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Torses with Two Curves in Intersecting Planes and with Parallel Axes

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Abstract. Methods for designing torse surfaces with two specified plane directrix curves in intersecting planes, which are projected onto the opposite sides of an arbitrary plane quadrilateral base, are studied and implemented. The other two sides of the base coincide with the two genertrix lines. The theoretical construction techniques are illustrated and visualized using computer graphics for four torse surfaces. Parabolas of the second and fourth orders and a hyperbola are chosen as the directrix curves. The geometric and strength studies on torse surfaces and shells that have been conducted by scientists over the past 10 years are briefly described. They show that interest in their study is not fading, but is moving towards computer modeling and comparative strength calculations using the finite element method. The directions in research of various torses, that are desirable to extend to the torse surfaces proposed for implementation, are illustrated.

Keywords: torse surface, arbitrary quadrilateral base, torse design, coplanarity of three vectors, torse implementation, developable helicoid

Conflicts of interest. The author declares that there is no conflict of interest.

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Торсы с двумя кривыми в пересекающихся плоскостях и с параллельными осями

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Аннотация. Исследованы и реализованы методики проектирования торсовой поверхности с двумя заданными направляющими плоскими кривыми на пересекающихся плоскостях, которые проецируются на противоположные стороны произвольного плоского четырехугольника. Две другие стороны плана совпадают с двумя образующими прямыми. Теоретические построения проиллюстрированы и визуализированы с помощью компьютерной графики на четырех торсовых поверхностях. В качестве направляющих кривых выбраны параболы второго и четвертого порядков и гипербола. Кратко

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описаны геометрические и прочностные исследования по торсовым поверхностям и оболочкам, которые проводились учеными за последние 10 лет. Они показывают, что интерес к их изучению не проходит, но перемещается в сторону компьютерного моделирования и сравнительных расчетов на прочность при помощи метода конечных элементов. Проиллюстрированы направления в исследованиях разнообразных торсов, которые желательно распространить на предлагаемые к внедрению торсовые поверхности.

Ключевые слова: торсовая поверхность, произвольный четырехугольный план, конструирование торсов, компланарность трех векторов, использование торсов, развертывающийся геликоид

Заявление о конфликте интересов. Автор заявляет об отсутствии конфликта интересов.

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1. Introduction

The studies of geometrical problems of torse surfaces with an edge of regression, which were started by G. Monge, still have not stopped. Torse shells are very convenient for covering large arbitrary areas with rectilinear or curved contour sides [1]. In [2], torse surfaces with rectangular base are considered (Figure 1), while in [3] torse surfaces cover an equilateral trapezoidal area (Figure 2). In both cases, plane curves in the form of second-order algebraic curves and superellipses are located at the ends, and the sides of the horizontal base coincide with the rectilinear generatrices of the projected ruled surface of zero Gaussian curvature (torse surface), which will be neither a cone nor a cylinder (Figures 1, 2).

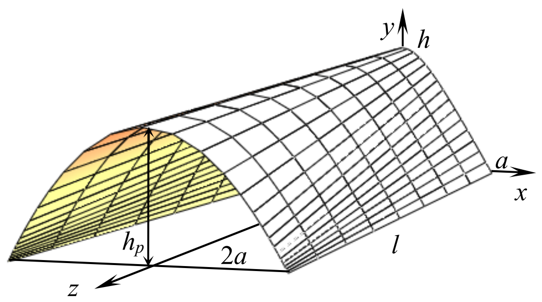


Figure 1. A torse with a parabola and a hyperbola at the ends
S o u r c e: compiled by S.N. Krivoshapko

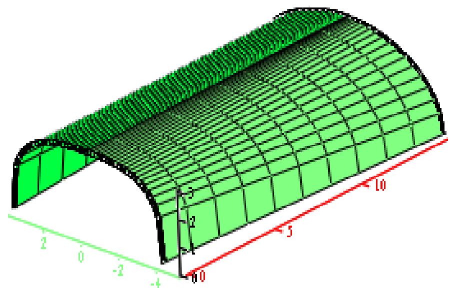


Figure 2. A torse with superellipses at the parallel ends
S o u r c e: compiled by S.N. Krivoshapko

In scientific literature, several techniques for designing analytical surfaces with rectangular and trapezoidal bases and with specified plane curves at the ends are proposed, but these techniques yield cylindrical [4] or conical surfaces, cylindroids [5] or kinematic ruled surfaces [6]. And only in works [2; 3] parametric equations of torse surfaces with specified plane directrix curves in parallel planes and with additional imposed geometric requirements are obtained.

