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Development of Analytical Method for Cable-stayed Bridges Considering Local Damages Caused by Failure of Supporting Cables

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Abstract. Bridge structures are often subjected to extreme conditions such as rough weather, earthquakes, impacts from traffic accidents, and even blasts. Such extreme loads can cause damage to the anchorage zones as a result of high stress concentration and can lead to cable loss. Such extreme loads can cause dam-age to the anchorage zones as a result of a high-stress concentration and can lead to cable loss. One of the main targets of this study is to develop an analytical method that increases our understanding of the behavior of long-span cable-supported bridges in the case of the failure of one or several cables,through this method, a formula can be deduced to calculate dynamic amplification factor (DAF) more accurately, which could be useful for academic research. In this study, a parallel-load bearing system is considered as a conceptual model of long-span cable-supported bridges. The objective is to investigate the structural robustness of long-span cable-supported bridges in a cable-loss scenario. The conceptual model consists of a beam suspended from cables (tension elements). A simplified model is intentionally selected to make the analytical approach easier. If examining the simplified model shows a certain phenomenon, a similar phenomenon in more sophisticated models can also be expected. The study considers multiple cable failures and employs an analytical approach, developing an approximation function for stress magnification factor in cable break scenarios, using least squares method. The proposed approximation function is accurate and less than 5% error-free in all tested systems, except for minor β values, and increasing β reduces stress magnifica-tion factor. The parameter β influences the calculation of the cable load. For systems with high β values, smaller design loads are necessary, allowing long-span cable-stayed bridges to be segmented into zones with varying β values. This approach enables the determination of minimum design loads for each zone, ultimately reducing cable design costs in cases of cable loss.

Keywords: bridge structures, progressive collapse, cable-stayed bridges, load conditions, analytical method, cable-loss scenario

Conflicts of interest. The authors declare that there is no conflict of interest.

Authors' contribution. *Ahmed A.R.* — data collection, analysis and processing. *Qais A.A.Q.* — concept and development of research topics, data collection, analysis and processing. *Yermoshin N.A.* — concept and scientific guidance

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Разработка аналитического метода для вантовых мостов с учетом локальных повреждений, вызванных обрывом несущих тросов

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Аннотация. Мостовые сооружения часто подвергаются воздействию экстремальных условий, таких как непогода, землетрясения, дорожно-транспортные происшествия, а также взрывы. Экстремальные нагрузки могут привести к повреждению зон крепления в результате высокой концентрации напряжений и могут привести к повреждению стальных тросов. Основная цель исследования — разработка аналитического метода, расширяющего понимание поведения мостов с длинными пролетами на вантовых опорах в случае отказа одного или нескольких вант. С помощью этого метода можно вывести формулу для более точного расчета коэффициента динамического усиления. Система с параллельной нагрузки рассмотрена как концептуальная модель длиннопролетных мостов на вантовых опорах. Также исследована надежность конструкции мостов с длинными пролетами, опирающихся на ванты, в случае потери вантов. Концептуальная модель состоит из балки, подвешенной на тросах (натяжных элементах). Выбрана модель, упрощающая аналитический подход. Если изучение упрощенной модели показывает возможность ожидать аналогичного явления в более сложных моделях. Рассмотрены множественные отклонения кабеля. Использован аналитический подход в разработке функции аппроксимы для коэффициентов увеличения напряжения при обрыве кабеля с использованием метода наименьшего квадрата. Предложенная аппроксимирующая функция является точной и безошибочной менее чем на 5 % во всех протестированных системах, за исключением незначительных значений β, а увеличение β уменьшает коэффициент усиления напряжения. Параметры β влияют на расчёт нагрузки на кабель. Системы высоких значений β требуют меньших проектных нагрузок, которые позволяют сегментировать вантовый мост с большим пролетом на зоны различных значений β. Такой подход дает возможность определить минимальную проектную нагрузку на каждую зону, что в результате снижает затраты на монтаж кабеля при потере кабелей.

Ключевые слова: мостовые конструкции, прогрессирующее обрушение, вантовые мосты, условия нагрузки, аналитический метод, сценарий обрыва кабеля

Заявление о конфликте интересов. Авторы заявляют об отсутствии конфликта интересов.

Вклад авторов. *Ахмед А.Р.*— сбор, анализ, обработка данных. *Кайс К.А.А.* — концепция и разработка емы исследования, сбор, анализ и обработка данных. *Ермошин Н.А.* — концепция и научное руководство

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1. Introduction

Long spans characterize cable-stayed bridges. They are widely used due to their aesthetic typology and economic efficiency. As a result of constant improvements in design and construction technology over the past decades, the number of cable-stayed bridges and the length of their spans have increased rapidly. For example, the Russian Bridge is the longest cable-stayed bridge with a main span of 1.104 meters. At the

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same time, it should be noted that cable-stayed bridges are not highly resistant to destructive factors. This is because the cable-stayed bridge framework has more load-bearing structural elements. Failure of each of them due to the impact of dangerous natural processes (landslides, flooding, avalanches, seismicity, abrasion, cryogenic processes, etc.), as well as the lack of proper maintenance, can cause failure of the bridge. As a result of extreme external loads, progressive collapse may occur, caused by the loss of operability of one or more load-bearing structural elements. The progressive collapse in this context is described as an initial local failure that propagates from element to element until the entire structure or a disproportionately large part collapses [1]. The issue of progressive failure of cable-supported bridges has been studied in some works recently [2–12], the facts and circumstances leading to structural failure have been examined (see Table 1).

