

## Phase synthesis method in multi-aperture optical systems based on iterative image processing algorithms

Petr A. Semenov, Viacheslav V. Zemlyakov, Svetlana A. Vyatkina 

Southern Federal University  
105/42, Bolshaya Sadovaya Street,  
Rostov-on-Don, 344006, Russia

**Abstract – Background.** Modern technologies in the field of optics and photonics place high demands on the quality and output power of the radiation source. The fulfillment of these requirements can currently be achieved by creating multi-aperture laser systems with coherent beam addition. The main problem of creating such systems is the development of phase synthesis methods in a multichannel optical system. **Aim.** In this paper, we consider a phase synchronization method without a reference beam for a coherent addition system with active feedback, which is based on the Gerchberg–Saxton algorithm. This algorithm makes it possible to reconstruct complex field amplitudes in the aperture and focal planes from intensity distributions in these fields. The application of this algorithm for multichannel systems is analyzed and its features, such as the occurrence of stagnation and ambiguity of the solution, are revealed. **Methods.** Solutions are proposed to eliminate the problem of convergence of the algorithm to side solutions and getting the iterative procedure into the local extremum using global optimization algorithms, methods of reducing the dimension of the problem and the introduction of antisymmetric amplitude modulation. **Results.** The paper demonstrates the results of phase field reconstruction for a large number of optical sources. For a seven-aperture system, a physical simulation was performed to restore phase information, confirming the results of numerical modeling. **Conclusion.** The proposed approach is a reasonable alternative to currently existing methods and can be used in the problem of coherent addition in multichannel optical systems.

**Keywords –** Gerchberg–Saxton algorithm; phase field reconstruction; multi-aperture laser systems; physical modeling; global optimization.

### Introduction

The development of new coherent radiation sources for competitive optoelectronic devices and location systems requires an increase in power while minimizing losses and preserving radiation quality [1]. However, such a set of requirements is difficult to achieve, since increasing the peak power of the radiation source leads to an increase in losses and a significant decrease in the quality of the laser beam [2]. The solution to these problems can currently be achieved by creating multi-aperture laser systems with coherent beam combining; in this case, there is no need to impose higher power requirements on individual emitters [3].

Coherent beam combining is a complex scientific and technical problem requiring the solution of phase, frequency, and polarization synchronization problems. This paper presents approaches to solving the phase synchronization problem. There are two main methods of solving this problem: the active and the passive phase synchronization of emitters. Passive phase matching utilizes the internal physical pa-

rameters of the system, forming a distributed optical feedback in which a portion of the total beam is returned to all the radiation sources of the laser ensemble. Such systems have a more complex design and are often limited by the total output radiated power [4].

Active phase synchronization methods in most cases use a distributed adaptive optical system with quality function optimization. The most commonly encountered system to date is one based on the stochastic gradient approximation algorithm [5]. Recently, methods based on neural network technologies have also become popular, but in relation to the problem of analyzing spatial characteristics in multi-aperture systems, such methods do not provide a guaranteed solution as of present and require more in-depth study [6].

In this paper, we consider a method for recovering phase information without a reference beam for a coherent combining system with active feedback, which is based on the Gerchberg–Saxton algorithm [7]. This algorithm allows us to reconstruct complex fields on the lens aperture and in its focal plane from their intensity distributions.