The *aim of study* is to obtain parametric equations of torse surfaces with two plane curves lying in intersecting planes and with parallel axes, i.e., developable surfaces covering an arbitrary quadrilateral area. At present, only one torse surface with two parabolas with parallel axes is known, and the parabolas lie in intersecting planes [7].

2. Methods

2.1. Adopted Methodology for Determining Parametric Equations of Midsurfaces of Torse Shells Covering Arbitrary Quadrilateral Areas

Consider the case when the plane directrix curves lie in intersecting planes. Without restricting the generality of the problem, it is assumed that the first curve lies in the *xOy* plane, and both planes intersect at the *Oy* axis, then the equations of the curves can be written as

$$x_1 = x_1(u), \quad y_1 = y_1(u), \quad z_1 = 0;$$

$$x_2 = x_2(v), \quad y_2 = y_2(v), \quad z_2 = z_2(v). \tag{1}$$

This pair of directrix curves can be expressed in vector form as

$$\boldsymbol{r}_1 = \boldsymbol{r}_1(u) \text{ and } \boldsymbol{r}_2 = \boldsymbol{r}_2(v) \tag{2}$$

with respect to origin *O*, where *u, v* are the corresponding parameters, then the torse equation can be presented in the form of [7]:

$$\boldsymbol{r}(u,\lambda) = \boldsymbol{r}_1(u) + \lambda[\boldsymbol{r}_2(v) - \boldsymbol{r}_1(u)], \tag{3}$$

where λ is a dimensionless parameter, $0 \le \lambda \le 1$. By adopting

$$\boldsymbol{r}_2(v) = \boldsymbol{r}_2[v(u)] = \boldsymbol{R}(u),$$

and by defining $\boldsymbol{m}(u) = \boldsymbol{R}(u) - \boldsymbol{r}_1(u)$, the torse equation (3) can be written as

$$\boldsymbol{r}(u,\lambda) = \boldsymbol{r}_1(u) + \lambda \boldsymbol{m}(u). \tag{4}$$

By defining the torse in the form of (4), coordinate lines λ = 0 and λ = 1 coincide with the directrix curves (1). The relationship between parameters *u* and *v* can be obtained from the condition of coplanarity of three vectors:

$$\left(\boldsymbol{r}_2 - \boldsymbol{r}_1, \boldsymbol{r}_1^{/}, \boldsymbol{r}_2^{/}\right) = 0,$$

which in expanded form is written as

$$y_1(u) - uy_1'(u) = y_2(v) - vy_2'(v), \tag{5}$$

by assuming that $x_1(u) = u$, $x_2(v) = v$, $x_1^{/}(u) = 1$, $x_2^{/}(v) = 1$, $z_2 = v\text{tg}\varphi$, $z_2^{/} = \text{tg}\varphi$, φ is the angle between the intersecting planes.

The meaning of condition (5) is that the correspondence between points P_1 and P_2 (Figure 3) involves the requirement that the tangents to the curves at these points intersect at the line of intersection of the planes containing the given curves [8].

Vector equation (4) can be expressed in parametric form:

$$x = x(u, \lambda) = x_1(u)(1 - \lambda) + \lambda x_2[v(u)],$$

$$y = y(u, \lambda) = y_1(u)(1 - \lambda) + \lambda y_2[v(u)],$$

$$z = z(u,\lambda) = \lambda z_2[v(u)] = \lambda x_2[v(u)]\text{tg}\varphi. \tag{6}$$

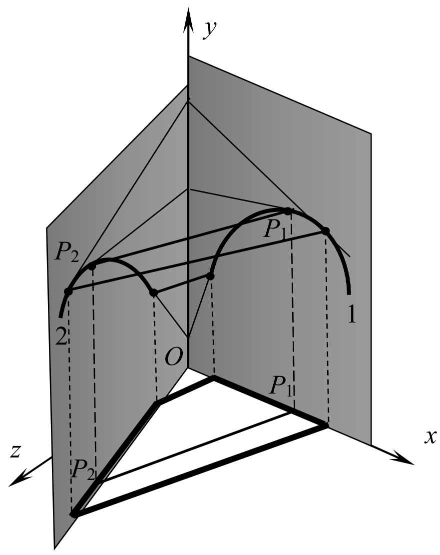


Figure 3. Construction of a torse with curves in intersecting planes
S o u r c e: compiled by S.N. Krivoshapko

