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Variant Design of Girder-Slab Structure with Different Geometric Cells Under Flexural Vibrations

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Abstract. Girder-slab structures are widely used in industrial buildings, bridge decks, complex combined engineering structures and other objects of construction and mechanical engineering. An important task in their design is to find the most economical structural solution with the least amount of material while ensuring the necessary strength and rigidity. Therefore, the development of methods and algorithms for searching of the most rational and optimal design solutions is of great significance. The authors offer a technique of variant design of girder-slab structures with various cell shapes: rectangular, triangular, rhombic, trapezoidal and other, when analyzing vibrations. The technique is based on the principles of physicomechanical analogies and geometrical methods of structural mechanics. For a numerical example, a cantilever girder-slab structure on trapezoidal base is studied. The bars are of typical sections, the flooring is smooth steel. It is shown that cell geometry affects flexural vibrations of the girder-slab structure and material consumption.

Keywords: flexural vibrations, girder-slab structure, fundamental frequency of vibration, material consumption, metal structure

Conflicts of interest. The authors declare that there is no conflict of interest.

Authors' contribution: *Chernyaev A.A.*— supervision, conceptualization, review and editing, experimental investigation, conclusions; *Marfin K.V.* — literature review, data processing, writing the text.

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Вариантное проектирование пластинчато-стержневой конструкции с различной геометрической ячейкой при изгибных колебаниях

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Аннотация. Пластинчато-стержневые конструкции широко используются в качестве перекрытий зданий, пролетных строений мостов, сложных комбинированных инженерных сооружениях и других объектов строительства и машиностроения. Важной задачей при их проектировании является поиск наиболее экономичного конструктивного решения, на выполнение которого затрачивалось бы наименьшее количество материала при обеспечении необходимой прочности и жесткости. В связи с этим большое значение при проектировании придают разработке методов и алгоритмов поиска рациональных и оптимальных конструктивных решений. Предложена авторская методика вариантного проектирования пластинчато-стержневых конструкций с различной геометрической ячейкой в плане: прямоугольной, треугольной, ромбической, трапециевидной и другой при исследовании колебаний. Методика основана на использовании принципов физико-механических аналогий и геометрических методов строительной механики. В качестве объекта исследования для численного примера рассматривается консольная пластинчато-стержневая конструкция на трапециевидном плане. Сечения стержней из типовых профилей, настил стальной гладкий. Показано, что геометрия ячейки влияет на изгибные колебания пластинчато-стержневой конструкции и материалоемкость.

Ключевые слова: изгибные колебания, пластинчато-стержневые конструкции, основная частота собственных колебаний, материалоемкость, металлическая конструкция

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1. Introduction

Girder-slab structures are widely used in industrial buildings, bridge decks, complex combined engineering structures and other construction objects and machine structures. An important task in their design is to find geometric grids with the least amount of material and while ensuring the strength and rigidity, also in case of vibrations [1–3]. Now, computer programs for the finite element analysis of the stress-strain state of structures are a common tool of engineers. In tasks of variant engineering and optimization of designs of girder-slab structures, numerical methods are also used [4–6]. A set of methods, which can be used for girder-slab design is very broad: from rather universal, such as nonlinear mathematical programming and genetic algorithms, to problem-oriented [7–9].

All methods have advantages and disadvantages and means for setup, appropriate application of which can strongly influence the speed of operation of the methods and even correctness of the results.

Moreover, direct borrowing of universal numerical optimization methods, which in some works are referred to as “search optimization methods”, from mathematics leads to the increase in dimensionality of the problems and significant growth in calculation amount in case of the increase in the number of design

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variables. Development of the methodology of nonlinear mathematical programming should be pointed out from mathematical works on geometrical optimization methods for the purposes of structural design and construction [10]. It requires accurate formalization of the optimization problem statement.

In this work, the authors propose a technique of variant design of girder-slab structures with various horizontal cell shapes, i.e. rectangular, triangular, rhombic, trapezoidal and other, with the choice of the most rational structural solution in terms of the lowest material costs for its production and while ensuring the rigidity under vibrations. The technique is based on use of the principles of physicomechanical analogies and geometrical methods of structural mechanics. To implement this technique, it is planned to develop software products that will visually represent the entire calculation procedure.