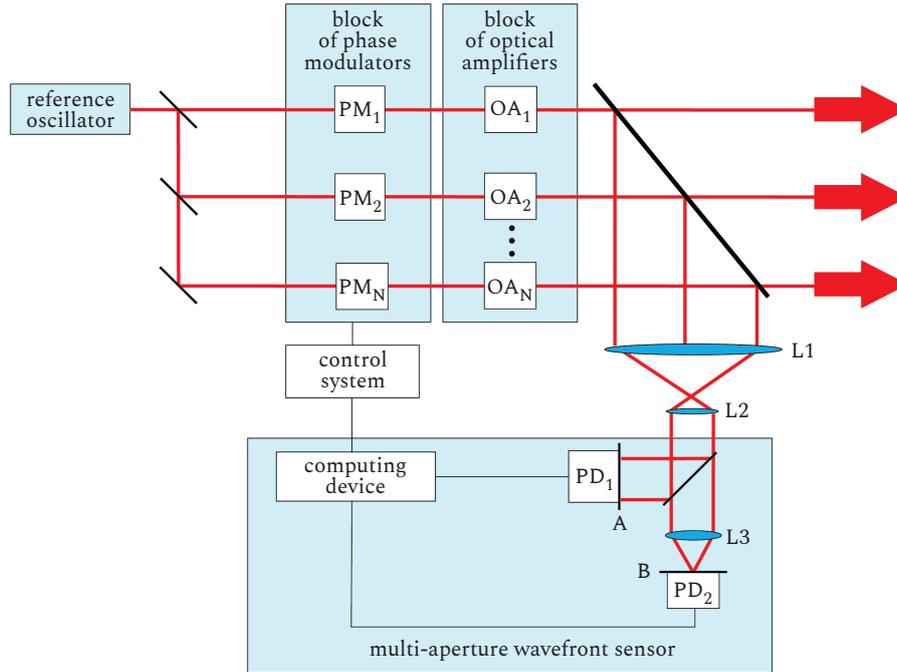


Fig. 1. Diagram of a multichannel laser emitter (OA – optical amplifiers; PM – phase modulators, PD – photodetector)  
 Рис. 1. Структурная схема многоканального лазерного излучателя (ОУ – оптические усилители; ФМ – фазовые модуляторы, ФП – фотоприемник)

## 1. Scheme of active coherent beam combining

The structural diagram of the active phase synchronization system using the Gerchberg–Saxton algorithm is shown in Fig. 1.

The optical signal from a single reference oscillator is divided into  $N$  laser beams. After passing through a block of phase modulators and single-mode optical amplifiers, the radiation is split into two beams. The phase at the output of each amplifier is random due to the different optical paths in the individual laser channels. The main beam hits the output aperture and the second beam is focused on the photodetector, which registers the image at the aperture and focal image planes. The measured distributions are fed to the computing device, where in the course of implementation of the iterative algorithm the phase distribution is determined, on the basis of which control signals to the phase modulators for synchronization of the laser channels are formed.

## 2. Analysis of the Gerchberg–Saxton algorithm

The mathematical formulation of the Gershberg – Saxton problem consists in constructing a complex function  $\tilde{E}(\xi)$  based on its modulus  $|\tilde{E}(\xi)|$  and the modulus of its Fourier transform  $|E(\mathbf{r})|$ , where

$E(\mathbf{r})$  is the Fourier transform of the function  $\tilde{E}(\xi)$ . At the first iteration, for the phase selected as the initial approximation and measured in the aperture plane of the distribution of the modulus  $|\tilde{E}(\xi)|$ , the complex amplitude in  $E(\mathbf{r})$  the focal plane is calculated. Then, the modulus of this amplitude is replaced by the known modulus  $|E(\mathbf{r})|$ . Next, the reverse propagation of the beam is calculated, the modulus replacement operation is performed in the aperture plane, and the phase obtained in this way is selected as the next approximation.

Thus, the Gerchberg–Saxton algorithm is written as the following iterative procedure

$$\begin{aligned} \tilde{E}_0(\xi) &= \tilde{M}(\xi) \exp[i\varphi_{\tilde{E}}^{(0)}(\xi)]; \\ \tilde{E}_k(\xi) &= P_2 FT\{P_1 FT^{-1}[\tilde{E}_{k-1}(\xi)]\}, \quad k = 1, 2, \dots, \end{aligned} \quad (1)$$

where  $FT$  and  $FT^{-1}$  are the forward and inverse Fourier transforms, respectively;  $P_1$  and  $P_2$  are the modulus substitution operations in the focal and aperture planes.

The convergence of the Gerchberg–Saxton algorithm was analyzed for a model of fiber laser sources with hexagonal channel packing and Gaussian amplitude distribution in each beam.