2.2. Examples of Determining Parametric Equations of Midsurfaces of Torse Shells Covering Arbitrary Quadrilateral Areas

E x a m p l e 1. Two square parabolas lying in planes intersecting at angle φ are given (Figure 4):

$$x_2 = x_2(v) = v, \quad y_2 = y_2(v) = H\left[1 - (v - c)^2 / \left(b^2\cos^2\varphi\right)\right], \quad z_2 = z_2(v) = v\text{tg}\varphi.$$

$$x_2 = x_2(v) = v, \quad y_2 = y_2(v) = H\left[1 - (v - c)^2 / \left(b^2\cos^2\varphi\right)\right], \quad z_2 = z_2(v) = v\text{tg}\varphi. \tag{7}$$

The relationship between parameters u and v is determined using formula (5):

$$v^2 + \left(b^2\cos^2\varphi - c^2\right) = \frac{hb^2\cos^2\varphi\left(u^2 + a^2 - c^2\right)}{a^2 H},$$

or

$$v = \sqrt{\frac{hb^2\cos^2\varphi\left(u^2 + a^2 - c^2\right)}{a^2 H} - b^2\cos^2\varphi + c^2}\,. \qquad (8)$$

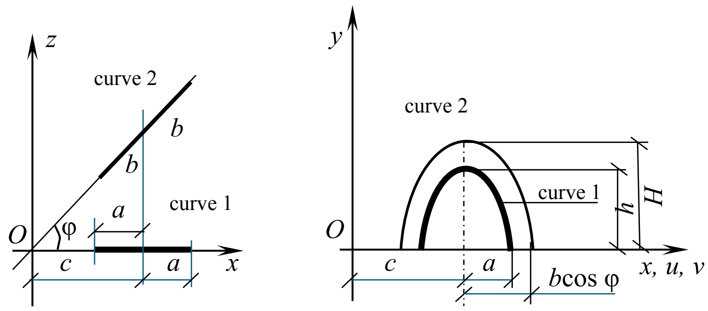
The parametric equations of the sought torse surface (Figure 5) are determined by formulas (6) taking into account relation (8):

$$x = x(u,\lambda) = u(1-\lambda) + \lambda v;$$

$$y = y(u,\lambda) = h\left[1 - (u-c)^2 / a^2\right](1-\lambda) + \lambda H\left[1 - (v-c)^2 / \left(b^2\cos^2\varphi\right)\right];$$

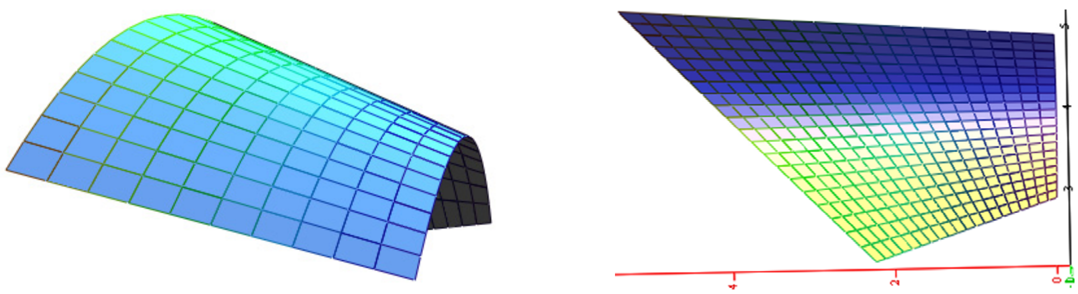
$$z = z(u,\lambda) = \lambda v \operatorname{tg}\varphi. \qquad (9)$$

The following notation (see Figure 4) in adopted in equation (6): h is the height of the first parabola with respect to the xOz plane, H is the same for the second parabola, c is the distance from axis Oz to the vertical axes of the two parabolas, $2b$ is the span of the second parabola in the xOz plane, $2a$ is the span of the first parabola in the xOz plane, $c - a \le u \le c + a$.



view of the curves in the xOz plane　　view of the curves in the xOy plane

Figure 4. Plane directrix curves in intersecting planes
S o u r c e: compiled by S.N. Krivoshapko



a　　*b*

Figure 5. A torse with two parabolas in intersecting planes:
a — general view; *b* — view in the xOz plane
S o u r c e: compiled by S.N. Krivoshapko