Table 1

A detailed analysis references

Sources	Studied	The results
[2]	Applied three earthquake accelerations to the structure and simulated the failure of two critical cables simultaneously	The study indicates that the conditions experienced during the Tabas and Loma Prieta earthquakes could result in progressive collapse, whereas the structure was able to endure the removal of two cables during the Bam earthquake. To prevent such destruction, six base isolations were incorporated beneath the structure. Analyses demonstrate that this strategy can reduce the axial force amplitude below its ultimate strength, thus preventing progressive collapse
[3]	Studied the modeling and analysis of a typical cable-stayed bridge through two important analytical procedures, i.e., nonlinear static and nonlinear dynamic. Furthermore, the response of the structural model is discussed for multiple types of cable loss cases	The study identifies two distinct progressive collapse patterns for nonlinear static and dynamic procedures, particularly in the context of dynamic analysis incorporating a dynamic unloading function. Findings reveal that the likelihood of failure progression in the cable-stayed model diminishes when the failed cables are situated closer to the pylon
[4]	Describe a methodology for performing probabilistic progressive collapse analyses and calibrating incremental analysis criteria for highway bridges accounting for the uncertainties in the applied loads and the load-carrying capacities of the members as well as the system	In the future, such criteria can be used to propose progressive collapse analysis guidelines for bridges that are compatible with the principles of Load and Resistance Factor Design (LRFD) methods
[5]	Discussed the cause of the failure of the Hongqi Bridge and better understood it based on numerical results	The model was used to simulate the bridge collapse caused by demolition, and the domino-type progressive collapse of the bridge was captured. Possible mitigation methods for such progressive collapses of multi-span bridges were proposed
[6]	Studied the development of a practical method for the optimization of cable distance in cable-supported bridges using the robustness index	The results showed that the optimum cable distance fundamentally depends on the assumed number of failed cables. As the cable distance decreases, the construction cost decreases. This cost reduction continues until the cable distance becomes shorter than 5 or 10 m corresponding to each case
[7]	Proposed an empirical equation that allows for the computation of the dynamic amplification factor (DAF) from the maximum norm stress in the static linear elastic analysis of the damaged model with a member removal	A total of 30 illustrative cases for two typical steel truss bridges are investigated to obtain the data points for the empirical equation. The proposed empirical equation is the enveloped line offset from the best-fit line for the data points in illustrative cases
[8]	The failure due to the loss of a cable	Can be prevented by designing the bridge for the loss of cable loads and presenting the corresponding nonlinear dynamic analyses. This procedure should be complemented by protecting the cables from vehicle collisions and malicious action
[9]	Presented a study of the structural answer of cable-stayed bridges when the sudden loss of one of their stays takes place. The analysis has been accomplished through two different methods: utilizing a dynamic analysis and carrying out a procedure, included in the P.T.I. (2000) and S.E.T.R.A. (2001) recommendations	The different results of bending moments in deck and pylons and tension force in stays obtained by these two methods are compared and analyzed
[10; 11]	Studied the dynamic response of cable-stayed bridges to the sudden loss of a stay. Its objectives are to quantify the relative importance of the accidental ultimate limit state of failure of a stay in the design of the bridge and to determine the safety level provided by the simplified procedure of using static analysis with a D.A.F. of 2.0	A static analysis with a D.A.F. equal to 2.0 can be considered as a safe method for evaluating the stress on the stays not only because of the sign of the error but also because of its small magnitude
[12]	Review the facts and circumstances leading up to the failure so that readers will better appreciate the problems that arose from the complex interactions between design, specifications, construction, and quality control	Good engineering design and execution often goes unnoticed; mistakes can result in dire consequences, and the Hyatt skyway collapse is an excellent example of the consequences of a "simple mistake"

Accordingly, the Post-Tensioning Institute (in united states) PTI (2007) guideline recommends to thoroughly investigate the implications of different cable loss scenarios by equivalent static analyses in conjunction with DAF. The typical DAF value for building and bridge structures adopted by existing standards and guidelines is DAF = 2.0, however, for cable-stayed bridges with high degrees of redundancy, applying a constant DAF = 2.0 in conjunction with equivalent static analysis has been questioned. The analysis of the sudden loss of cables in cable-stayed and suspension bridges is very important and has caught the attention of researchers in recent years [13]. Research related with the response of long-span cable-supported bridges to cable failure is infrequent, this study as aim to build up an analogue way by which we can have a good perception about the phenomenon behaviour and derive an expression for DAF more precisely

2. Materials and Methods

2.1. Cable Failure in Mathematical Models of Cable-supported Bridges

Figure 1 shows the conceptual model, which consists of a continuous beam suspended from tension elements (cables).

The parallel load system is a conceptual model for cable-supported, long-span bridges. The aim is to investigate the structural durability of long-span cable-supported bridges in a cable loss scenario. The conceptual model consists of a beam suspended from cables (tension elements). Parallel load-bearing systems are structural systems with load-bearing members that are similar in type and function. These systems are characterized by their ability to configure alternative load paths. Cable-supported bridges, including suspension and cable-stayed bridges, are good examples of such a structural system. In suspension and cable-stayed bridges, the hangers and anchor cables are parallel elements that carry loads, respectively [15].

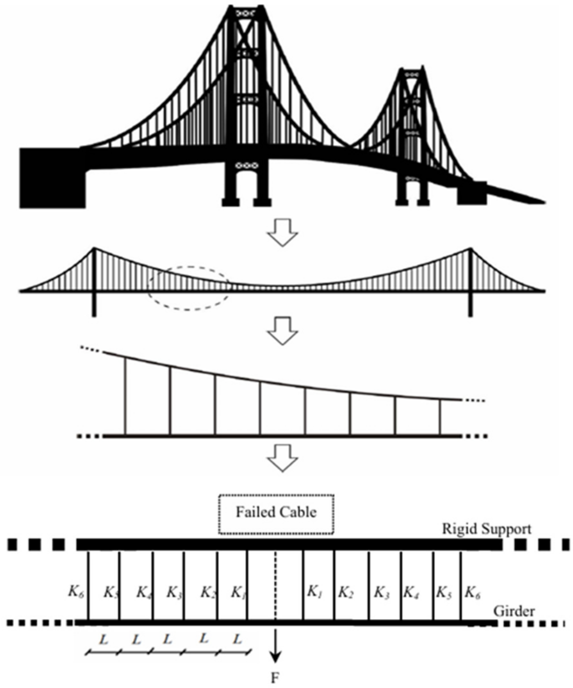


Figure 1. The system of eight cables and its design features
Source: made by M. Haberland et al. [14]