The phase in each channel was randomly selected within  $\pm\pi$  radian. Such a phase distribution mimics a multichannel system consisting of single-mode laser emitters, the phase in each of which is due to the dif-

ferent optical paths in the individual channels. The quality of the recovery of complex functions  $\tilde{E}(\xi)$  and  $E(\mathbf{r})$  was assessed by two characteristics. By normalized integral modulus error in the aperture plane

$$\delta_{\text{mod}k} = \sqrt{\int |\tilde{E}(\xi) - \tilde{M}(\xi)|^2 d\xi}, \quad (2)$$

$\delta_{\text{mod}1}$  is a modulus error in the first iteration. In addition to the modulus error, the image quality was also evaluated by the values of the normalized image sharpness function

$$S = \int I_k^2(\bar{r}) d\bar{r}, \quad (3)$$

where  $S_{PH}$  is the sharpness function of the phased system (with equal phases in each channel). The analysis for a 7-channel laser system was performed in [8]. We demonstrated that with a centrally symmetric initial modulus, the convergence of the algorithm is not guaranteed and depends on the initial conditions. The problem of collateral side solutions can be solved by introducing antisymmetric amplitude modulation. In this case, an amplitude mask is placed in the system, whose transmission coefficient is a specially chosen two-dimensional function that has no central symmetry. The implementation of this constraint provides guaranteed convergence of the algorithm for the 7-laser system.

However, as the number of phased channels increases, the algorithm starts to fall into local extrema or so-called stagnation states [9]. Physically, the formation of local extrema is related to the search for an energy balance between the different components of the error signal.

### 3. Technique for phase information recovery

The characteristic property of the algorithm, consisting in the presence of stagnation states, severely limits its applications. Although this method is quite simple and fast, it converges to the correct solution only under special conditions, such as a good initial approximation or a small number of channels for phasing. A good solution to ensure reliable convergence of the algorithm under such conditions is to use global optimization methods.

There is no single technique for solving problems with multiple extrema that would be recognized as universal, as the choice of an appropriate method depends on a variety of conditions and parameters of a particular system. For the Gerchberg–Saxton algorithm, it is important to provide fast and easy local

optimization without the need to compute partial derivatives. The high iteration speed is ensured by a simple modulus replacement operation, which is not affected by the dimensionality of the problem. Thus, when synthesizing a global optimization method, it is worth using the random multi-start algorithm: multiple searches from random starting points, followed by selection of the best result [10]. This global search algorithm has a high speed of approximations and allows simultaneous search for local extrema from different starting points.

However, when the number of phase synchronization channels is large, increasing the number of parallel processors has little effect on the result. Even with the number of channels  $N = 37$ , the convergence percentage of the algorithm is only 50 %. As the number of channels is further increased, there comes a point where the algorithm stops converging from any starting point. Therefore, when the number of channels is large, it is necessary to use some methods to reduce the dimensionality of the problem. Based on the above, to use this algorithm for phase synchronization of an arbitrary number of optical sources, the following optimization methods should be used [11]:

1. Implementation of antisymmetric amplitude modulation to eliminate symmetric solutions.
2. Division of a system with more than 19 channels into blocks and synchronization of individual blocks.
3. The use of global optimization algorithms, in particular of the random multi-start algorithm.
4. The use of additional a priori information based on system properties, single-mode laser sources, their packing and phase shift recovery only.

Thus, the Gerchberg–Saxton algorithm is part of a numerical procedure that performs global optimization, which can be defined as a strategy for recovering phase information in a multi-aperture wavefront sensor.

### 4. Experimental recovery of phase information

A physical experiment to restore phase information was carried out on a model of a seven-channel laser system with a uniform intensity distribution and a different phase shift  $\varphi$  in each channel. The general view of the laboratory bench is shown in Fig. 2.