Example 2. The two square parabolas, which were considered in the previous example, lie in the planes intersecting at an angle φ, but the vertical axis of the second parabola is located at a distance of $b\cos\varphi$ from the coordinate axis Oz (Figure 6):

$$x_1 = x_1(u) = u, \quad y_1 = y_1(u) = h\left[1-(u-c)^2/a^2\right], \quad z_1 = 0;$$

$$x_2 = x_2(v) = v, \quad y_2 = y_2(v) = H\left[1-(v-b\cos\varphi)^2/\left(b^2\cos^2\varphi\right)\right], \quad z_2 = z_2(v) = v\,\mathrm{tg}\varphi. \tag{10}$$

In this case, relationship (5) between parameters u and v allows to obtain:

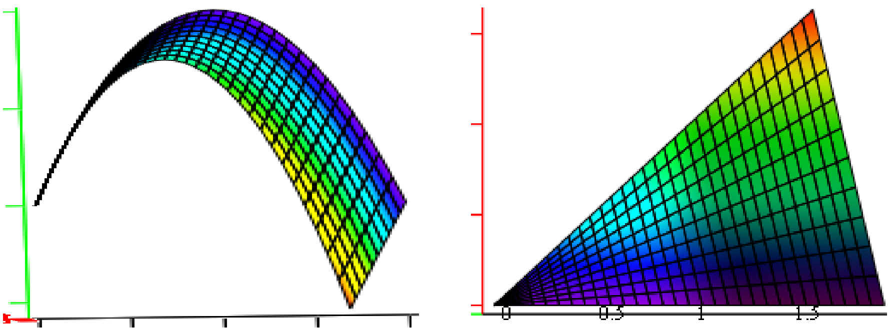
$$v = \frac{b\cos\varphi}{a}\sqrt{\frac{h\left(u^2+a^2-c^2\right)}{H}}, \tag{11}$$

and to write the parametric equations of the torse surface (6) in the form:

$$x = x(u,\lambda) = u(1-\lambda)+\lambda v;$$

$$y = y(u,\lambda) = h\left[1-(u-c)^2/a^2\right](1-\lambda)+\lambda H\left[1-(v-b\cos\varphi)^2/\left(b^2\cos^2\varphi\right)\right];$$

$$z = z(u,\lambda) = \lambda v\,\mathrm{tg}\varphi. \tag{12}$$



a *b*

Figure 6. A torse with two parabolas in intersecting planes, but with their axes at different distance from the *yOz* plane:
a — general view; *b* — view in the *xOz* plane
Source: compiled by S.N. Krivoshapko

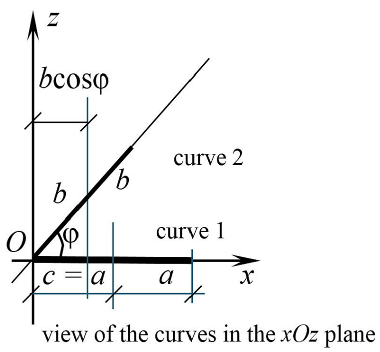


Figure 7. Plane directrix curves in intersecting planes for the 2nd example
Source: compiled by S.N. Krivoshapko

Figure 7 shows the geometrical parameters for example 2, which are used in formulas (10)–(12). For the construction of the surface shown in Figure 6 it is assumed that $a = c = 1$ m, $h = 2$ m, $0 \le u \le 2$ m, $0 \le \lambda \le 1$, $b = 1$ m.

Example 3. If a biquadratic and a quadratic parabola are placed in intersecting planes (see Figure 4), then

$$x_1 = x_1(u) = u, \quad y_1 = y_1(u) = h\left[1-(u-c)^4/a^4\right], \quad z_1 = 0;$$

$$x_2 = x_2(v) = v, \quad y_2 = y_2(v) = H\left[1-(v-c)^2/\left(b^2\cos^2\varphi\right)\right];$$

$$z_2 = z_2(v) = v\,\mathrm{tg}\varphi. \tag{13}$$

Relationship (5) between parameters u and v takes the following form:

$$v=\sqrt{\frac{hb^2\cos^2\varphi\left[\left(u-c\right)^3\left(3u+c\right)+a^4\right]}{a^4H}-b^2\cos^2\varphi+c^2}. \tag{14}$$