2. Methods

Geometrical methods of structural mechanics are based on mathematical analogy and the functional correlation, individual physicomechanical characteristics of the stress-strain state of plane elements of structures (pressures, deflections, vibration frequencies, critical buckling forces and others) in the shape of plates, membranes, bar cross-sections with their geometrical parameters (sizes, angles, side ratio and so on). For this, it is necessary to choose some characteristic of the geometric shape for plates and membranes or of the cross-section for the bars. And if it is proved that it is related to the parameters of the stress-strain state by some function or expression, then it is possible to study the change in the stress-strain state parameters using the chosen geometric characteristic [11; 12] et. al.

In this work, it is proposed to use the geometrical parameter representing the relation of the inner mapping radius to the external mapping radius of the areas restricted to the contour of plates, membranes or bar cross-sections [13].

During the research, the authors' technique of physical-geometric analogy is used, see references to this technique in [14–16], according to which the integral geometric characteristic of a flat area with a convex contour (the ratio of mapping radii) is analogous to the integral physical characteristics in the considered problems, that is, the laws of change of physical characteristics in these problems are modeled by the corresponding changes in the shape of the area.

2.1. Definition of Terms

"The following quantities are involved with a plane domain $\underline{D}$:

A is the area of $\underline{D}$," …. "r_a is the inner radius of $\underline{D}$ with respect to point *a;* point *a* lies necessarily inside $\underline{D}$. The interior of $\underline{D}$ is mapped conformally onto the interior of a circle, so that point *a* corresponds to the center of the circle and the linear magnification at point *a* is equal to 1; the radius of the obtained circle is r_a. The value of r_a varies with point *a.* $\dot{r}$ is the maximum inner radius." … "$\overline{r}$ is the outer radius of $\underline{D}$. The complementary domain of $\underline{D}$, exterior to $\underline{C}$, is mapped conformally onto the exterior of a circle, so that the points at infinity correspond and the linear magnification at infinity is equal to 1; the radius of the obtained circle is $\overline{r}$." [13, p. 2].

"… $\underline{C}$ a closed plane curve without double points surrounding plane domain $\underline{D}$" [13, p. 1].

2.2. Formulae

The formulas for finding internal $\dot{r}$ and external $\overline{r}$ mapping radii for some singly connected domains, which are considered in the work, take the following form [13; 17]:

- for a circle of radius a

$$\dot{r} = a, \quad \overline{r} = a; \tag{1}$$

- for perfect n-squares

$$\dot{r} = \frac{G\left(1-\frac{1}{n}\right)}{2^{1-\frac{2}{n}} G\left(\frac{1}{2}\right) G\left(\frac{1}{2}-\frac{1}{n}\right)} L, \quad \overline{r} = \frac{G\left(1+\frac{1}{n}\right)}{2^{1+\frac{2}{n}} G\left(\frac{1}{2}\right) G\left(\frac{1}{2}+\frac{1}{n}\right)} L, \tag{2}$$

where n is the number of the sides; L is the perimeter; $G(x)$ is the Gamma-function;

- for arbitrary triangles with angles $\pi\alpha$, $\pi\beta$, $\pi\gamma$:

$$\dot{r} = 4\pi \cdot f(\alpha) f(\beta) f(\gamma) \cdot \rho, \qquad \overline{r} = \frac{A}{\pi \dot{r}}, \tag{3}$$

where

$$f(x) = \frac{1}{G(x)} \left\{ \frac{x^x}{(1-x)^{1-x}} \right\}^{\frac{1}{2}}; \tag{4}$$

ρ is the large radius; A is the square; x is α or β or γ; $G(x)$ is the same as in (2);

- for isosceles triangles with angles $\alpha = \beta$ expressions (3) will take the following form:

$$\dot{r} = 4\pi \cdot f^2(\alpha) f(\gamma) \cdot \rho; \quad \overline{r} = \frac{\mathrm{ctg}\alpha \cdot h^2}{\pi \dot{r}}, \tag{5}$$

where α is the equal base angle; h is the height;

- for rectangular triangles ($\alpha = \pi/2$), it follows from expression (3) that

$$\overline{r} = \frac{\sin 2\alpha \cdot c^2}{4\pi \dot{r}}, \tag{6}$$

where α is the angle in case of hypotenuse; c is the hypotenuse;