It should be noted that in some cases torsion can be neglected. For example, in single-cable flat systems with box girders or double-cable flat systems with edge girders, the effect of torsion is negligible. In this study, torsion is neglected. It is worth emphasizing that although the main idea comes from a suspension bridge, the simplified model can be used for any parallel load-bearing system, including cable-stayed bridges. It is assumed that all cables have the same axial stiffness, and the stiffness of the girders is the same in all cross sections. The axial stiffness of the cables must be determined considering the entire structural system of a real bridge. The target is to find a general equation for the stress increase ratio of a critical element due to cable failure. Thus, the number of cables may vary. In the first step, it is assumed that only one cable fails, and then an equation is derived for the stress increase ratio of the critical cable. In the second stage, the number of failed cables increases. Finally, an equation is obtained for a system including $2n$ cables in case of failure of m cables. In the simplified model, the distance between two adjacent cables is L, the axial stiffness of the cable is K, and the flexural stiffness of the beam is $K_b = 12EI / L^3$. The failing cable is in the center and the whole system is symmetrical. The load carried by the failing cable is F, and the absorbed load in the critical cable due to cable rupture is F_1, and the corresponding absorbed loads in other cables on either side of the center are F_2 to F_n

(corresponding to K_2 to K_n). The calculated forces in the cables, and therefore the calculated bending moment in the girder, are the increased force in the cable and the increased bending moment due to cable rupture [16; 17].

2.2. Analytical Approach for Determining Coefficient of Stress Increase in Critical Cable due to Cable Loss

Considering the system's symmetry, Figure 1 shows a straight symmetric system that can be solved using boundary conditions and the overlay principle. To further explain the mathematical method, an eight-cable system will be used as an example. The elastic properties of the beam are explained as follows:

$$M(x) = EI\frac{d^2v}{dx^2}, \tag{1}$$

where *EI* is the girder flexural stiffness; *N* m^2, *I* is the girder moment of inertia, m^4, *v* is the vertical displacement, *m*, and *x* is the distance of the section from the left end of the beam, *m*. *M(x)* is the bending moment depending on *x*, arising due to damage to the central cable, *N m*, which can be determined as follows:

$$0 \le x \le L; \qquad M(x) = F_n x; \tag{2}$$

$$L \le x \le 2L; \qquad (x) = F_n x + F_{n-1}(x - L); \tag{3}$$

$$2L \le x \le 3L; \qquad M(x) = F_n x + F_{n-1}(x - L) + F_{n-2}(x - 2L); \tag{4}$$

$$(n-1)L \le x \le nL; \qquad M(x) = F_n x + F_{n-1}(x - L) + F_{n-2}(x - 2L) + \cdots + F_1(x - (n-1)L). \tag{5}$$

The solution to the Eight-Cable Problem:

$$0 \le x \le L; \qquad M(x) = F_4 x = EI\frac{d^2v}{dx^2}. \tag{6}$$

Integrating Equation 6:

$$\int M(x)dx = \int F_4 x dx = F_4\frac{x^2}{2} + C_1 = EI\frac{dv}{dx}. \tag{7}$$

Integrating Equation 7:

$$\iint M(x)dx = \iint F_4 x dx = F_4\frac{x^3}{6} + C_1 x + C_2 = EIv, \tag{8}$$

where C_1 and C_2 are constants of integration and are determined by the boundary conditions of the system. The boundary conditions are the vertical displacements at the locations of the corresponding cables v_i.

Boundary Condition 1: $v_{x=0} = -y_4$.

Boundary Condition 2: $v_{x=0} = -y_3$.

$$C_1 = \frac{EIy_4 - EIy_3 - \dfrac{F_4 L^3}{6}}{L}, \tag{9}$$

$$C_2 = -EIy_4. \tag{10}$$

Repeating this method for the other parts leads to a system of linear equations. The solution to the resulting system of linear equations gives the value of the axial force in each cable.

$$\begin{cases} F_1 + F_2 + F_3 + F_4 = \dfrac{F}{2} \\ 6F_4L^3 + F_3L^3 + 6EIy_4 - 12EIy_3 + 6EIy_2 = 0 \\ 12F_4L^3 + 6F_3L^3 + F_2L^3 + 6EIy_3 - 12EIy_2 + 6EIy_1 = 0 \\ 29F_4L^3 + 20F_3L^3 + 11F_2L^3 + 3F_1L^3 + 6EIy_2 - 6EIy_1 = 0 \end{cases} \tag{11}$$

where F is the external force at the cable fault location and F is the axial force in each cable. Only half of the system is considered since it is symmetrical. The final set of equations for the eight-cable system is then developed.

By defining parameter β as the system stiffness coefficient $\left(\beta = \dfrac{EI}{kL^3}\right)$ and substituting the corresponding value $y_i = \dfrac{F_i}{k}$ the above system of equations can be rewritten as follows:

$$\begin{cases} F_1 + F_2 + F_3 + F_4 = \dfrac{F}{2} \\ F_4(6+6\beta) + F_3(1-12\beta) + F_2(6\beta) = 0 \\ F_4(12) + F_3(6+6\beta) + F_2(1-12\beta) + F_1(6\beta) = 0 \\ F_4(29) + F_3(20) + F_2(11+6\beta) + F_1(3-6\beta) = 0 \end{cases} \tag{12}$$

In order to find a mathematical pattern in the final system of equations, the analytical method was applied to evaluate several tiny systems. If such a mathematical model is found, then the final set of equations for any given large system can be obtained by a simple procedure. In the following analytical example, many systems with different numbers of cables can be used. The goal is to determine the final set of equations for each system.