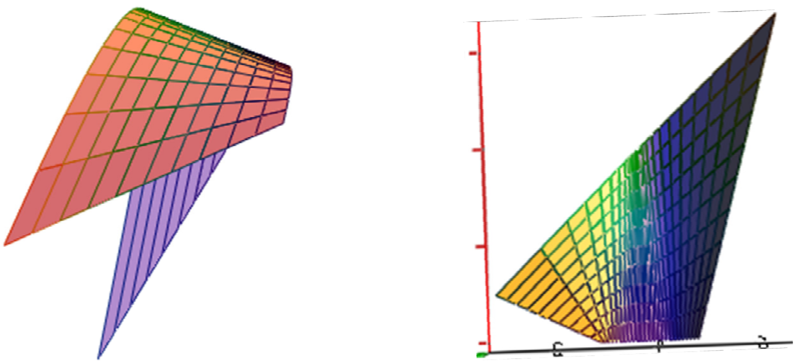
The parametric equations of the sought torse surface (Figure 8) are determined by formulas (6) taking into account relation (14):

$$x=x(u,\lambda)=u(1-\lambda)+\lambda v;$$

$$y=y(u,\lambda)=h\left[1-(u-c)^4/a^4\right](1-\lambda)+\lambda H\left[1-(v-c)^2/\left(b^2\cos^2\varphi\right)\right];$$

$$z=z(u,\lambda)=\lambda v\,\mathrm{tg}\varphi, \tag{15}$$

where $c-a\le u\le c+a$, h is the height of the biquadratic parabola above the xOz plane, H is the same for the second order parabola, c is the distance from axis Oz to the vertical axes of the two parabolas, $2b$ is the span of the quadratic parabola in the xOz plane, $2a$ is the span of the biquadratic parabola in the xOz plane (Figure 4).



a *b*

Figure 8. A torse with the second and the fourth orders parabolas in intersecting planes:
a — general view; *b* — view in the *xOz* plane
Source: compiled by S.N. Krivoshapko

Example 4. Let a hyperbola and a parabola of the second order with axes parallel to the coordinate axis Oy be located in two intersecting planes (Figure 4):

$$x_1=x_1(u)=u,\quad y_1=y_1(u)=d+h-\sqrt{d^2+\frac{h(u-c)^2(2d+h)}{a^2}},\quad z_1=0,$$

$$x_2=x_2(v)=v,\quad y_2=y_2(v)=H\left[1-(v-c)^2/\left(b^2\cos^2\varphi\right)\right],\quad z_2=z_2(v)=v\,\mathrm{tg}\varphi. \tag{16}$$

Relationship (5) between parameters u and v takes the following form:

$$v=\sqrt{\frac{b^2\cos^2\varphi}{H}\left[d+h-H-\frac{d^2-hc(u-c)(2d+h)/a^2}{\sqrt{d^2+\frac{h(u-c)^2(2d+h)}{a^2}}}\right]+c^2}. \tag{17}$$

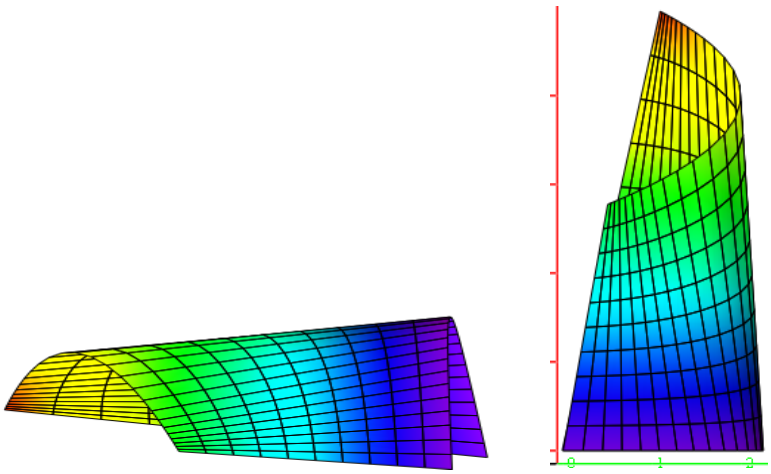
The constant geometric parameters are shown in Figure 4, the constant parameter d associated with the bisector of the hyperbola is chosen arbitrarily as needed, but $d \neq 0$. When constructing the surface shown in Figure 9, it is assumed that $d = 1$ m.