- for rhombs with angle $\pi\alpha$

$$\dot{r} = \frac{\pi^{\frac{1}{2}}}{G\left(\frac{\alpha}{2}\right) G\left(\frac{1-\alpha}{2}\right)} L, \quad \overline{r} = \frac{\pi^{\frac{1}{2}}}{8 G\left(1-\frac{\alpha}{2}\right) G\left(\frac{1+\alpha}{2}\right)} L; \tag{7}$$

where $G(x)$ and L is the same as in (2);

- for ellipses with semiaxes a and b ($a \geq b$)

$$\dot{r} = \overline{r} \left\{ \sum_{n=0}^{\infty} q^{n(n+1)} \right\}^{-1} \left\{ 1 + 2\sum_{n=1}^{\infty} q^{n^2} \right\}^{-1}, \quad \overline{r} = \frac{a+b}{2}, \tag{8}$$

where

$$q = \frac{(a-b)^2}{(a+b)^2}; \tag{9}$$

▪ for rectangles with the sides *a* and *b (a* ≥ *b*)

$$\dot{r} = \frac{2}{\pi} b \left(1 + 2\sum_{n=1}^{\infty} q^{n^2}\right)^{-2}, \quad \begin{cases} \dfrac{a}{\overline{r}} = \pi \cos^2 \alpha \displaystyle\sum_{k=0}^{\infty} \frac{\left((2k-1)!!\right)^2}{2^{2k}(k+1)!k!} \cos^{2k} \alpha; \\ \dfrac{b}{\overline{r}} = \pi \sin^2 \alpha \displaystyle\sum_{k=0}^{\infty} \frac{\left((2k-1)!!\right)^2}{2^{2k}(k+1)!k!} \sin^{2k} \alpha, \end{cases} \tag{10}$$

where $q = e^{-\frac{\pi a}{b}}$, α is the argument of complex numbers (circle points which images are rectangle vertices in case of conformal mapping).

For other complex domains, for example, parallelogram, trapezoid and other, mapping radii can be obtained using the Schwarz-Christoffel formula [18] and expanding the mapping function in a Taylor series.

These formulas are also given in the authors' works [14; 15], among other.

2.3. Mathematical Functional Correlation

Mathematical analogy and the functional correlation of mapping radiuses with the characteristics of the stress-strain state, vibrations and stability of structural elements in the shape of plates, membranes, bar cross-sections are defined in earlier works [19].

Since the stress state of a plate under vibrations is considered, then, the resulting relationship for the fundamental frequency of vibration of a plate should be also considered. The fundamental frequency of vibration of a plate ω_0 is related to mapping radii $\dot{r}$ and $\overline{r}$ by expression [19]:

$$\omega_0 \le k \cdot \left(\frac{\dot{r}}{\overline{r}}\right)^{-1} \cdot \frac{\sqrt{D/m}}{A}, \tag{11}$$

where *k* is a numerical constant turning the expression into an equality for round plates, which depends on the type of boundary conditions, *m* is the mass (weight) of the plate with an area of 1 m^2; *A* is the area of the plate; *D* is the flexural rigidity of the plate:

$$D = \frac{Et^3}{12(1-\nu^2)}, \tag{12}$$

where *E* is the modulus of elasticity; *t* is thickness of the plate; ν is the Poisson's ratio.

In case of fixed supports of the plate, *k* = 32.08 in (11).

In case of pinned supports of the plate, the value of *k* can be indicated only for a specific plate material. Since the transition of a regular n-angle plate into a round one with pinned supports leads to the well-known Sapondzhyan — Babuska paradox [20], according to which the value of the vibration frequency will depend on the Poisson's ratio. For such a case of boundary conditions, for comparison, the value of *k* = 17.8 corresponding to a plate in the shape of a regular 16-gon is specified, which in shape is close enough to round.

Expression (11) was obtained using the variational representation of the eigenvalue of the differential equation of free vibrations of the plate and the conformal representation of the inner part of its region when mapped onto a unit circle. And also with the help of many mathematical transformations.

Expression (11) shows that fundamental frequency of vibration of a plate ω_0 is inversely proportional to the ratio of mapping radii $\dot{r}/\overline{r}$, or directly proportional to the ratio of mapping radii $\overline{r}/\dot{r}$. It means that, the change in fundamental frequency of vibration ω_0 for plates of various shapes can be studied by defining the change in the ratio of mapping radii $\dot{r}/\overline{r}$. The ratio of mapping radii $\dot{r}/\overline{r}$ in expression (11) characterizes the geometric shape of the plate in dimensionless form.

