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АНАЛИТИЧЕСКИЕ И ЧИСЛЕННЫЕ МЕТОДЫ РАСЧЕТА КОНСТРУКЦИЙ
ANALYTICAL AND NUMERICAL METHODS OF STRUCTURAL ANALYSIS

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Natural Frequency Spectrum and Fundamental Frequency Formula for Plane Periodic Lattice Truss

Mikhail N. Kirsanov

National Research University “MPEI”, *Moscow, Russian Federation*
✉ c216@ya.ru

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Abstract. The goal is to determine the free vibration natural frequency spectrum for a plane statically determinate truss with a cross-shaped lattice. The truss members are elastic and have the same stiffness. Both truss supports are pinned; the truss is externally statically indeterminate. A model, in which the mass of the structure is uniformly distributed over its nodes, and their vibrations occur vertically, is considered. The Maxwell-Mohr method is used to determine the stiffness of the truss. The member forces included in the formula are calculated by the method of joints using the standard operators of Maple mathematical software in symbolic form. The eigenvalues of the matrix for trusses with different numbers of panels are determined using the Maple system operators. Spectral constants are found in the overall picture of the frequency distribution constructed for trusses of different orders. A formula for the relationship between the first frequency and the number of panels is derived from the analysis of the series of analytical solutions for trusses of different orders. A simplified version of the Dunkerley method is used for the solution, which gives a more accurate approximation in a simple form. The relationship between the truss deflection under distributed load and the number of panels was found. Spectral constants were found in the frequency spectrum.

Keywords: Dunkerley method, first frequency, deflection, Maxwell-Mohr formula, periodic structure, analytical solution, frequency spectrum

Conflicts of interest. The author declares that there is no conflict of interest.

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Mikhail N. Kirsanov, Doctor of Physical and Mathematical Sciences, Professor of the Department of Robotics, Mechatronics, Dynamics and Strength of Machines, National Research University “MPEI”, 14/1, Krasnokazarmennay St, Moscow, 111250, Russian Federation; eLIBRARY SPIN-code: 8679-6853, ORCID: 0000-0002-8588-3871; e-mail: c216@ya

Спектр собственных частот и формула для основной частоты плоской регулярной фермы решетчатого типа

М.Н. Кирсанов

Национальный исследовательский университет «Московский энергетический институт», *Москва, Российская Федерация*
✉ c216@ya.ru

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Аннотация. Для плоской статически определимой фермы с крестообразной решеткой определяется спектр собственных частот свободных колебаний. Стержни фермы упругие и имеют одинаковую жесткость. Обе опоры фермы неподвижные шарниры, ферма внешне статически неопределима. Рассмотрена модель, в которой масса конструкции равномерно распределена по ее узлам, а их колебания происходят по вертикали. Для определения жесткости фермы применен метод Максвелла — Мора. Усилия в стержнях, входящие в формулу, рассчитывались методом вырезания узлов с применением стандартных операторов системы компьютерной математики Maple в символьной форме. Собственные числа матрицы для ферм с различным числом панелей разыскиваются с помощью операторов системы Maple. В общей картине распределения частот, построенной для ферм различного порядка, обнаружены спектральные константы. Из анализа последовательности аналитических решений для ферм разного порядка выведена формула зависимости первой частоты от числа панелей. Для решения использован упрощенный вариант метода Донкерлея, дающий более точное приближение в простой форме. Найдена зависимость прогиба фермы под действием распределенной нагрузки от числа панелей. В спектре частот обнаружены спектральные константы. Выведена формула зависимости прогиба фермы от числа панелей.

Ключевые слова: метод Донкерлея, первая частота, прогиб, формула Максвелла — Мора, регулярная конструкция, аналитическое решение, спектр частот

Заявление о конфликте интересов. Автор заявляет об отсутствии конфликта интересов.