The parametric equations of the sought torse surface (Figure 9) are determined by formulas (6) taking into account relation (17):

$$x = x(u,\lambda) = u(1-\lambda) + \lambda v;$$

$$y = y(u,\lambda) = \left[d + h - \sqrt{d^2 + \frac{h(u-c)^2(2d+h)}{a^2}} \right](1-\lambda) + \lambda H \left[1 - \frac{(v-c)^2}{b^2 \cos^2\varphi} \right].$$

$$z = z(u,\lambda) = \lambda v \operatorname{tg}\varphi. \qquad (18)$$



a *b*

Figure 9. A torse with a parabola and a hyperbola in intersecting planes:
a — general view, *b* — view in the *yOz* plane
S o u r c e: compiled by S.N. Krivoshapko

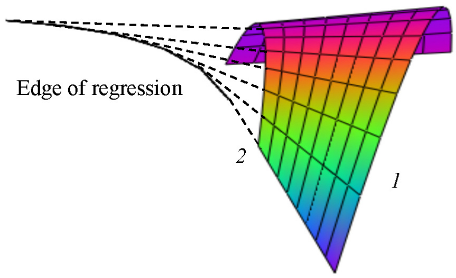


Figure 10. A torse surface with two parabolas in the intersecting planes, formed by tangent lines to its edge of regression
S o u r c e: compiled by S.N. Krivoshapko

E x a m p l e 5. Four examples of finding parametric equations (6) of torse surfaces with specified second-order algebraic curves lying in intersecting planes are presented above. A review of relevant literature on the methods of constructing torse surfaces with curves in intersecting planes has shown that there is only one more publication [7; 9], which studies a torse surface with two parabolas in intersecting planes through the determination of parametric equations of its edge of regression (Figure 10).

This surface is also considered in example 1.

3. Research on Torse Surfaces and Shells Performed in the last 10 Years (2015–2025)

The most complete information on torse surfaces and shells is given in monograph [9] with 386 used sources. This monograph describes the studies on the history of the appearance of the results, geometry, applications, parabolic bending of torse surfaces, and strength analysis of torses. Research performed up to 2009 is described. More recent new research will be summarized in the paper. As the review of the studies related to torse surfaces and thin shells has shown, these studies were mainly progressing in 6 directions.

Application of Torses in Construction and Mechanical Engineering

Architects used them mostly as objects of small architecture or for sculptural compositions [10], but there are examples where they were used to decorate the facades of large civil structures, such as the Luxembourg pavilion at Expo 2020 [1]. In [11] it is recommended to use a fragment of a torse of the same slope as a reinforced concrete canopy with a thickness of 5 cm over the entrance to a public building.

In paper [12] it is proposed to perform segmentation of non-developable surfaces using torse fragments. The authors prove with specific examples that it will reduce the cost of products in the textile industry, in the production of thin-walled metal structures of mechanical engineering, in the construction of lattice shell pavilions. Similar problems of approximation of complex surfaces by torse surfaces in the form of ribbons are solved in [13]. There are proposals for the use of torse surfaces for tent structures [14]. The authors of article [15] state that torse surfaces are of great interest to experts of various branches of human activity, such as architecture, engineering, textile industry, replacement of free-form surfaces by torse surfaces.

Design of Torses with Predetermined Geometric Conditions

To date, 10 methods of designing torse surfaces are known [9], of which the most popular are designing a torse surface using two given plane directrix curves and using a specified edge of regression [16]. The shaping of new torse surfaces containing two specified plane curves can be illustrated by examples of torses on rectangular [2], trapezoidal [3], and arbitrary quadrilateral (Figures 5–9) bases. Torse surfaces can be easily computer modeled [17]. Paper [18] uses a well-known technique to obtain equations of torse surfaces of equal slope with curvilinear coordinates in the lines of curvature. The name of the surface itself indicates that its rectilinear generatrices are inclined to a plane with a directrix curve at the same angle. Researchers are also interested in the method of designing torses with a predetermined line of curvature and a single straight generatrix [19]. Quite a lot of studies are devoted to the construction of developable Bézier surfaces [20]. The method of torse shaping with two plane curves in parallel planes is widely used in shipbuilding for the formation of external hulls [1; 15].

Construction of Torse Surface Developments

Currently, the construction of torse developments onto a plane is not difficult. Having more than a dozen analytical and numerical methods of torse develop-ments [9], geometers have decided that the issue with the construction of torse developments is finally solved. Over the last 10 years, only one paper on the construction of torse developments with parabolas of the second and sixth orders in parallel planes has appeared [21].