2.4. Graphs and Functions

In work [21], instead of numerical constant k (11), correcting polynomial functions of the form $k_\omega = f(\dot{r}/\overline{r})$ were defined for all plates of simple shapes: rectangular plates, isosceles triangular plates, rectangular triangular plates, rhombic plates, regular polygonal plates, elliptical plates. k_ω represents the fundamental frequency of vibration of a plate in general form. Graphs were constructed (Figures 1, 2).

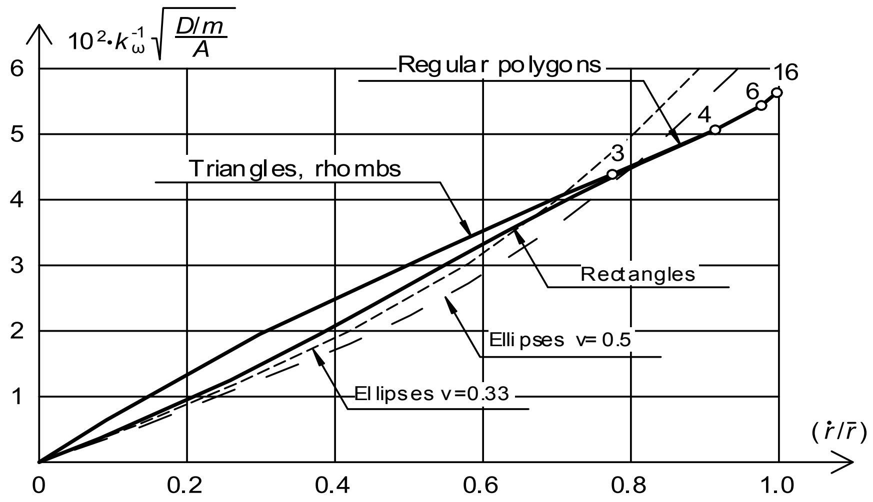

Figure 1. Graphs of the fundamental frequency of vibration of plates with pinned supports along the perimeter, presented depending on the ratio of mapping radii
S o u r c e: made by A. A. Chernyaev

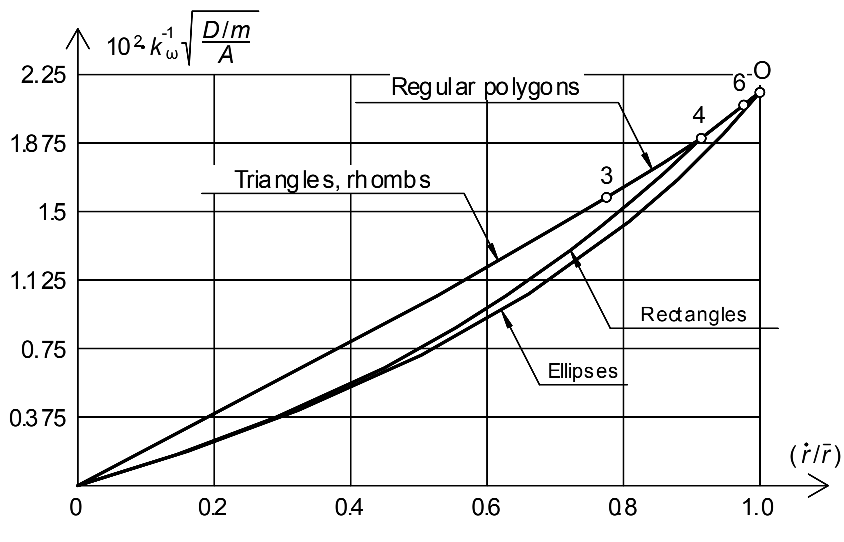

Figure 2. Graphs of the fundamental frequency of vibration of plates with fixed supports along the perimeter, presented depending on the ratio of mapping radii
S o u r c e: made by A. A. Chernyaev

For elliptical plates with pinned supports along the perimeter, the graphs and functions $k_\omega = f(\dot{r}/\overline{r})$ will be different for different values of the Poisson's ratio due to the indicated paradox [20]. Figure 1 shows, for example, graphs for two values of the Poisson's ratio: ν = 0.33, ν = 0.5.

As shown in Figures 1, 2 points 3, 4, 6, 16 correspond to k_ω values for plates in the shapes: regular triangle (point 3), square (points 4), regular hexagon (points 6), regular hexadecagon (points 16), circle (point O).