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1. Introduction

One of the problems of dynamics of engineering structures is the calculation of natural vibration frequencies. Most often, standard programs based on the finite element method are used for this purpose [1; 2]. This allows to obtain results for rather complex systems with many parameters, both geometric, related to the dimensions of the calculated object, and physical, characterizing various material properties of its elements. An alternative method of calculation of natural frequencies was developed mainly after the appearance of mathematical computer software. This method is applicable principally for simplified models of objects. Its main advantage manifests in case of periodic structures, for analytical solution of which the number of periodicity elements in the model does not affect the accuracy and complexity of calculations. R.G. Hutchinson and N.A. Fleck [3; 4] were first engaged in the theory of existence and calculation of periodic statically determinate truss structures. Later, this issue was addressed by F.W. Zok, R.M. Latture and M.R. Begley [5]. A simplified analysis of civil engineering structures using graph theory methods is proposed in [6]. The problem of optimizing the size, layout and topology of truss structures with the use of special algorithms is considered in [7]. In [8], approximate analytical solutions of statics of thin elastic plates in Maple are given. The superposition method for analyzing the stress state of an isotropic rectangle is proposed in [9]. A.S. Manukalo [10] analytically solved the problem of the first frequency of natural vibrations of a plane girder truss. The author's reference books [11; 12] contain analytical solutions of problems on the deflection of plane periodic trusses with an arbitrary number of panels. A simple formula for the lower estimate of natural vibrations of a plane periodic beam truss with a rectilinear upper chord is obtained in [13] by the Dunkerley method in the Maple mathematical software system. Analytical solutions

Кирсанов Михаил Николаевич, доктор физико-математических наук, профессор кафедры робототехники, мехатроники, динамики и прочности машин, Национальный исследовательский университет «МЭИ», Российская Федерация, 111250, г. Москва, ул. Красноказарменная, д. 14, стр. 1; eLIBRARY SPIN-код 8679-6853, ORCID: 0000-0002-8588-3871; e-mail: c216@ya

for spatial truss structures are also known. In [14], an estimate of the fundamental frequency of vibration of an L-shaped spatial truss with an arbitrary number of panels in the cross-girder was obtained by the induction method. Similar problems of statics and dynamics were solved in analytical form for spatial trusses in [15; 16].

In this paper, a new configuration of an externally statically indeterminate lattice truss is considered, and the analytical relationship between the deflection and the first frequency of natural vibration and the mass, dimensions, and number of panels is derived. The obtained formulas are compared with the results of traditional numerical calculations taking into account all degrees of freedom of the structure. In the joint frequency spectrum of a number of trusses of different orders, patterns are determined, which can be used to simplify practical calculations.

2. Structure

A plane truss consists of $2n$ panels of length $2a$ in its middle part and has the total height of $3h$ (Figure 1). A special feature of the truss is the pinned supports on its ends. At the same time, the truss remains statically determinate: the truss contains $\eta = 8n + 20$ members, including four members modelling the supports, and $K = 4n + 10$ internal hinge joints, $\eta = 2K$.

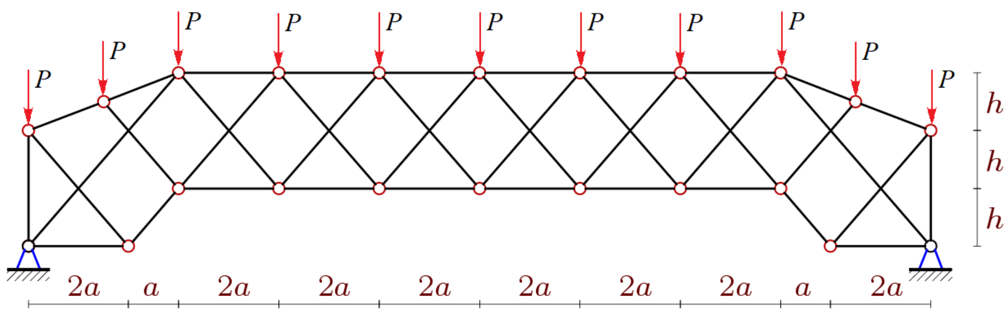

Figure 1. Truss under uniform load, *n* = 3

Source: made by M.N. Kirsanov

The mass of the truss in the problem of determining the natural frequency is uniformly distributed over all internal joints. In the model under consideration, the masses vibrate along the vertical *y*-axis. Under this assumption, the number of degrees of freedom of the truss is equal to the number of nodes *K*. It is assumed that the hinges of the truss are ideal and the material of the members is elastic.