Parabolic Bending of Thin Metal Pieces

When bending a thin metal piece into a desired torse, normal stresses emerge in it, unlike a surface that has no thickness and Poisson's ratio. Several papers [22–24] are devoted to the study of this phenomenon. As a result of parabolic bending of a metal sheet, plastic deformations can also occur.

Some researchers propose to consider that bending flat metal sheets into a torse uses minimum energy [15], but it should be known that, practically, a real flat piece made of plywood, aluminum or steel cannot be bent exactly into the designed torse with preservation of straight generatrices due to the presence of Poisson's ratio of the used material [22].

Strength Analysis of Thin Torse Shells

The main works on determining the strength of torse shells defined in non-orthogonal conjugate curvilinear coordinates were published before 2015 [9; 25]. After that, only torse surfaces of equal slope on an elliptic base [18; 26] and developable helicoids [27] were studied.

Investigation of Helical Developable Products and Building Shells

The helical developable surface (torse-helicoid, helical torse, developing helicoid, involute helicoid) is the best known torse surface (Figure 11). This surface was used as early as Archimedes, who created a screw to lift water to a height of 0.5 meters. This surface has always been on the radar of geometers, architects and engineers [25]. The increased introduction of numerical methods and the growing interest in computer modeling has also affected all areas of research on developable helicoids. Five types of linear helicoidal surfaces have been established, of which the torse helicoid is a surface of zero Gaussian curvature and the other four are surfaces of negative Gaussian curvature, with the straight helicoid being the only minimal surface [27]. In [28], the optimal mesh sizes for FEM analysis of straight and developable helicoids are determined.

The analysis of modern software packages for constructing helicoid models and the possibilities of their application in 3D modeling is carried out in article [29]. It is noted that it is not possible to implement the method of parametric modeling in all software packages.

It is possible to list even more scientific articles devoted to helical developable products and building shells, but in this paper the discussion is restricted to the cited sources, since the purpose of the section is to show the interest of researchers in the application of torse surfaces and shells.

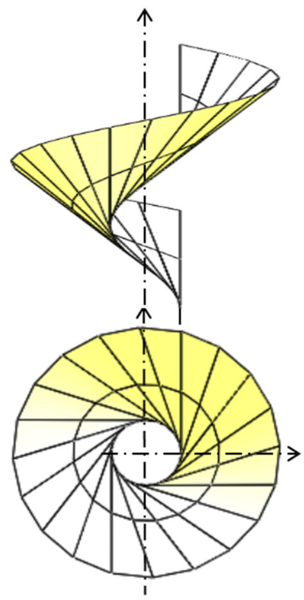


Figure 11. Developable helicoid
S o u r c e: compiled by S.N. Krivoshapko

4. Results

1. The parametric equations (6) of torse surfaces with two plane curves lying in intersecting planes and with parallel axes are obtained. These developable surfaces allow to cover an arbitrary quadrilateral area.

2. It is found that up to the present time, only one torse surface with two parabolas and with parallel axes, defined by analytical formulas, is known, and the parabolas lie in intersecting planes, although the method of constructing the surfaces under consideration was proposed as recently as by G. Monge.

3. Parametric equations of four torse surfaces with two second-order directrix curves lying in intersecting planes are derived. The proposed four torses are visualized using computer graphics.

4. It is shown that the four considered developable surfaces can be applied in construction, mechanical engineering, textile industry, and in the production of thin-walled metal structures.

5. A literature review on the subject over the last 10 years has shown that there are currently no published studies of the stress-strain state of thin shells with the considered torse middle surfaces defined in curvilinear non-orthogonal conjugate coordinates u, λ in the form of (6) with the application of the moment theory of shells.

5. Conclusion

Algebraic second-order surfaces are the most widespread in various fields of human activity. The next most widespread surfaces are surfaces of zero Gaussian curvature (cylindrical, conical and torse surfaces). Next are transfer surfaces, oblique linear surfaces, and tubular surfaces. This short list of the most common analytical surfaces implemented in real products and structures and the given list of used sources shows that torse surfaces have a wide range of application in various branches of economy. With specific examples, the possibility of determining parametric equations of torse surfaces containing two predetermined directrix curves in intersecting planes is shown. Directions in the research of a variety of torses, which are desirable to extend to the torse surfaces proposed for implementation, are illustrated.

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