As the main source of radiation, a Zygo interferometer was used, the laser beam of which has a uniform polarized distribution. From the output of the inter-

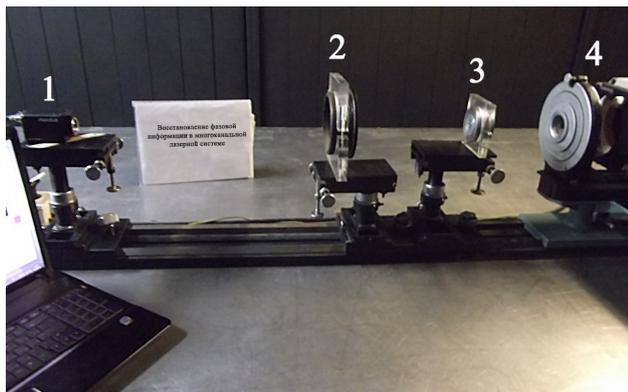


Fig. 2. Laboratory stand for the restoration of phase characteristics in a seven-channel system: 1 – camera; 2 – beam splitter; 3 – mask; 4 – interferometer

Рис. 2. Лабораторный стенд по восстановлению фазовых характеристик в семиканальной системе: 1 – видеокамера; 2 – светоделитель; 3 – маска; 4 – интерферометр

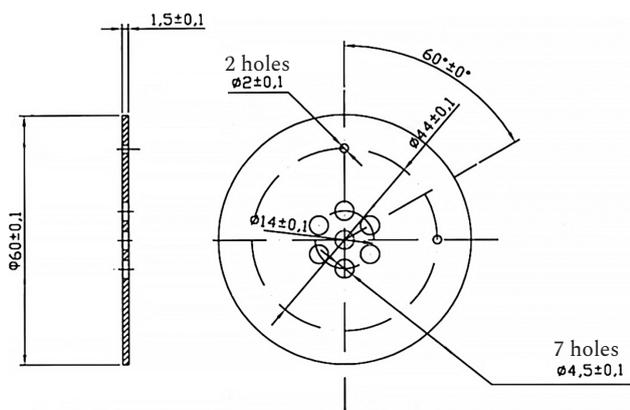


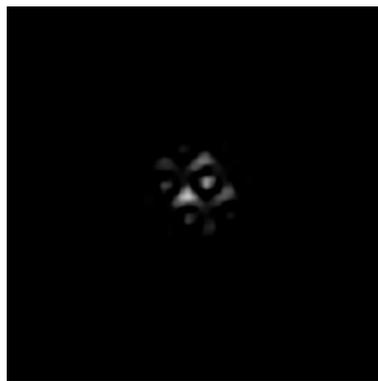
Fig. 3. Design of a seven-channel laser system model

Рис. 3. Конструкция макета семиканальной лазерной системы

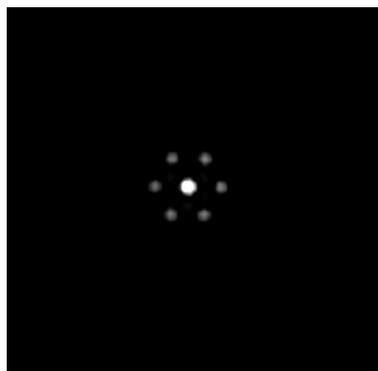
ferometer, the radiation falls on a mockup of a seven-channel laser system [12] specially made for the experiment, with a different phase shift in each channel (Fig. 3).

Using a beam splitter, a portion of the radiation is diverted to register the distribution in the image plane. An energy and spatial characteristic measurement system was used as an intensity distribution meter.

The intensity distribution in the focal plane plays the main role, since the distribution in the aperture plane did not change as a function of phase shifts, and was replaced by numerical modeling in a number of experiments. The bulk of the radiation is focused by a lens onto the surface of the receiver, where an image in the focal plane of the beam is recorded. The obtained distributions are processed by specialized software, in which the recovery of the phase distribution using the Gerchberg–Saxton algorithm is im-



a



b

Fig. 4. Images recorded in the focal plane: a – system with a random shift in each channel; b – phased system

Рис. 4. Зарегистрированные изображения в фокальной плоскости: а – система со случайным сдвигом в каждом канале; б – сфазированная система

plemented. Examples of the registered images for the phased and unphased systems are shown in Fig. 4.