3. Methods

3.1. Calculation of Forces

For the analytical calculation of forces in the truss members, a program written in the Maple symbolic mathematics language is used. The algorithm of this program was previously used in [10–12]. The nodal coordinates of the truss with a span of $L_0 = (4n + 6)a$ are defined using loops. The coordinate origin is located in the left pinned support (Figure 2):

$$x_1 = 0, y_1 = 0, x_2 = 2a, y_2 = 0;$$

$$x_{i+2} = x_{2n+7+i} = a(2i+1), \ y_{i+2} = h, \ y_{2n+7+i} = 3h, \ i = 1, ..., 2n+1;$$

$$x_{2n+4} = L_0 - 2a, \ y_{2n+4} = 0, \ x_{2n+5} = L_0, \ y_{2n+5} = 0;$$

$$x_{2n+6} = 0, \ y_{2n+6} = 2h, \ x_{2n+7} = 3a / 3, \ y_{2n+7} = y_{4n+9} = 5h / 2;$$

$$x_{4n+9} = L_0 - 3a / 2, \ x_{4n+10} = L_0, \ y_{4n+10} = 2h.$$

The order of connecting the members into nodes is specified by special lists containing the end numbers of the corresponding members Φ_i , $i=1, ..., \eta$. The members of the lower chord, for example, are coded by the lists: $\Phi_i=[i,i+1], \quad i=1, ..., 2n+4.$

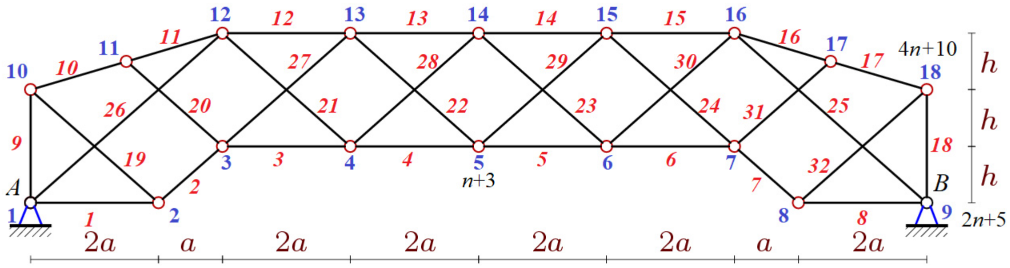

Figure 2. Numbering of nodes and members, n = 2

S o u r c e: made by M.N. Kirsanov

The matrix form is used to write the equations of equilibrium of the nodes projected onto the coordinate axes: $\mathbf{GS}=\mathbf{T}$. Here $\mathbf{G}$ is the matrix of directional cosines of the member forces, $\mathbf{S}$ is the vector of unknown forces and support reactions, $\mathbf{T}$ is the vector of the applied nodal loads. Odd elements T_{2i-1} of this vector contain the load components along the x-axis, even T_{2i} — along the y-axis. The elements of matrix $\mathbf{G}$ are calculated in terms of nodal coordinates and the data of lists Φ_i, $i=1,..,\eta$. Maple uses a relatively fast inverse matrix method to solve the matrix equation in symbolic form: $\mathbf{S}=\mathbf{G}^{-1}\mathbf{T}$.

3.2. Deflection

The deflection due to a uniform nodal load applied on the top chord is calculated using the Maxwell — Mohr formula:

$$\delta=\sum_{j=1}^{\eta} S_j^{(n+3)} S_j^{(P)} l_j \,/\, (EF),$$

where $S_j^{(P)}$ is the force in member j due to load applied at nodes $i=2n+6,..,4n+10$ of the top chord; $S_j^{(n+3)}$ is the force in the same member due to a unit force, which is applied at the middle node of the lower chord, where the deflection is measured. Stiffness EF of all members is assumed to be the same.

The deflection is sought for an arbitrary number of panels in the truss, so first a series of solutions is compiled for individual trusses of different orders:

$$\delta_1=P\frac{53a^3+15c^3+21h^3}{6h^2EF};$$

$$\delta_2=P\frac{3232a^3+438c^3+11d^3+964h^3}{72h^2EF};$$

$$\delta_3=P\frac{220a^3+17c^3+11h^3}{2h^2EF};$$

$$\delta_4=P\frac{20064a^3+1038c^3+19d^3+1364h^3}{72h^2EF};$$

$$\delta_5=P\frac{4036a^3+111c^3+45h^3}{6h^2EF}, ...,$$

where lengths $c=\sqrt{a^2+h^2}$, $d=\sqrt{9a^2+h^2}$ are used. The required number of members of the series to determine its common term in this problem is twelve. Using computer mathematics methods, the common term of the series is obtained:

