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Artificial Neural Network for Downward Continuation of Anomalous Magnetic Fields

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Abstract: The downward continuation of an anomalous magnetic field is used for many applications in geophysics. However, such a problem is ill-posed, so it does not have a unique and stable solution. In this paper, we propose an artificial neural network architecture for the downward continuation of the vertical component of an anomalous geomagnetic field measured in a plane at a given height. The inverse problem is solved here by a direct method: the neural network is trained to reconstruct such a distribution of the magnetic field B_{down}, which after a stable upward continuation corresponds to the measured field B_{up}. The performance of the neural network was demonstrated using the example of an anomalous geomagnetic field obtained using the Enhanced Magnetic Model.

Keywords: stray magnetic field, magnetic anomaly, untrained neural networks, inverse modeling, downward continuation.

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Research Article

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1. Introduction

The downward continuation of the magnetic field is an important applied task, primarily for general navigation and for directional drilling [*Buchanan et al.*, 2013; *Kaji et al.*, 2019]. For example, during aeromagnetic surveying, the magnetic field is measured at a given height above the Earth's surface, but for technical applications the magnetic field must be known at the ground level and several kilometers deeper.

The geometry of the problem is schematically illustrated in Figure 1. Magnetized bodies create an anomalous magnetic field B_{up} in the plane (x, y) at a certain height above the ground level. The problem is to calculate, from the measured magnetic field B_{up}, the magnetic field B_{down} at a depth ΔZ, closer to the magnetized sources.

The magnetic fields in the upper and lower planes are related through the well-known integral equation [*Blakely*, 1995]

$$B_{\text{up}}(x,y) = \frac{\Delta z}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{B_{\text{down}}(x',y')}{\left[(x-x')^2+(y-y')^2+\Delta z^2\right]^{3/2}}\mathrm{d}x'\mathrm{d}y' = A\left[B_{\text{down}}\right],\ \Delta z > 0. \tag{1}$$

Here, if the magnetic field below is known, B_{down}, then the calculation of the field in the upper plane, B_{up}, is performed directly using the operator of upward continuation, $B_{\text{up}} = A[B_{\text{down}}]$. However, in order to calculate the magnetic field in the lower plane B_{down} from the known distribution in the upper plane B_{up}, it is necessary to construct the inverse operator of downward continuation, $B_{\text{down}} = A^{-1}[B_{\text{up}}]$.

An equation of type (1) is a Fredholm integral equation of the first kind. If the magnetic field B_{down} is unknown, then this is a linear inverse problem, which, generally speaking, does not have a unique and stable solution [*Blakely*, 1995; *Tikhonov et al.*, 1977].

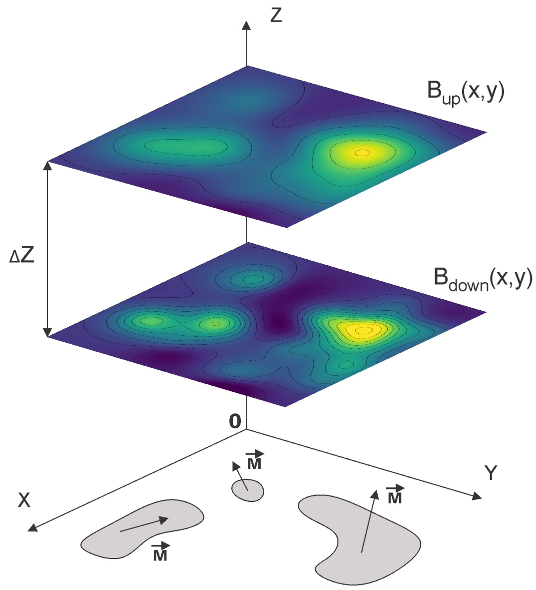

Figure 1. Schematic representation of the magnetic field downward continuation problem.

Although it is possible to obtain an analytical solution specifically for an equation of type (1) using the Fourier transform, however, recalculating the magnetic field downwards using such a solution is unstable [*Blakely*, 1995]. Therefore, to find an approximate solution to equation (1), some numerical methods are usually used.

2. Neural network architecture

One way to find an approximate solution is to construct a downward continuation operator A^{-1} using a stable solution of the forward problem. Artificial neural networks, which are universal approximators, may be well suited for this purpose [*Nielsen*, 2015]. Neural network models have already been successfully applied to solve a number of inverse problems in magnetism and stray magnetic field modeling [*Coskun et al.*, 2022; *Dubois et al.*, 2022; *Pollok et al.*, 2021; 2023].