Eight-cable system:

$$\begin{cases} F_1 + F_2 + F_3 + F_4 + F_5 = \dfrac{F}{2} \\ F_5(6+6\beta) + F_4(1-12\beta) + F_3(6\beta) = 0 \\ F_5(12) + F_4(6+6\beta) + F_3(1-12\beta) + F_2(6\beta) = 0 \\ F_5(18) + F_4(12) + F_3(6+6\beta) + F_2(1-12\beta) + F_1(6\beta) = 0 \\ F_5(38) + F_4(29) + F_3(20) + F_2(11+6\beta) + F_1(3-6\beta) = 0 \end{cases} \tag{13}$$

16-cable system:

$$\begin{cases} F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 = \dfrac{F}{2} \\ F_8(6+6\beta) + F_7(1-12\beta) + F_6(6\beta) = 0 \\ F_8(12) + F_7(6+6\beta) + F_6(1-12\beta) + F_5(6\beta) = 0 \\ F_8(18) + F_7(12) + F_6(6+6\beta) + F_5(1-12\beta) + F_4(6\beta) = 0 \\ F_8(24) + F_7(18) + F_6(12) + F_5(6+6\beta) + F_4(1-12\beta) + F_3(6\beta) = 0 \\ F_8(30) + F_7(24) + F_6(18) + F_5(12) + F_4(6+6\beta) + F_3(1-12\beta) + F_2(6\beta) = 0 \\ F_8(36) + F_7(30) + F_6(24) + F_5(18) + F_4(12) + F_3(6+6\beta) + F_2(1-12\beta) + F_1(6\beta) = 0 \\ F_8(65) + F_7(56) + F_6(47) + F_5(38) + F_4(29) + F_3(20) + F_2(11+6\beta) + F_1(3-6\beta) = 0 \end{cases} \tag{14}$$

By comparing the systems of equations obtained for different structural systems, the mathematical pattern is demonstrated. The equilibrium equation serves as the basis for the final system of equations, the final equation represents the boundary condition at the location of the cable downtime, and the remaining equations can be easily derived from the boundary conditions of other intact cables. The following system of linear equations is created as a general representation of a structural system with an arbitrary number of cables after the mathematical rule is established in the final system of equations:

$$\begin{cases} F_n + F_{n-1} + F_{n-2} + \cdots + F_1 = \dfrac{F}{2} \\ F_n(6+6\beta) + F_{n-1}(1-12\beta) + F_{n-2}(6\beta) = 0 \\ \quad F_n(12) + F_{n-1}(6+6\beta) + F_{n-2}(1-12\beta) + F_{n-3}(6\beta) = 0 \\ F_n(18) + F_{n-1}(12) + F_{n-2}(6+6\beta) + F_{n-3}(1-12\beta) + F_{n-4}(6\beta) = 0 \\ F_n(24) + F_{n-1}(18) + F_{n-2}(12) + F_{n-3}(6+6\beta) + F_{n-4}(1-12\beta) + F_{n-5}(6\beta) = 0 \\ F_n(6n-12)) + F_{n-1}(6(n-1)-12) + \cdots + F_3(6+6\beta) + F_2(1-12\beta) + F_1(6\beta) = 0 \\ F_n(9n-7) + F_{n-1}(9(n-1)-7) + \cdots + F_2(11+6\beta) + F_1(3-6\beta) = 0 \end{cases} \quad (15)$$

It is worth emphasizing that since the system is symmetrical, all the calculations above only consider half of the system.

Figure 2 shows the critical cable stress magnification factor, also called the relative force magnification, for different systems.

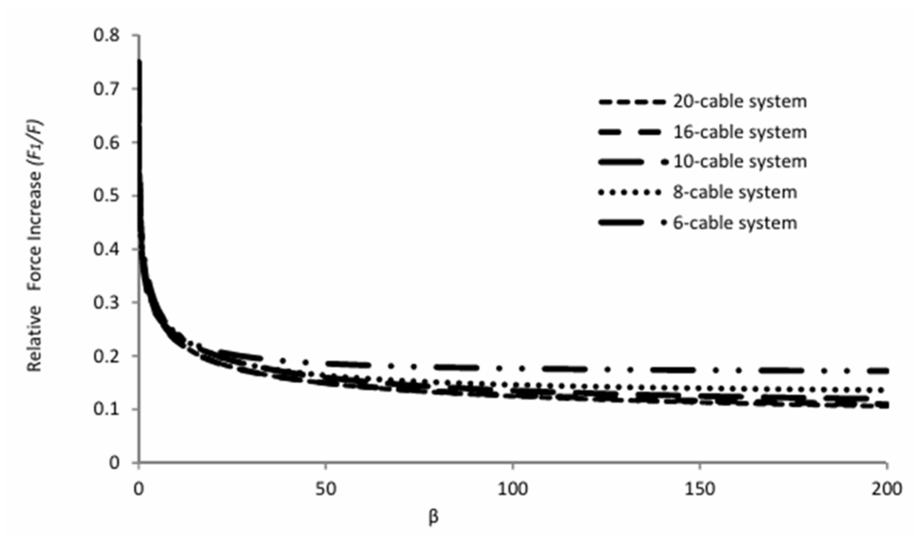


Figure 2. Stress magnification factor in the main cables for different systems
S o u r c e: made by R.A. Ahmed, N.A. Yermoshin

The stress magnification factor in the main cable decreases as β increases, as shown in Figure 2. Next, we take the resulting system of linear equations and determine the general solution. On the main cable *F1*, the goal is to calculate the stress magnification factor as a function of β.

The system analysis method is used to gradually solve this system of equations. The basic method is the same as in the previous step. The idea is to identify a mathematical pattern by solving a system of linear equations for several small systems. If this pattern is found, it can be applied to any system. The stress magnification factors in the critical cable, which depend on β, were calculated for many systems, and the results are shown below.

Eight-cable system:

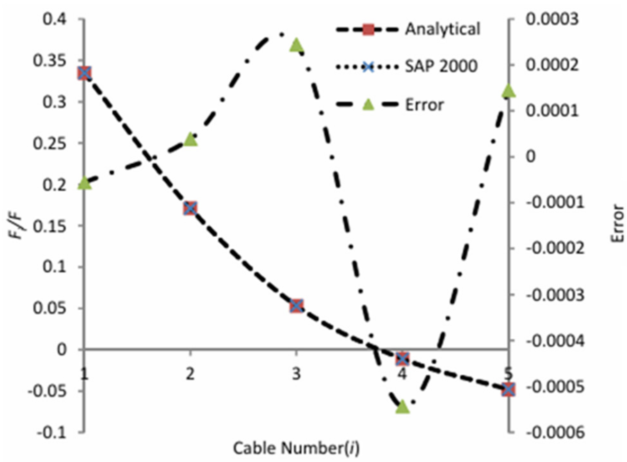
$$\frac{F_1}{F}=\frac{173+3540\beta+6264\beta^2+216\beta^3}{232+5976\beta+18288\beta^2+1728\beta^3}, \tag{16}$$