$$\delta_n = P\frac{C_1a^3+C_2c^3+C_3d^3+C_4h^3}{h^2EF}.$$

The coefficients in this formula are obtained from the solution of homogeneous recurrence equations, which are constructed using the operator `rgf_findrecur` from the `genfunc` package in Maple:

$$C_1=\left(30n^4+4\left(5-4(-1)^n\right)n^3+4\left(2(-1)^n+5\right)n^2+\left(110(-1)^n+218\right)n+105(-1)^n+231\right)/36;$$

$$C_2=\left(12n^2+2\left((-1)^n+13\right)n+9(-1)^n+33\right)/24;$$

$$C_3=(3+4n)\left(1+(-1)^n\right)/144;$$

$$C_4=\left(\left(16(-1)^n+34\right)n+48(-1)^n+93\right)/18.$$

3.3. First Frequency Estimate

Two simple methods for estimating the first natural frequency of vibration are the most popular. They are the Dunkerley method for the lower estimate and the Rayleigh method for the upper one [17; 18]. The Rayleigh estimate is more accurate, however, its analytical expression is generally cumbersome. In [19], a modified version of the Dunkerley method is presented, which gives a simpler and more accurate solution than the original Dunkerley method. According to this method, the following formula is valid for the approximate expression of the first frequency ω_*:

$$\omega_*^{-2}=m\sum_{p=1}^{K}\delta_p=mK\delta^{\max}/2=mK\Delta_n, \tag{1}$$

where by the Maxwell — Mohr formula each term has the form: $\delta_p=m\sum_{\alpha=1}^{\nu}\left(S_\alpha^{(p)}\right)^2 l_\alpha/(EF)$, $S_\alpha^{(p)}$ is the force in member α due to a unit vertical force applied at node *p*, l_α is the length of the corresponding member, *m* is the mass of the truss node. Here, unlike the original Dunkerley method, the sum is computed by the mean value theorem. In this formula: $\delta^{\max}$ is the maximum value of δ_p, which is calculated for a particular node, having the maximum deflection from the individual vertical force applied to the same node. Obviously, for this problem such a node is node $n+3$ at the mid-span of the lower chord. For numerical calculation there is no difficulty in computing the sum in (1), however, summation difficulties arise when seeking an analytical solution. That is why in the simplified method the sum is replaced by its average value. In this case, of course, the property of the lower bound of the first frequency according to the Dunkerley method is lost.

Similar to the deflection calculation, expressions Δ_n for different numbers of truss panels are calculated step by step:

$$\Delta_1=\frac{9a^3+5c^3+h^3}{4h^2EF};$$

$$\Delta_2=\frac{233a^3+66c^3+d^3+41h^3}{36h^2EF};$$

$$\Delta_3=\frac{65a^3+9c^3+h^3}{4h^2EF};$$

$$\Delta_4 = \frac{897a^3 + 102c^3 + d^3 + 41h^3}{36h^2EF};$$

$$\Delta_5 = \frac{233a^3 + 13c^3 + h^3}{4h^2EF}, \dots .$$

By finding the common terms of the coefficient series before the exponentials of dimensions in these series, an expression for coefficient Δ_n is obtained:

$$\Delta_n = \frac{B_1a^3 + B_2c^3 + B_3d^3 + B_4h^3}{h^2EF},$$

where

$$B_1 = \left(12n^3 + 6\left(1 - 2(-1)^n\right)n^2 + 4\left((-1)^n + 7\right)n + 35(-1)^n + 62\right)/36;$$

$$B_2 = \left(12n + (-1)^n + 19\right)/24;$$

$$B_3 = \left((-1)^n + 1\right)/72,\ B_4 = \left(16(-1)^n + 25\right)/36.$$

Finally, according to (1):

$$\omega_* = h\sqrt{\frac{EF}{m(4n+10)\left(B_1a^3 + B_2c^3 + B_3d^3 + B_4h^3\right)}}. \qquad (2)$$

3.4. Natural Frequency Spectrum

For numerical calculation of all frequencies of the truss, the same Maple operators as for the analytical results can be used, by entering their numerical values into the program instead of the symbolic geometric and physical characteristics of the structure. The results of calculations of frequencies of trusses of different orders are presented in Figure 3. The analysis was performed at $E = 2.1 \cdot 10^5$ MPa, $F = 9\,\text{cm}^2$, $m = 200$ kg, $a = 3$ m, $h = 2$ m. Each point on the graph corresponds to the value of frequency plotted on the vertical axis, and in the corresponding place on the horizontal axis the number of this frequency in the spectrum is marked. All frequencies of the same truss are connected by a broken line of individual color. The figure shows the calculations of the spectra of 12 trusses.