In works [19; 21] it was proved that the values of k_ω for all plates with a convex contour, for example parallelogram, trapezoid, are inside between these graphs.

2.5. Computational Model and Algorithm

The computational model of the girder-slab structure represents a cross system of bars of the first, second and third levels, which support the slabs (plates). The bars of the first level are called the main girders or spars. The bars of the second level are called secondary beams. The bars of the third level are called floor beams. Beams and girders can be made in the shape of bars of constant or variable cross-section. They can be located in space not only at right angles, but also inclined with a different geometric cell (Figure 3) [14; 15].

The mass (weight) from the plates is transmitted along the perimeter to the floor beams and secondary beams. The plates can be loaded, then the mass (weight) takes this load into account. The mass (weight) from the floor beams is transferred to the secondary girders at the connection points. The mass (weight) from the secondary beams is transferred to the main girders (spars) at the connection points. The girder-slab structure vibrates freely from the mass (weight). The main parameter to determine is the fundamental frequency corresponding to the first mode of vibration. The main factor that affects the fundamental frequency of vibrations is the mass (weight) of the girder-slab structure. If the horizontal outline of the structure is assumed to be constant or specified to be unchanged according to the design assignment, then with different geometric cells (geometric shape of plates) the main vibration frequency will be different (see similar problems, for example [22–24]). Therefore, the main task in the design of girder-slab structures is to determine the geometry of the cells.

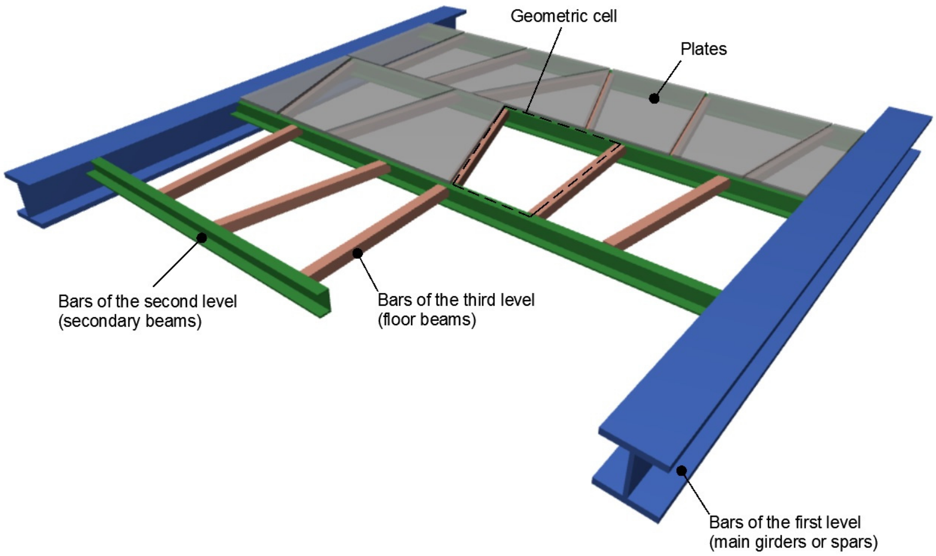

Figure 3. Computational model of the girder-slab structure
S o u r c e: made by A.A. Chernyaev

An algorithm is being developed for determining the geometry of cells of various shapes in the girder-slab system at a given fundamental frequency of vibration of the plates based on the established mathematical and graphical relationship between the fundamental frequency of vibration of the plates and the ratio of the mapping radii of the region bounded by the contour of the plates (Figure 4).

This algorithm uses a custom designed computer program: patent No. 2016615454 (RU) (Figure 5).