10-cable system:

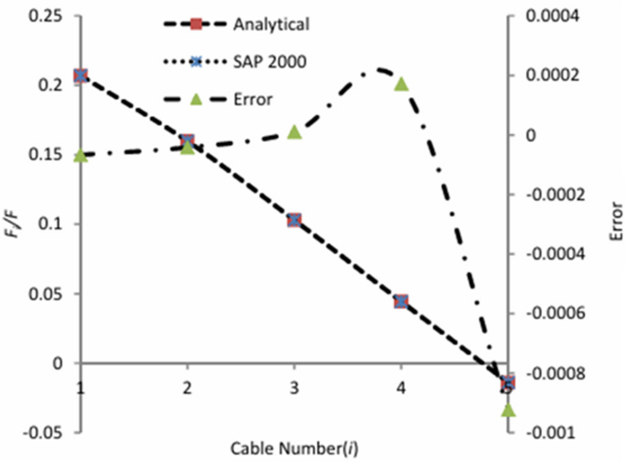
$$\frac{F_1}{F}=\frac{323+14100\beta+168021\beta^2+649836\beta^3+399168\beta^4+5832\beta^5}{433+21237\beta+294408\beta^2+1407996\beta^3+1472256\beta^4+58320\beta^5}. \tag{17}$$

It is important to note that the stress magnification factor for any cable can be determined taking into account the methodology used. However, in this case, only the stress magnification factor in the main cable is significant.

The analytical and numerical results for the 10-cable system are compared using the SAP2000 software package, confirming the accuracy of the analytical solution. The differences between the two solutions are minimal due to the geometric simplicity of the model and the use of linear static analysis (Figure 3).



a



b

Figure 3. Comparison of analytical and numerical solutions for a 10-cable system:
a — β = 1.83; *b* — β = 18.3

S o u r c e: made by R.A. Ahmed, N.A. Yermoshin

The previous equations (16 and 17) show the general form:

$$\frac{F_1}{F}=\frac{a'+b'\beta+c'\beta^2+d'\beta^3+\cdots}{a''+b''\beta+c''\beta^2+d''\beta^3+\cdots}. \tag{18}$$

The objective is to find a mathematical method to determine the critical coefficient of increase in cable stress for each specific system.

The relevant parameters (*a, b, c*, and *d*) are known. It was possible to reduce the total number of unknown coefficients to four using this method. The value *a* indicates the coefficient for the minimum stress increase in a system with 2*n* cables, which occurs at β = ∞ and is equal to 1/2*n*, while the coefficient for the maximum stress increase is indicated by the parameter *b*, which occurs at β = 0. From equations 16 and 17, it can be concluded that for any system with 2*n* ≥ 6, the maximum stress increase is determined at a ratio $\frac{F_1}{F}$ close to 0.75.

Therefore, the general form of the approximation function will be:

$$\frac{F_1}{F}=\frac{1}{2n}+\frac{\frac{3}{4}-\frac{1}{2n}}{1+\left(\frac{\beta}{c}\right)^d}. \quad (19)$$

The line that best fits the collected data is determined using linear regression. LSM, or theleast squares method, was used in the study. A value is considered an estimate of the unknown parameters (parameters *c* and *d*) if it minimizes the sum of squares between the exact and approximate values, in this case, function *T*, according to the LSM technique. Equations 26 and 27, which derive *T* equal to zero in terms of parameters *c* and *d*, achieve this. The following equations show the calculation process applied to a data set with *x*-match points (y_i и f_i):

$$f_i=a+\frac{b-a}{1+\left(\frac{\beta}{c}\right)^d}\ ; \quad (20)$$

$$\Delta_i=y_i-f_i=y_i-\left(a+\frac{b-a}{1+\left(\frac{\beta}{c}\right)^d}\right)\ ; \quad (21)$$

$$\Delta_i^2=\left(y_i-a\right)^2+\frac{(b-a)^2}{1+\left(\frac{\beta}{c}\right)^{2d}+2\left(\frac{\beta}{c}\right)^d}-\frac{2\left(b-a\right)\left(y_i-a\right)}{1+\left(\frac{\beta}{c}\right)^d}; \quad (22)$$

$$\frac{\partial\left(\Delta_i^2\right)}{\partial d}=\frac{-(b-a)^2\left(2\left(\frac{\beta}{c}\right)^{2d}\mathrm{Ln}\left(\frac{\beta}{c}\right)+2\left(\frac{\beta}{c}\right)^d\mathrm{Ln}\left(\frac{\beta}{c}\right)\right)}{\left(1+\left(\frac{\beta}{c}\right)^{2d}+2\left(\frac{\beta}{c}\right)^d\right)^2}-\frac{-2\left(b-a\right)\left(y_i-a\right)\left(\frac{\beta}{c}\right)^d\mathrm{Ln}\left(\frac{\beta}{c}\right)}{\left(1+\left(\frac{\beta}{c}\right)^d\right)^2}, \quad (23)$$

$$\frac{\partial\left(\Delta_i^2\right)}{\partial c}=\frac{(b-a)^2\left(2d\,\beta^{2d}c^{-2d-1}+2d\,\beta^d c^{-d-1}\right)}{\left(1+\left(\frac{\beta}{c}\right)^{2d}+2\left(\frac{\beta}{c}\right)^d\right)^2}-\frac{2\left(b-a\right)\left(y_i-a\right)d\,\beta^d c^{-d-1}}{\left(1+\left(\frac{\beta}{c}\right)^d\right)^2}; \quad (24)$$

$$T=\sum\nolimits_{i-1}^{x}\Delta_i^2\ , \quad (25)$$

$$\frac{\partial T}{\partial d}=0\ ; \quad (26)$$

$$\frac{\partial T}{\partial c}=0, \quad (27)$$

where y_i and f_i are the actual and calculated values of stress magnification factor F_1 for different values of β.