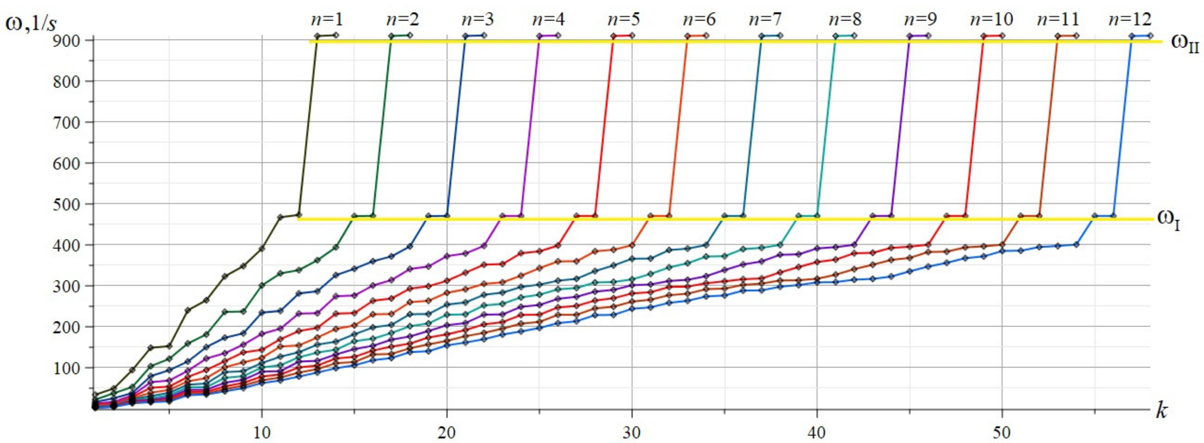

Figure 3. Spectra of trusses of order $n = 1, .., 12$
S o u r c e: made by M.N. Kirsanov

The overall picture of the frequency distribution of trusses of different orders reveals a number of patterns. First, given the dimensions of panel length, truss height, mass and stiffness, the frequencies of vibration are limited from above. Secondly, a jump of frequency values in the upper part of the spectrum is evident, which is the same for trusses of different orders. There are at least two horizontal lines connecting the points and representing almost constant values (with very small errors) of natural frequency for trusses of different orders. These number lines are labeled ω_{I} and ω_{II} respectively, and are the spectral constants of the truss [20; 21]. The practical meaning of these constants is obvious. To calculate this frequency in any truss with a large number of panels, when due to the amount of calculations there are often problems with accuracy and time-consumption, it is possible to use the solution for a similar truss with a small number of panels by adopting in this solution the desired value lying on the same horizontal straight line as the desired solution. For example, the value of the highest frequency for a truss with one panel (n = 1) coincides with great accuracy for a truss with 12 panels, $\omega = 911.08\,\mathrm{s}^{-1}$, the calculation of natural frequencies of which is much more complicated than for a truss with one panel.

4. Results and Discussion

The analytical solution (2) is approximate. It should be compared with the numerical solution obtained in Maple using the Eigenvalues operator from the linear algebra package LinearAlgebra, designed to calculate the eigenvalues of a matrix. For the numerical solution, the algorithm for calculating the forces, the input of node coordinates and the order of connecting the members to the nodes is the same as for the derivation of formula (2). The numerical characteristics of the trusses are taken the same as in the construction of the frequency spectra. Figure 4 shows the curves of the relationship between the first frequency and the number of panels, obtained numerically and analytically using formula (2). The first frequency found numerically (dashed line) is denoted as ω_1, the analytical solution is denoted as ω_*. For small number of panels the error is quite noticeable. Starting from about n = 4, the error decreases. The method used is characterized by the fact that the curve of the analytical solution crosses the numerical solution, nominally assumed as the exact one, several times. It is possible to estimate the error of the method more accurately by introducing relative value $\varepsilon = |\omega_1 - \omega_*|/\omega_1$ (Figure 5). It was obtained that the error decreases with increasing number of panels starting from n = 5. At the same time, the relationship between the error and the truss height h is uncertain: for small n the error decreases or increases when h changes. Starting from n = 5, the error is almost independent of the truss height.