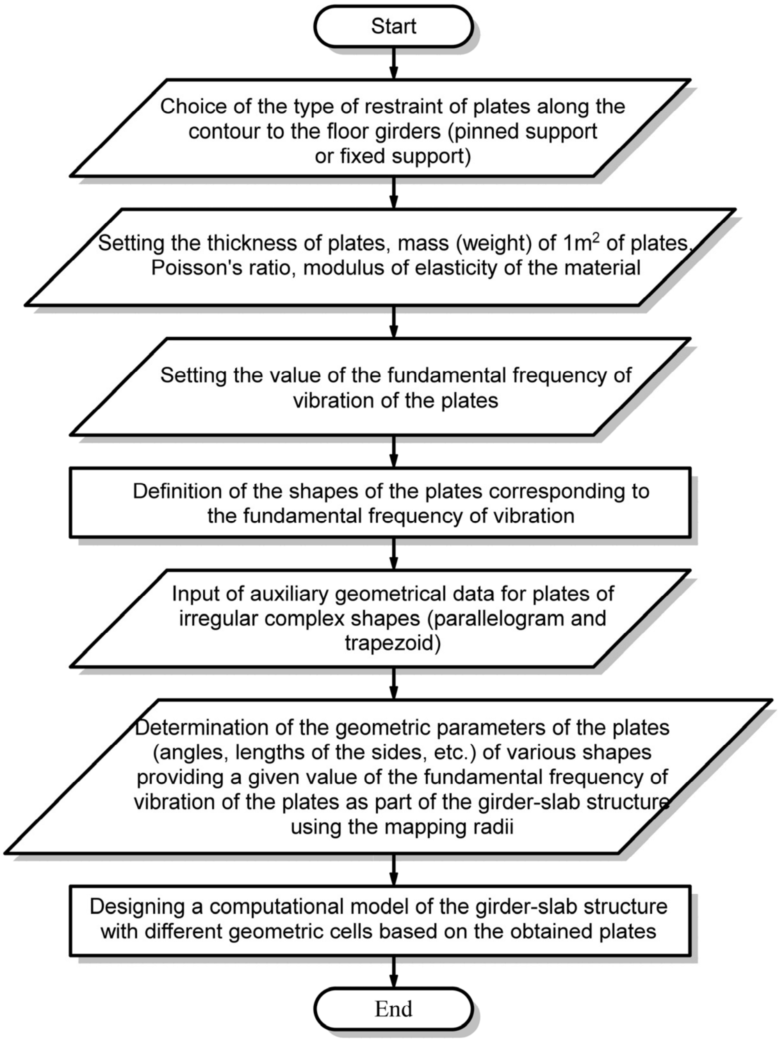

Figure 4. Algorithm for determining the geometry of cells of various shapes in the girder-slab structure at a given fundamental frequency of vibration of the plates

S o u r c e: made by A. A. Chernyaev

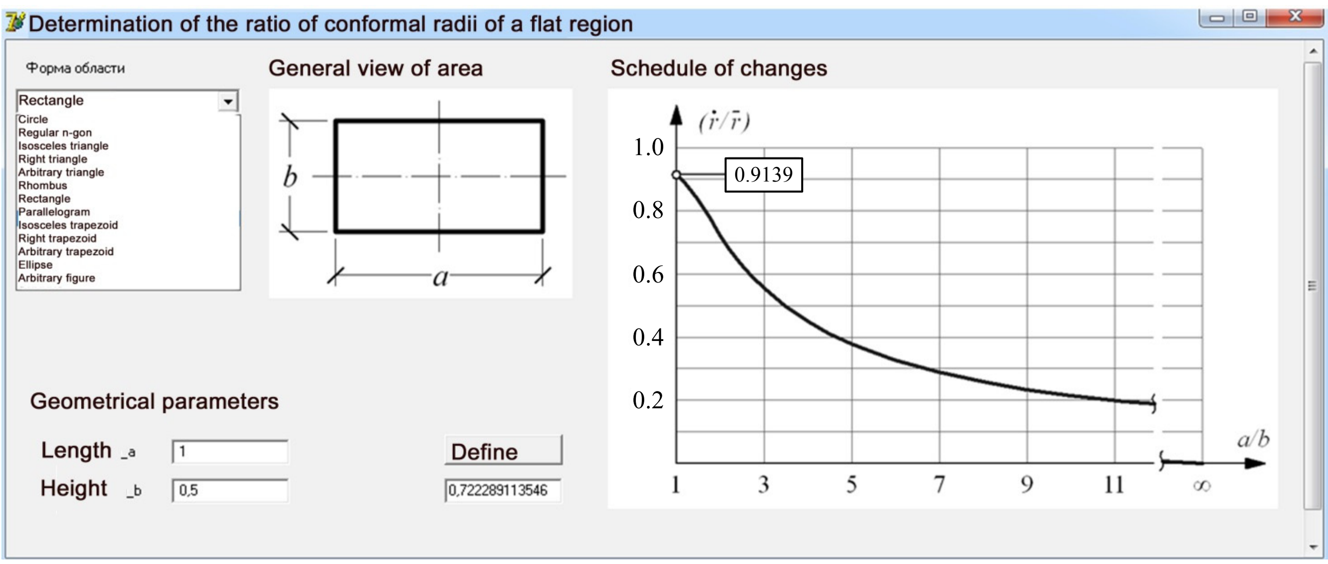

Figure 5. Working window of the computer program: patent № 2016615454 (RU)