Recovery occurred from both null and random starting points. The reconstructed distribution was compared with numerical simulations at the minimum of the modulus error in the focal plane for 10,000 different random variations of phase shifts in each channel. The phase field was reconstructed in an average of 15–20 consecutive iterative procedures. The accuracy of the wavefront measurement was of the order of  $\lambda/100$ , while the main contribution to the error was the accuracy of the alignment and installation of the phase plates.

## Conclusion

The approach to the construction of a wavefront sensor for phase synchronization of single-mode emitters in a multi-aperture laser system considered in this paper is based on image processing methods, in particular, on the Gerchberg–Saxton algorithm. The use of the developed wavefront restoration

technique in conjunction with global optimization methods allows to remove the main problem of the algorithm used, related to the lack of guaranteed convergence in the general case. It can be concluded that this approach would be a sensible alternative to current methods.

## References

1. E. R. Milyutin, "The irradiance fluctuations signal in atmospheric optical of systems transmit of informations," *Physics of Wave Processes and Radio Systems*, vol. 19, no. 1, pp. 12–15, 2016, url: <https://journals.ssau.ru/pwp/article/view/7152>. (In Russ.)
2. J. W. Dawson et al., "Power scaling analysis of fiber lasers and amplifiers based on non-silica materials," *Proc. SPIE*. 2010. Vol. 7686, p. 768611, doi: <https://doi.org/10.1117/12.852393>.
3. D. A. Veden'kin and Yu. E. Sedel'nikov, "Properties and technical applications of antenna arrays focused on a broadband signal," *Physics of Wave Processes and Radio Systems*, vol. 26, no. 4, pp. 88–94, 2023, doi: <https://doi.org/10.18469/1810-3189.2023.26.4.88-94>. (In Russ.)
4. M. L. Minden, "Passive coherent combining of fiber oscillators," *Proc. of SPIE*, vol. 6453, p. 64530P, 2007, doi: <https://doi.org/10.1117/12.714437>.
5. M. A. Vorontsov and S. L. Lachinova, "Laser beam projection with adaptive array of fiber collimators. I. Basic considerations for analysis," *Journal of the Optical Society of America A*, vol. 25, no. 8, pp. 1949–1973, 2008, doi: <https://doi.org/10.1364/JOSAA.25.001949>.
6. M. A. Vorontsov and G. A. Filimonov, "The self-learning AI controller for adaptive power beaming with fiber-array laser transmitter system," *Advances in Artificial Intelligence and Machine Learning*, vol. 3, no. 1, pp. 731–760, 2023, doi: <https://doi.org/10.54364/AAIML.2023.1148>.
7. R. W. Gerchberg and W. O. Saxton, "A practical algorithm for the determination of phase from image and diffraction plane pictures," *Optik*, vol. 35, pp. 237–250, 1971.
8. S. D. Pol'skikh and P. A. Semenov, "Multi-aperture wavefront sensor for coherent laser beam combining system," *Opticheskij zhurnal*, vol. 83, no. 6, pp. 7–13, 2016, url: <https://opticjourn.ru/ru/abstract/2016-83-6-7-13>. (In Russ.)
9. J. R. Fienup, "Phase retrieval algorithms: a comparison," *Applied Optics*, vol. 21, no. 15, pp. 2758–2769, 1982, doi: <https://doi.org/10.1364/AO.21.002758>.
10. A. A. Zhiglyavskiy and A. G. Zhilinskas, *Methods for Searching for a Global Extremum*. Moscow: Nauka, 1991. (In Russ.)
11. P. A. Semenov, V. V. Zemlyakov, and S. A. Vyatkina, "Analysis of the efficiency of phase field reconstruction in multi-aperture optical systems based on image processing algorithms," *Infokommunikatsionnye i radioelektronnye tekhnologii*, vol. 7, no. 2, pp. 268–279, 2024, url: <https://www.elibrary.ru/item.asp?id=68623783>. (In Russ.)
12. P. A. Semenov, "Development of a seven-channel laser system layout for physical modeling of a multi-aperture wavefront sensor," *Inzhenernyy vestnik Dona*, no. 10, pp. 701–708, 2023, url: <http://www.ivdon.ru/ru/magazine/archive/n10y2023/8764>. (In Russ.)