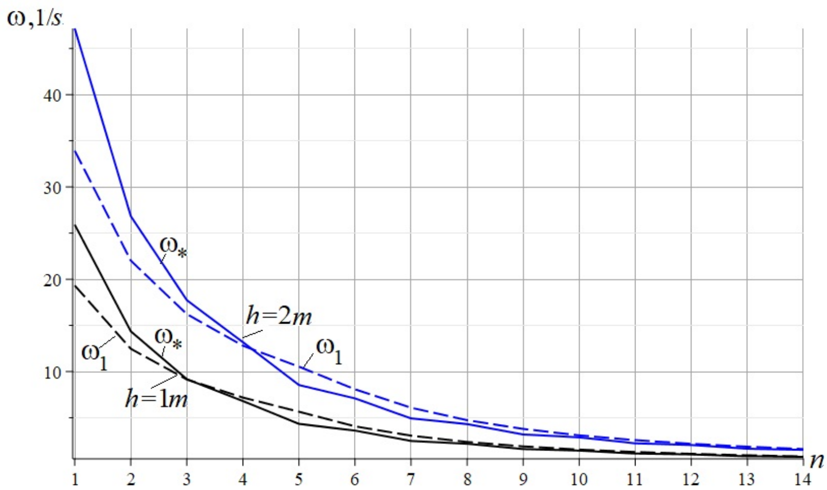

Figure 4. Relationship between the fundamental frequency of vibration and the number of panels

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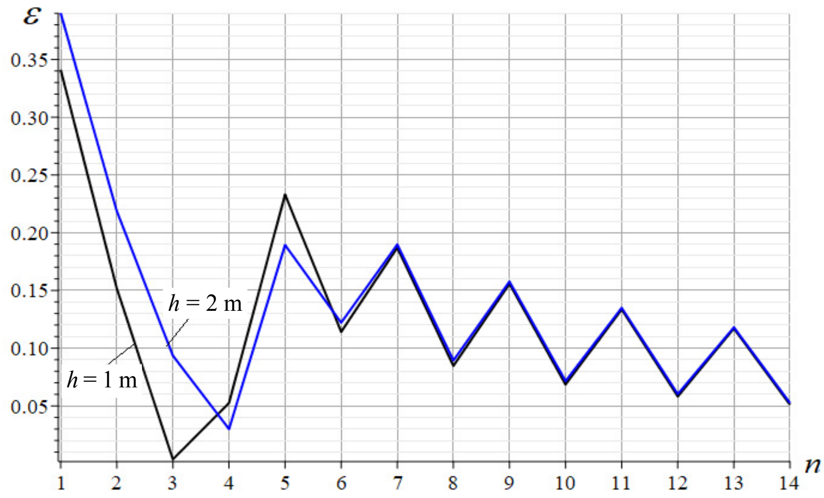

Figure 5. Relationship between the relative error and the number of panels

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5. Conclusion

The considered truss configuration is more complicated than a conventional beam truss due to uncommon supports, more typical for frames and arches: both truss supports are pinned. This complicates the calculation of the structure, since the reactions of the supports cannot be found from the equations of equilibrium of the truss as a whole in the general way. However, in this formulation of the problem, an analytical solution is determined using mathematical computer software, for which it is not difficult to solve the complete system of equilibrium equations of all nodes, including the support ones, both in numerical and symbolic mode. On the basis of these solutions, the formula for the relationship of the fundamental frequency of vibration and the number of panels (this is the main result) is derived in a compact form in this paper. The deflection problem is solved in analytical form and the joint spectrum of all frequencies of natural vibrations of trusses of different orders is numerically constructed. Characteristic features are noticed in the spectra, the use of which in practice can significantly simplify and refine the solution.

The main results are:

1. A model of statically determinate periodic truss is proposed.
2. A formula for the first natural frequency as a function of the number of panels is derived.
3. Comparison of the analytical result with the numerical solution shows their good agreement. The accuracy of the estimates increases with increasing number of panels.
4. Spectral constants are found in the spectrum of the natural frequencies of the truss.

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