The previous equations were solved iteratively. For equation 26, multiple values of the parameter *c* and their associated parameter values are determined.

The values of parameter c are determined by equation 27 using the values of parameter *d*. A single set of parameters *c* and *d* can only fit both equations according to the systems under consideration. The calculations of parameters *c* and *d* for 10- and 20-cable systems are shown in Figure 4. The function approximation parameters for the different systems are shown in Table 2 after splitting into a single table. *c* and *d* for several systems are shown in Figure 5. When *c* is one, it means that *c* is somewhat elevated. This will be verified in the next section.

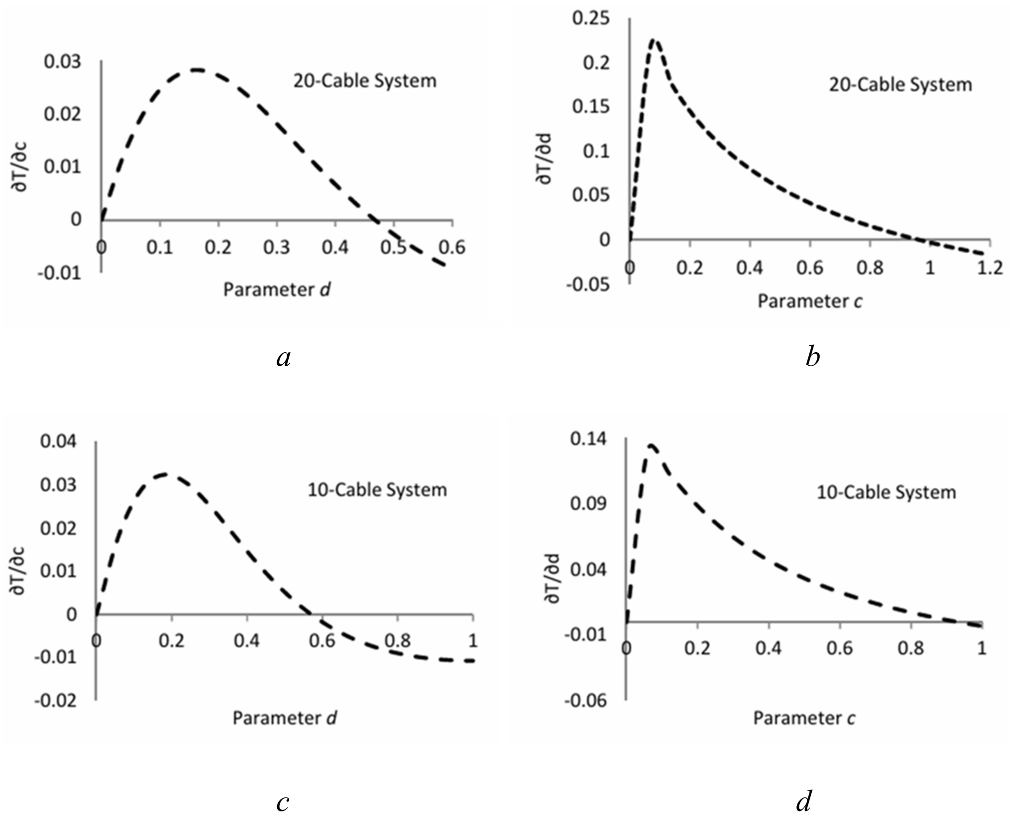
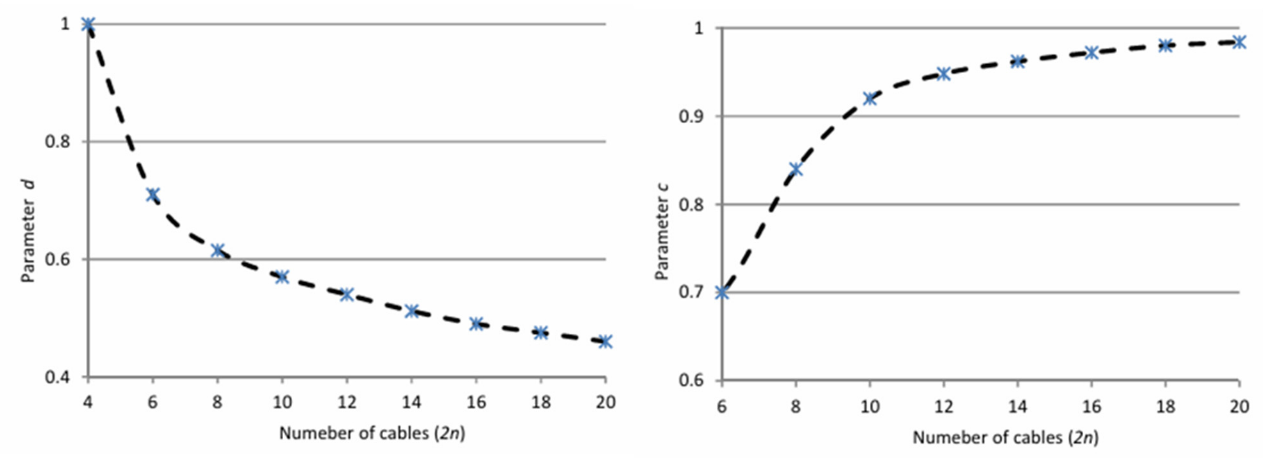


Figure 4. Values of *c* and *d* for systems with 10 and 20 cables calculated:
a — the calculation of coefficient *d* for Cable 20; *b* — the calculation of coefficient *c* for Cable 20; *c* — the calculation of coefficient *d* for Cable 10; *d* — the calculation of coefficient *c* for Cable 10
S o u r c e: made by R.A. Ahmed, N.A. Yermoshin

Table 2

Calculated parameters of the approximation function — one failed cable

Cable number	*a*	*b*	*c*	*d*
4-cable system	0.250	0.69	0.666	1.000
6-cable system	0.167	0.75	0.700	0.710
8-cable system	0.125	0.75	0.840	0.620
10-cable system	0.100	0.75	0.920	0.580
12-cable system	0.083	0.75	0.948	0.540
14-cable system	0.071	0.75	0.962	0.510
16-cable system	0.063	0.75	0.972	0.490
18-cable system	0.056	0.75	0.980	0.475
20-cable system	0.050	0.75	0.985	0.460



a *b*

Figure 5. Parameters c and d for several systems are shown:
a — the coefficient d for a number of cables $2n$; *b* — the coefficient c for a number of cables $2n$
Source: made by R.A. Ahmed, N.A. Yermoshin