## Information about the Authors

**Petr A. Semenov**, postgraduate student of Faculty of Physics, Southern Federal University, Rostov-on-Don, Russia.  
*Research interests:* computer modeling, image processing, adaptive optical systems.  
*E-mail:* piter@bk.ru

**Viacheslav V. Zemlyakov**, Doctor of Physical and Mathematical Sciences, associate professor, professor of the Department of Applied Electrodynamics and Computer Modeling, Faculty of Physics, Southern Federal University, Rostov-on-Don, Russia.  
*Research interests:* physics of radio waves, numerical methods of applied electrodynamics, infocommunication systems and technologies.  
*E-mail:* vvezemlyakov@sfsu.ru

**Svetlana A. Vyatkina**, Candidate of Physical and Mathematical Sciences, associate professor of the Department of Radiophysics, Faculty of Physics, Southern Federal University, Rostov-on-Don, Russia.  
*Research interests:* microwave electrodynamics, spin electronics, magnetostatic wave devices, microwave electronics.  
*E-mail:* svyatkina@sfsu.ru  
*ORCID:* <https://orcid.org/0000-0001-7858-8607>

## Физика волновых процессов и радиотехнические системы 2024. Т. 27, № 3. С. 40–46

DOI 10.18469/1810-3189.2024.27.3.40-46  
УДК 535.41  
Оригинальное исследование

Дата поступления 28 июня 2024  
Дата принятия 29 июля 2024  
Дата публикации 30 сентября 2024

## Метод фазового синтеза в мультиапертурных оптических системах на основе итерационных алгоритмов обработки изображений

П.А. Семёнов, В.В. Земляков, С.А. Вяткина 

Южный федеральный университет  
344006, Россия, г. Ростов-на-Дону,  
ул. Большая Садовая, 105/42

**Аннотация – Обоснование.** Разработка современных оптико-электронных систем предъявляет повышенные требования к качеству и выходной мощности источника излучения. Одним из возможных вариантов выполнения указанных требований является создание систем когерентного сложения нескольких оптических пучков. Основной проблемой при этом стала разработка методов фазового синтеза в мультиканальной оптической системе. **Цель.** В статье рассмотрен метод восстановления фазового поля для мультиапертурной системы когерентного сложения пучков с активной обратной связью, в основе работы которого лежит алгоритм Гершберга – Сэкстона. Данный алгоритм позволяет восстанавливать комплексные амплитуды поля в апертурной и фокальной плоскостях по распределениям интенсивности в данных полях. Проанализировано применение данного алгоритма для мультиканальных систем, и выявлены его особенности, такие как возникновение стагнации и неоднозначность решения. **Методы.** Предложены решения для устранения проблемы сходимости алгоритма к побочным решениям и попадания итерационной процедуры в локальный экстремум с помощью алгоритмов глобальной оптимизации, методов редукции размерности задачи и введении антисимметричной модуляции амплитуды. **Результаты.** В работе продемонстрированы результаты по восстановлению фазового поля для большого числа оптических источников. Для семиапертурной системы осуществлено физическое моделирование по восстановлению фазовой информации, подтверждающее результаты численного моделирования. **Заключение.** Предложенный подход является разумной альтернативой существующим в настоящее время методам и может быть использован в задаче когерентного сложения в мультиканальных оптических системах.

**Ключевые слова** – алгоритм Гершберга – Сэкстона; восстановление фазового поля; мультиапертурные лазерные системы; физическое моделирование; глобальная оптимизация.