S o u r c e: made by A. A. Chernyaev using PrintScreen

3. Results and Discussion

In this section, a numerical example is presented. A cantilever girder-slab structure on trapezoidal plane with sizes: larger base 3 m, smaller base 1.5 m, height 4.5 m, larger inner angle 90^0, is studied. The bars are made of typical sections, plates (flooring) are steel and smooth. The bars of the first level (main girders or spars) are made with a linearly variable section. The bars of the second level are parallel.

The bars of the first level (main girders or spars) with a larger base are fixed. The bars of the second level (secondary beams) are connected to the bars of the first level by means of steel covers using electric welding, such connection is considered to be rigid. The bars of the third level (floor beams) are connected to the bars of the second level in the same way. The steel flooring is welded to flooring beams by means of welding. Strictly speaking, such connection is considered rigid, however in calculations such connection is considered as a hinge for thin plates in safety margin of material.

Initial data. The material is steel, modulus of elasticity $E = 2.1\ 10^5$ MPa, Poisson's ratio $\nu = 0.3$, weight of steel 78.5 kN/m^3. Thickness of the plates (flooring) $t = 1.0$ mm.

The cross section of the first level bars is flange beam in accordance with Figure 6.

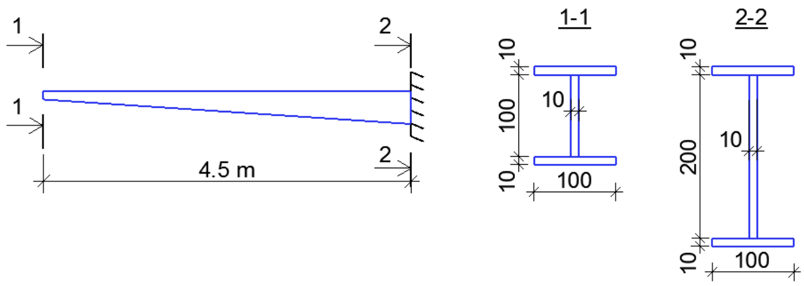

Figure 6. Bars of the first level (main girders or spars)
Source: made by A. A. Chernyaev

The cross section of the second level bars is channel No. 5 in accordance with the interstate standard GOST 8240-97[1].

The cross section of the third level bars is equilateral angle No. 30×3 in accordance with the interstate standard GOST 8509-93[2].

Solution. With the help of the computer program: patent No. 2016615454 (RU)[3] (see Figure 5), the geometric sizes of plates of various shapes are determined. All plates have identical values of the fundamental frequency of vibration

$$\omega_0 = 24.665 \frac{\sqrt{D/m}}{A},$$

absolute value $\omega_0 = 76.34$ 1/sec.

On their basis, girder-slab structures with a different geometric cell are designed. The structures numbered 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14 (see Figure 5) are selected for calculation (Figure 7).

Example result of flexural vibrations of girder-slab structures with geometric cell number 1 (Figure 7) in SCAD Office [25] is shown in Figure 8. Calculation of flexural vibrations of other girder-slab structures is performed similarly in SCAD Office. The results of the flexural vibration analysis of the structures (Figure 7) are shown in Table.

[1] GOST 8240-97. *Hot-rolled steel channels. Assortment*. Minsk, 1997.

[2] GOST 8509-93. *Hot-rolled steel equal-leg angles*. Assortment. Minsk, 1993.

[3] Chernyaev A.A. *Determination of the ratio of conformal radii of a flat region*. Certificate of registration of the computer program RU 2016615454, 05/25/2016. Application No. 2016612918. 2016.