The LSM method is used to obtain the equation for parameter d. For large systems, we only consider systems with more than 12 cables to simplify the equation and improve its accuracy. The following equation can be used to represent parameter d.

$$d = 0.35 + \frac{0.65}{1 + \left(\frac{2n}{5}\right)^{1.1}} \qquad 2n \geq 12. \tag{28}$$

For large values of n, parameter d is 0.35 according to the above equation. There is no need to repeat the mathematical calculations used in equations (20)–(27). Figure 6 shows how the values obtained by the LSM method confirm the correctness of equation 28. It is obvious that the proposed equation has a maximum error of less than 1% and can accurately describe the value of parameter d.

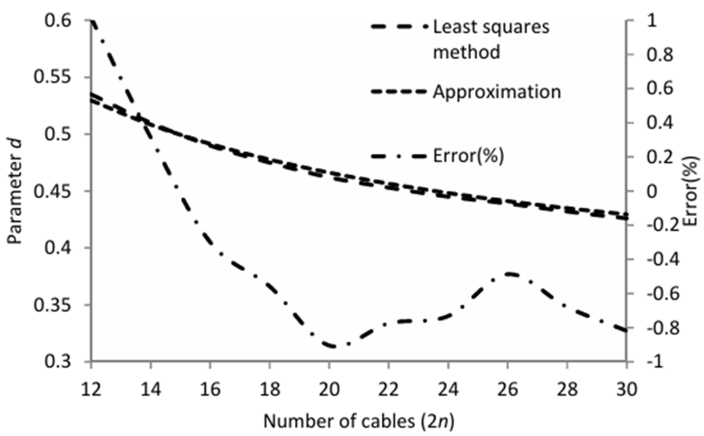


Figure 6. Comparison of the calculation of parameter d by two methods
Source: made by R.A. Ahmed, N.A. Yermoshin

Given the results mentioned earlier, the approximation function could be rewritten for the general system as follows:

$$\frac{F_1}{F} = \frac{1}{2n} + \frac{\frac{3}{4} - \frac{1}{2n}}{1 + \left(\frac{\beta}{c}\right)^d} \delta_i = 1(i = 1 \text{ to } n). \tag{29}$$

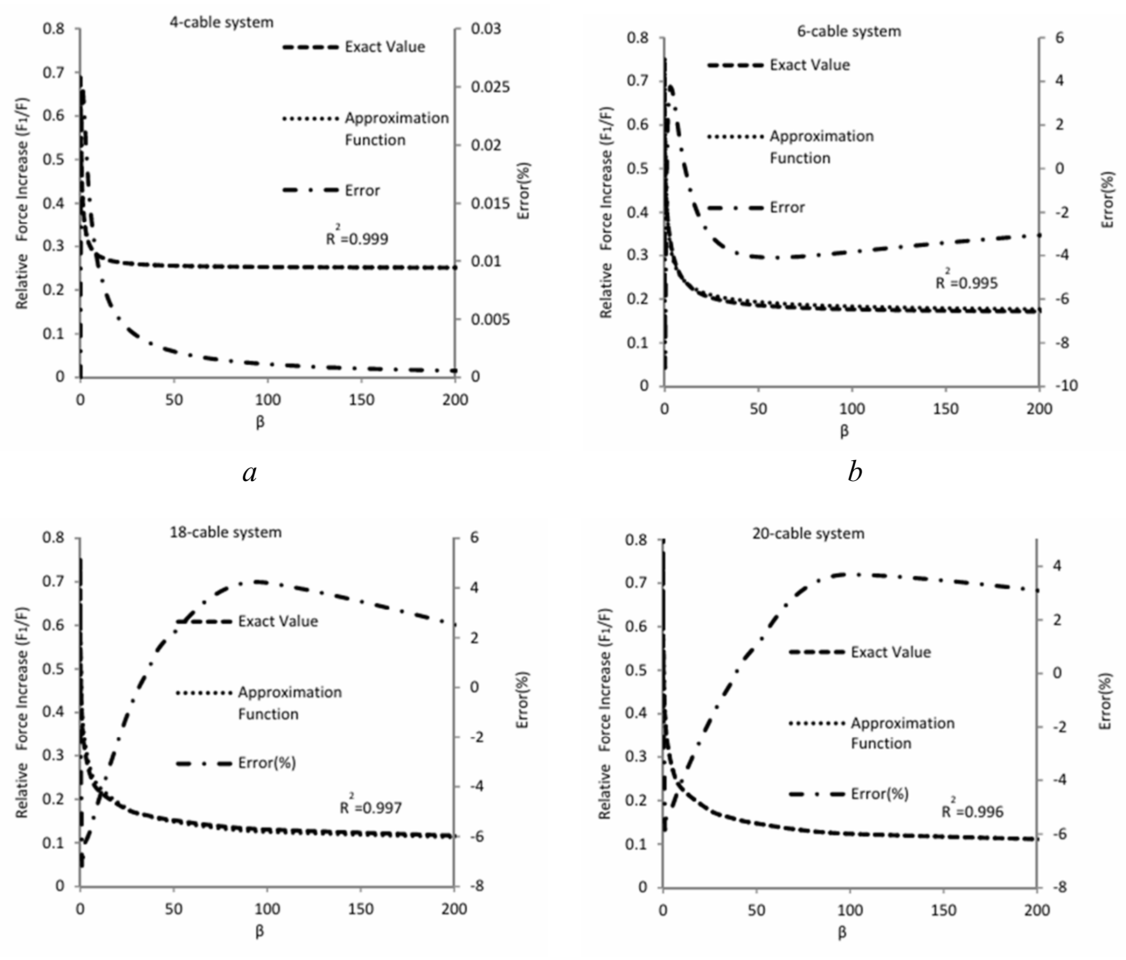
And for larger values of n :

$$\frac{F_1}{F} \cong \frac{\frac{3}{4}}{1 + \beta^d}, \tag{30}$$

where parameter d is to be calculated using equation 28.