✉ piter@bk.ru (Семёнов Петр Алексеевич)



© Семёнов П.А., Земляков В.В., Вяткина С.А., 2024

## Список литературы

1. Милютин Е.Р. Флуктуации интенсивности сигнала в атмосферных оптических системах передачи информации // Физика волновых процессов и радиотехнические системы. 2016. Т. 19, № 1. С. 12–15. URL: <https://journals.ssau.ru/pwp/article/view/7152>
2. Power scaling analysis of fiber lasers and amplifiers based on non-silica materials / J.W. Dawson [et al.] // Proc. SPIE. 2010. Vol. 7686. P. 768611. DOI: <https://doi.org/10.1117/12.852393>
3. Веденькин Д.А., Седелников Ю.Е. Свойства и технические приложения антенных решеток, сфокусированных по широкополосному сигналу // Физика волновых процессов и радиотехнические системы. 2023. Т. 26, № 4. С. 88–94. DOI: <https://doi.org/10.18469/1810-3189.2023.26.4.88-94>
4. Minden M.L. Passive coherent combining of fiber oscillators // Proc. of SPIE. 2007. Vol. 6453. P. 64530P. DOI: <https://doi.org/10.1117/12.714437>
5. Vorontsov M.A., Lachinova S.L. Laser beam projection with adaptive array of fiber collimators. I. Basic considerations for analysis // Journal of the Optical Society of America A. 2008. Vol. 25, no. 8. P. 1949–1973. DOI: <https://doi.org/10.1364/JOSAA.25.001949>
6. Vorontsov M.A., Filimonov G.A. The self-learning AI controller for adaptive power beaming with fiber-array laser transmitter system // Advances in Artificial Intelligence and Machine Learning. 2023. Vol. 3, no. 1. P. 731–760. DOI: <https://doi.org/10.54364/AAIML.2023.1148>
7. Gerchberg R.W., Saxton W.O. A practical algorithm for the determination of phase from image and diffraction plane pictures // Optik. 1971. Vol. 35. P. 237–250.
8. Польских С.Д., Семёнов П.А. Многоапертурный датчик волнового фронта для системы когерентного сложения лазерных пучков // Оптический журнал. 2016. Т. 83, № 6. С. 7–13. URL: <https://opticjourn.ru/ru/abstract/2016-83-6-7-13>
9. Fienup J.R. Phase retrieval algorithms: a comparison // Applied Optics. 1982. Vol. 21, no. 15. P. 2758–2769. DOI: <https://doi.org/10.1364/AO.21.002758>
10. Жиглявский А.А., Жилинскас А.Г. Методы поиска глобального экстремума. М.: Наука, 1991. 248 с.
11. Семёнов П.А., Земляков В.В., Вяткина С.А. Анализ эффективности реконструкции фазового поля в мультиапертурных оптических системах на основе алгоритмов обработки изображений // Инфокоммуникационные и радиоэлектронные технологии. 2024. Т. 7, № 2. С. 268–279. URL: <https://www.elibrary.ru/item.asp?id=68623783>
12. Семёнов П.А. Разработка макета семиканальной лазерной системы для физического моделирования мультиапертурного датчика волнового фронта // Инженерный вестник Дона. 2023. № 10. С. 701–708. URL: <http://www.ivdon.ru/magazine/archive/p10y2023/8764>

## Информация об авторах

**Семёнов Петр Алексеевич**, аспирант физического факультета Южного федерального университета, Ростов-на-Дону, Россия.  
*Область научных интересов:* компьютерное моделирование, обработка изображений, адаптивные оптические системы.  
*E-mail:* piter@bk.ru

**Земляков Вячеслав Викторович**, доктор физико-математических наук, доцент, профессор кафедры прикладной электродинамики и компьютерного моделирования физического факультета Южного федерального университета, Ростов-на-Дону, Россия.

*Область научных интересов:* физика радиоволн, численные методы прикладной электродинамики, инфокоммуникационные системы и технологии.

*E-mail:* vvezemlyakov@sfedu.ru