In this paper, we propose for the scalar magnetic field downward continuation the neural network architecture schematically shown in Figure 2. Here, the neural network is trained to act as a downward continuation operator A^{-1}. In the first step, the neural network calculates a trial magnetic field in the lower plane, B_{down}, using the known magnetic field distribution B_{up}. After the calculation, the trial field B_{down} is used to directly calculate the field in the upper plane, B_{trial}, at a height of ΔZ using the stable upward continuation operator A. Finally, the loss between the known field B_{up} and the calculated trial B_{trial} is calculated, which is then used to train the neural network using the backpropagation method. The neural network is trained until the error becomes less than a certain small value.

It is worth noting that a computationally efficient way to calculate convolution-type integrals (1) is to use the Fourier transform [*Blakely*, 1995]

$$F[B_{\text{up}}] = F[B_{\text{down}}]e^{-\Delta Z|k|}, \tag{2}$$

where $F[B]$ denotes the Fourier transform of the magnetic field B, and the modulus of the wave number $|k| = \sqrt{k_x^2 + k_y^2}$. The calculation of the field in the upper plane B_{up}, in the developed neural network training scheme, was carried out using the fast Fourier transform.

Numerical tests have shown that the neural network structure can be quite simple to implement the scheme in Figure 2, and its structure can be more or less arbitrary. Here, a convolutional neural network was created to work with 512×512 pixel images. It contains an input layer, three Conv2D layers with a 2×2 pixel kernel, 32 filters, and linear activation functions. Each convolutional layer is followed by an AveragePooling layer with a 2×2 pixel

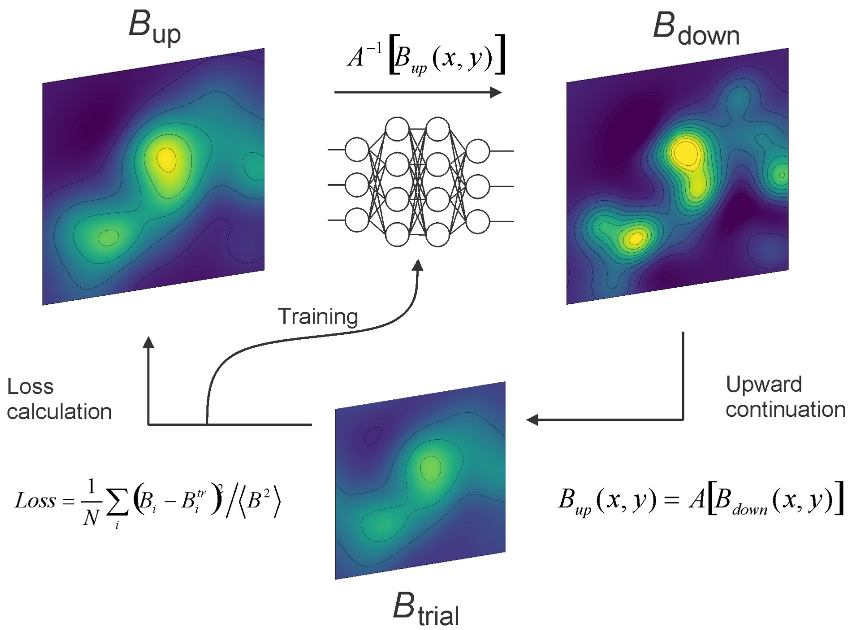

Figure 2. Architecture of a neural network for magnetic field downward continuation using stable upward continuation operator.

kernel, and an output layer. The standard normalized mean squared error function was used to calculate the loss Loss $= \frac{1}{N}\sum_i \left(B_i - B_i^{\mathrm{tr}}\right)^2 / \langle B^2 \rangle$, where B_i is the known field at point i, B_i^{tr} is the trial field at point i, $\langle B^2 \rangle$ is the mean square of the known anomalous field, N is the total number of measurement points.

The neural network was implemented using the tensorflow library [*Abadi et al.*, 2016] and the model was trained using the Adam stochastic optimization algorithm [*Kingma et al.*, 2014]. The training was carried out on a simple personal computer with an Intel processor Core i7-9700 and NVIDIA graphics card GeForce GTX 950.

This type of neural networks called untrained neural network in opposite to traditional trained on large datasets models [*Dubois et al.*, 2022]. Untrained neural network does not require a database for training, which is its advantage. Also, the accuracy of the magnetic field downward continuation is controlled while training.

This method assumes that magnetized bodies are located under the plane of magnetic field calculation. If the plane of the calculation field is located in the magnetized bodies area, this method may not work correctly.

3. Synthetic example

The performance of the neural network was tested on the anomalous magnetic field of the EMM model [*The National Centers...*, 2018]. The EMM model is compiled from satellite, marine, aeromagnetic and ground-based data. It represents both the main and anomalous magnetic fields of the Earth using the Gaussian model with 790 spherical harmonics, which corresponds to a spatial resolution of about 50 km. In order to isolate the anomalous magnetic field, the coefficients of the first 16 harmonics in the EMM model were set to zero. The model anomalous magnetic field was calculated at sea level on an equidistant grid in of 512×512 points with a step of 2 km. Then, using (2), the magnetic field was extended upward to a height of $\Delta Z = 4$–24 km, and then extended downward using the developed method.

