001007

9. Ovcharenko VN. *Identification of aerodynamic characteristics of aircraft by flight data*. MAI Publishing House, Moscow; 2017. (In Russ.)

Овчаренко В.Н. Идентификация аэродинамических характеристик воздушных судов по полетным данным. М.: Изд-во МАИ, 2017. 182 с. ISBN 978-5-4316-0403-4

10. Om MH, Latt KZ. Analysis of the influence of input signal forms on the accuracy of identification of aerodynamic parameters in the longitudinal motion of an aircraft. *Cloud of Science.* 2017;4(4):636–649. EDN: YSCXKM

11. Nikolaev SV, Om MH, Latt KZ. Determination of aerodynamic parameters in various conditions of input signal. *IOP Conference Series: Materials Science and Engineering. 2019 Workshop on Materials and Engineering in Aeronautics*. Moscow; 2020;714(1):012021. https://doi.org/10.1088/1757-899X/714/1/012021

12. Morelli EA, Klein V. Accuracy of Aerodynamic Model Parameters Estimated from Flight Test Data. *Journal of Guidance, Control, and Dynamics.* 1997;20(1):74–80. Available from: https://ntrs.nasa.gov/api/citations/19980048414/downloads/19980048414.pdf (accessed: 11.01.2024).

13. Maine RE, Iliff KW. *The Theory and Practice of Estimating the Accuracy of Dynamic Flight-Determined Coefficients*. NASA Reference Publication 1077, 1981. Available from: https://ntrs.nasa.gov/api/citations/19810019327/downloads/19810019327.pdf (accessed: 11.01.2024).

14. Korsun ON, Om MH. The practical rules for aircraft parameters identification based on flight test data. *Metascience in Aerospace.* 2024;1(1):53–65. https://doi.org/10.3934/mina.2024003

15. Efremov AV, Zakharchenko VF, Ovcharenko VN, Sukhanov VL. *Flight dynamics*. Byushgens GS (ed.). Moscow: Mashinostroenie Publ.; 2017. (In Russ.)

Ефремов А.В., Захарченко В.Ф., Овчаренко В.Н., Суханов В.Л. Динамика полета / под ред. академика РАН Г.С. Бюшгенса. М.: Изд-во Машиностроение, 2017. 776 p.

16. Korsun ON. *Methods of parametric identification of technical systems.* Moscow: Bauman Moscow State Technical University Publ.; 2011. (In Russ.) EDN: VQLLUZ

Корсун О.Н. Методы параметрической идентификации технических систем. М.: Изд-во: МГТУ им. Н.Э. Баумана, 2011. 69 с. EDN: VQLLUZ

17. Jategaonkar RV. *Flight vehicle system identification: A time domain methodology*. USA, Reston: AIAA Publ.; 2006. https://doi.org/10.2514/4.866852

18. Korsun ON, Nikolaev SV, Om M.H. Detection of dynamic errors in aircraft flight data. *IOP Conf. Series: Materials Science and Engineering*. Moscow; 2021;1027:012011. https://doi.org/10.1088/1757-899X/1027/1/012011

19. Vasilchenko KK, Kochetkov YuA, Leonov VA, Poplavskii BK. *Aircraft Flight Tests.* Moscow: Mashinostroenie Publ.; 1996. (In Russ.) ISBN 5-217-02574-3

Васильченко К.К., Леонов В.А., Пашковский И.М., Поплавский Б.К. Летные испытания самолетов. 2-е изд., перераб. и доп. М.: Машиностроение, 1996. 719 с. ISBN 5-217-02574-3

20. Li Z, Qiao Z, Tang T. *Numerical Solution of Differential Equations*. Cambridge University Press; 2017. https://doi.org/10.1017/9781316678725

About the authors

Oleg N. Korsun, Doctor of Technical Sciences, Head of the Scientific and Educational Center, State Scientific Research Institute of Aviation Systems (GosNIIAS), Moscow, Russia; Professor, Department of Design and Certification of Aircraft Engineering, Moscow Aviation Institute (National Research University), Moscow, Russia; eLIBRARY SPIN-code: 2472-6853, ORCID: 0000-0003-3926-1024; e-mail: marmotto@rambler.ru

Moung Htang Om, Ph.D., Post-doctoral Candidate, Department of Design and Certification of Aircraft Engineering, Moscow Aviation Institute (National Research University), Moscow, Russia; ORCID: 0000-0002-7770-2962; e-mail: mounghtangom50@gmail.com

Sekou Goro, Postgraduate Student, Department of Design and Certification of Aircraft Engineering, Moscow Aviation Institute (National Research University), Moscow, Russia; ORCID: 0000-0002-4397-9429; e-mail: gorosekoi@gmail.com

Сведения об авторах

Корсун Олег Николаевич, доктор технических наук, руководитель научно-образовательного центра, Государственный научно-исследовательский институт авиационных систем (ГосНИИАС), Москва, Россия; профессор кафедры проектирования и сертификации авиационной техники, Московский авиационный институт (Национальный исследовательский университет), Москва, Россия; eLIBRARY SPIN-код: 2472-6853, ORCID: 0000-0003-3926-1024; e-mail: marmotto@rambler.ru

Ом Моунг Хтанг, кандидат технических наук, докторант кафедры проектирования и сертификации авиационной техники, Московский авиационный институт (Национальный исследовательский университет), Москва, Россия; ORCID: 0000-0002-7770-2962; e-mail: mounghtangom50@gmail.com

Горо Секу, аспирант кафедры проектирования и сертификации авиационной техники, Московский авиационный институт (Национальный исследовательский университет), Москва, Россия; ORCID: 0000-0002-4397-9429; e-mail: gorosekoi@gmail.com