Figure 7 shows the exact stress increase factor curves for each system, as well as the curves obtained from the approximation function. It is evident that the curves of the approximation function accurately describe the exact values of the stress increase factor. The approximation error is less than 5% even for small values of β.

It is important to note that the maximum stress increase factor is greater than 0.50 with a value of 0.75. The ratio of the variance in the model that is explained by the approximation function to the total variance is called the *R*-squared parameter (R^2), which is sometimes called the coefficient of determination (Figure 7).



a *b*

c *d*

Figure 7. Exact and approximate stress increase factors in various systems — single failed cable:
a — the stress increase ratios for cable 4; *b* — the stress increase ratios for cable 6;
c — the stress increase ratios for cable 18; *d* — the stress increase ratios for cable 20
S o u r c e: made by R.A. Ahmed, N.A. Yermoshin

The accuracy of the fitting function is assessed using the R-squared, a measure of statistical significance. It indicates how well the regression method will fit the actual data points. For a perfect fit, the R-squared is equal to one, while for a poor fit, it approaches zero. The R-squared for a data set with a matched point x (y_i and f_i) is calculated as follows:

$$\overline{y} = \frac{1}{x}\sum_{i=1}^{x} y_i \,, \tag{31}$$

$$SS_{\text{tot}} = \sum_{i=1}^{x}\left(y_i - \overline{y}\right)^2 , \tag{32}$$

$$SS_{\text{res}} = \sum_{i=1}^{x}\left(y_i - f_i\right)^2 , \tag{33}$$

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}} , \tag{34}$$

where y_i and f_i are exact and approximate values, respectively. Table 3 provides a summary of R-squared calculations for various systems.

Equation (34) is an approximation function for the critical cable stress increase factor after a single cable failure.

Table 3

***R*-squared calculations for various systems**

Cable number	***R*-squared** (R^2)	$\sum_{i=1}^{x} y_i$	$\overline{y}$	SS_{tot}	SS_{res}
4-cable system	0.999	7.96	0.419	0.546	3.47E-07
6-cable system	0.994	7.30	0.386	0.76	0.0059
8-cable system	0.989	6.95	0.39	0.817	0.015
10-cable system	0.990	6.83	0.361	0.861	0.013
12-cable system	0.995	6.30	0.35	0.799	0.009
14-cable system	0.996	6.72	0.355	0.908	0.0051
16-cable system	0.998	6.69	0.38	0.921	0.0049
18-cable system	0.998	6.65	0.352	0.929	0.0048
20-cable system	0.998	6.64	0.351	0.936	0.007

3. Results

In cable-stayed bridges, the likelihood of occurrence of progressive collapse triggered by cable loss scenarios must be thoroughly investigated. The Post-Tensioning Institute (in united states) PTI (2007) recommends considering the probable cable loss scenarios during the design phase. Static analysis and application of a DAF of 2 is recommended to determine the effect of loss of cable.

There are two main approaches to preventing progressive collapse. First, ensure a high level of safety against local failure by using structural or non-structural strategies. Second, prevent failure from spreading by designing a robust structure that allows local failure. In the case of the failure of one of the parallel load-bearing elements (cables), the load carried by the failed member must be redistributed to the remaining structure. In this situation, the member adjacent to the failed member receives most of the redistributed load and becomes the critical member. If this member cannot tolerate the redistributed load, the collapse will progress to the subsequent members and, possibly, the entire structure.

When constructing bridges, the possibility of failure of all cables within a 10m radius should be taken into account. For example, if the distance between two adjacent cables is $5 < L \leq 10$ m, the failure of both lines should be taken into account. Therefore, the minimum design load of a cable includes its original load plus the load redistributed from adjacent failed cables in a cable loss scenario and can be calculated as follows:

$$\text{Cable Design Load} = F + 2F\left(\frac{0.105m^2 + 0.645m}{1+\beta^{0.35}}\right). \quad (35)$$

It can be seen that the design load of the cable depends on β. This means that for systems with larger β values, smaller design loads are required. Equation 35 shows that the design loads for two systems with β values of 50 and 500, assuming two cables fail are 1.69F and 1.35F, respectively. This shows a difference of 25%. Long-span cable-stayed bridges can be divided into zones based on β values (small, medium, and large) for calculating the design loads. Thus, using the proposed method reduces the cable design costs in the event of cable loss.

4. Conclusion

In modern bridges, the distance between two adjacent cables is much shorter than in old bridges. Therefore, in the event of car accidents or explosions on new bridges, more than one cable is likely to rupture. Accordingly, it was proposed to consider the rupture of all cables within 10 meters when designing bridges.

1. Several studies have been conducted to identify DAF in bridges. These studies show that a DAF equal to two is not safe in all cases. While recent research proves that the proposed DAF is safe for cable design, it is not safe for the design of pylons, as well as girders with negative moments.

2. A parallel load-bearing system, which is a long-span cable-stayed bridge, is considered and the "stress magnification factor" of the critical cable in a cable loss scenario is investigated. The design parameters of the system, such as the beam bending stiffness and the unique axial stiffness of each cable, are taken into account.

3. Failure of multiple cables is also considered. An analytical approach based on the differential equations of the system is applied and an approximation function is developed to calculate the stress magnification factor of the main cable in the event of a cable break. The least squares method is used to minimize the error of the approximation function.

The proposed approximation function is found to be accurate when compared with accurate values of the stress magnification factor. The proposed approximation function has an error of less than 5% in all the systems tested, except for minor values of β (system stiffness factor). Increasing the value of β reduces the stress magnification factor in the main cable. The parameter β influences the calculation of the cable load.

This means that for systems with large β values, smaller design loads are required. Therefore, in the case of long-span cable-stayed bridges, the bridge can be divided into different zones corresponding to different β values. The minimum design load for each zone can then be determined.

As a result, the use of the proposed method can reduce the cable design costs in the event of cable loss.

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