In Figure 3a, the region in which the vertical component of the anomalous magnetic field was calculated is highlighted. The dynamics of the loss function during neural network training is shown in Figure 3b for heights $\Delta Z = 4$–24 km. The series Figure 3c–3e show the distribution of the anomalous magnetic field at heights of 12, 8, 4 km, the results of the downward continuation of the vertical component of the magnetic field, and the error distribution between true and estimated magnetic fields. For a height of $\Delta Z = 4$ km the average error is 15 nT, for a height of $\Delta Z = 8$ km the average error is 30 nT, for a height of $\Delta Z = 12$ km the average error is 45 nT.

Figure 3b shows that the loss here quickly reaches values below 0.4, followed by a smooth decrease in the loss to values of 0.2 and below. The total calculation time of 5000 training iterations on a personal computer does not exceed 10 minutes.

In Figure 3c–3e it is evident that with increasing altitude the efficiency of the downward continuation of the field decreases. This is easily explained by equation (2), which shows that the decrease in the intensity of the magnetic anomaly exponentially depends on the product of the wave number of its constituent harmonics k and the altitude ΔZ.

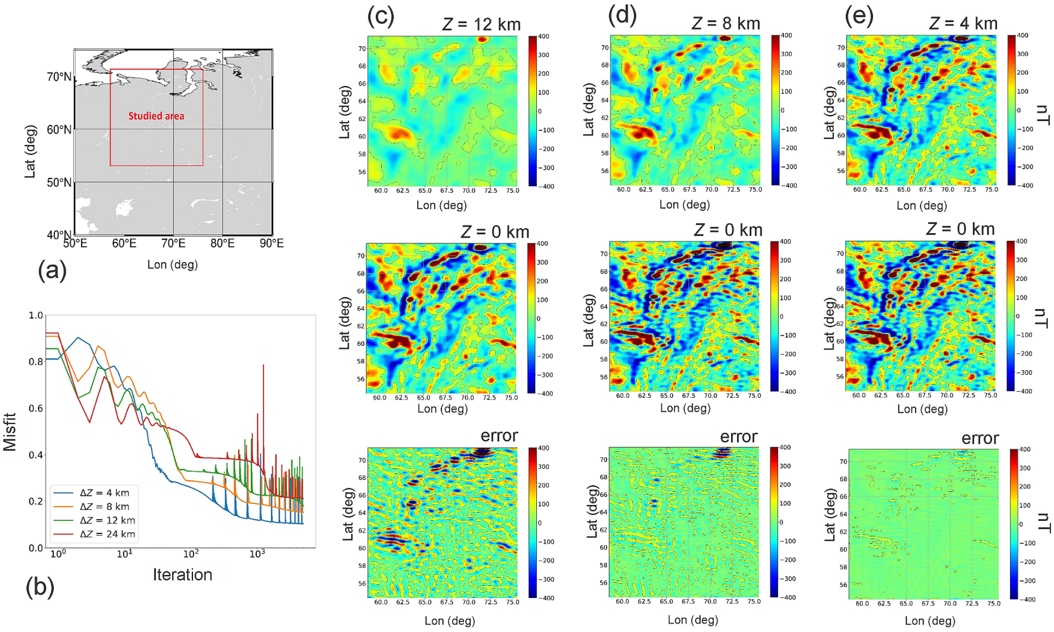

Figure 3. (a) The region in which the vertical component of the anomalous magnetic field was calculated; (b) The dynamics of the loss function during the training of the neural network for heights $\Delta Z = 4$–24 km; the distribution of the anomalous magnetic field at a given height, the result of magnetic field downward continuation and the error for $\Delta Z = 12$ km (c), $\Delta Z = 8$ km (d), $\Delta Z = 4$ km (e).

For example, the intensity of harmonics with a wavelength $l = 50$ km decreases by a factor of e at an altitude $\Delta Z \approx 8$ km. The high-frequency contribution to the anomalous field decreases first with increasing altitude ΔZ, which is reflected in the efficiency of the downward continuation of the anomalous magnetic field.

4. Conclusions

In this paper, a neural network model was developed for downward continuation of the anomalous magnetic field towards the location of magnetic sources. The performance of the neural network was demonstrated using a synthetic example. The accuracy of magnetic field reconstruction depends on the distance ΔZ, for the demonstrated examples the error did not exceed 45 nT for $\Delta Z = 4$–12 km. Further improvement of the algorithm is planned.

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