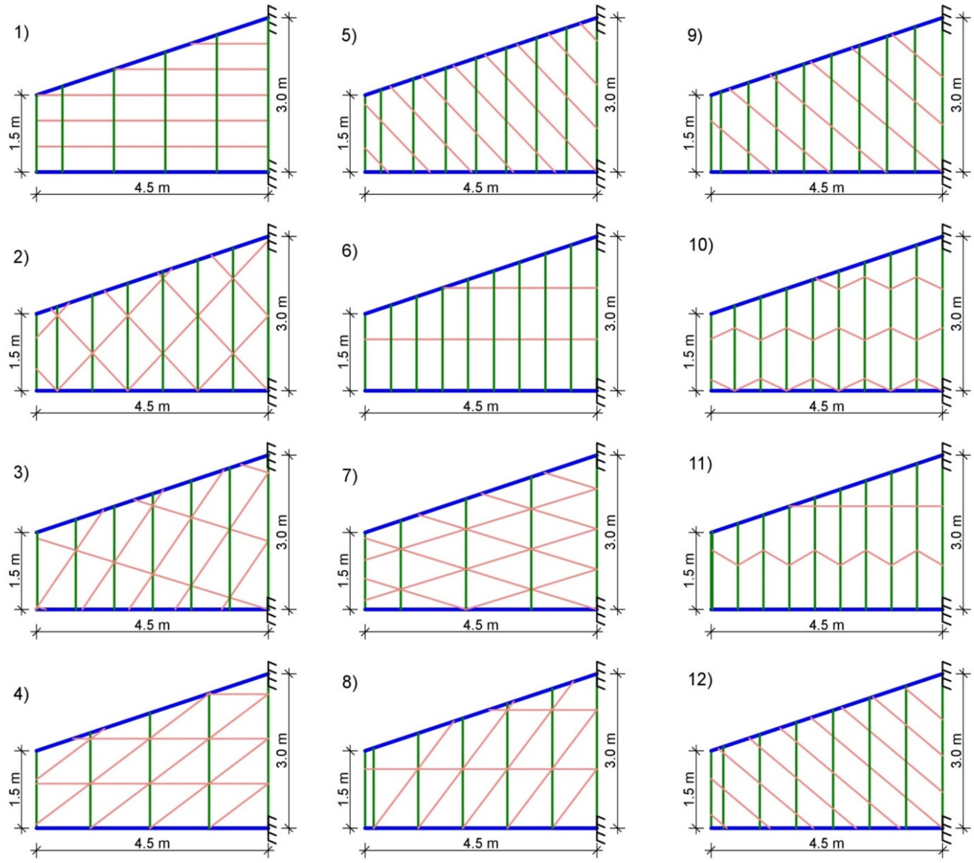

Figure 7. Girder-slab structures with different geometric cell

Source: made by A.A. Chernyaev

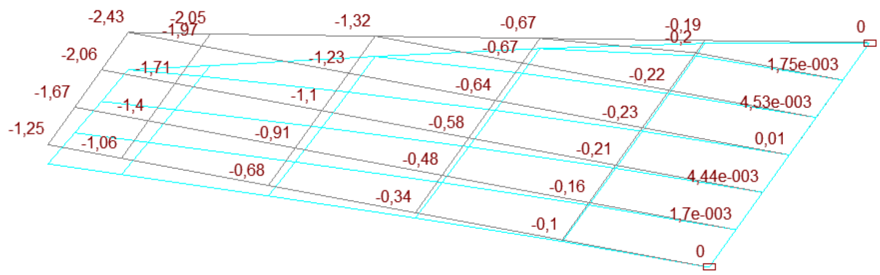

Figure 8. Example result of flexural vibration analysis of the girder-slab structure with geometric cell number 1 (Figure 7)

Source: made by A.A. Chernyaev using SCAD Office

Results of flexural vibration analysis of girder-slab structures with different geometric cell

Geometric cell (Figure 7)	Material consumption, kg	Fundamental frequencyof vibration, 1/sec	Maximum deflection, mm
1	421	37.13	2.43
2	446	36.67	2.42
3	438	37.22	2.35
4	420	37.30	2.28
5	453	36.44	2.46
6	452	37.01	2.41
7	421	36.77	2.33
8	434	36.46	2.40
9	459	36.15	2.50
10	457	36.8	2.44
11	453	36.98	2.42
12	443	36.49	2.42

Source: made by A.A. Chernyaev

4. Conclusions

The conducted research allowed to conclude the following:

1. Cell geometry affects the flexural vibrations of the girder-slab structure. Between the 12 types of cells considered in the work (see Figure 7), the difference in the results was: 9.2% by material consumption, 3.1% by fundamental frequency of vibration, 7.8% by maximum deflection (see Table);

2. The developed technique can be used for variant design and optimization of girder-slab structures.

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