Russian Journal of Earth Sciences

Special Issue: "Data Science, Geoinformatics and Systems Analysis in Geosciences"

Estimation of Parameters of Non-stationary Geophysical Signals Based on Two-Stage Approximations Using Local Models

V. G. Getmanov[1,2], **A. D. Gvishiani**[1,2], **V. A. Pilipenko**[1,2], **D. A. Stukov**[2]

[1]Geophysical Center of the Russian Academy of Sciences, Moscow, Russia
[2]Schmidt Institute of Physics of the Earth, Moscow, Russia
* **Correspondence to:** Victor Getmanov, vgetm2015@yandex.ru

Abstract: The authors have developed a technology for estimating the parameters of non-stationary geophysical signals, using a two-stage approximation with local approximation models at the first stage and weighted averaging at the second stage. This article considered an example of estimating the amplitude, frequency and trend parametric functions for geomagnetic pulsations Pc1.

Keywords: geophysical signals, geomagnetic pulsations, estimation, non-stationary parameters, two-stage approximation.

Citation: Getmanov, V. G., A. D. Gvishiani, V. A. Pilipenko, and D. A. Stukov (2025), Estimation of Parameters of Non-stationary Geophysical Signals Based on Two-Stage Approximations Using Local Models, *Russian Journal of Earth Sciences*, 25, ES2020, EDN: ZWLDSR, https://doi.org/10.2205/2025ES000979

1. Introduction

Geophysical signals (geomagnetic pulsations, seismic waves, wave processes in the atmosphere, etc.) in some cases have pronounced non-stationarity. The geomagnetic pulsations considered here are quasi-periodic oscillations, characterized by non-stationary parameters. Such pulsations occupy frequency ranges from thousandths to several Hz and amplitudes from hundredths to hundreds of nT [*Klejmyonova*, 2007]. Pulsations arise due to the resonant interaction of magnetospheric plasma and the geomagnetic field.

Estimates of non-stationary parameters of geomagnetic pulsations are used for diagnostics of the magnetosphere [*Guglielmi et al.*, 1973], can be used to determine the frequencies and amplitudes of "serpentine emission" signals [*Guglielmi et al.*, 2015], to find the parameters of Alfvén signals [*Potapov et al.*, 2021], etc.

The most widely used method of signal analysis is the discrete Fourier transform (DFT) [*Le'j*, 2007], which, however, is poorly suited for non-stationary signals due to its limited resolution. The fairly common spectral-temporal analysis using the Wigner transform [*Time-Frequency Analysis: Concepts and Methods*, 2008], which is actually based on the DFT, has errors in parameter estimates of approximately the same level as the errors of the DFT. When implementing these methods, important information about the properties and characteristics of signals may be lost.

The purpose of this work is to describe the developed technology for estimating parametric functions of geomagnetic pulsations based on two-stage approximations [*Getmanov*, 2021] using local approximation models [*Katkovnik et al.*, 2006]. The proposed technology is an alternative to the DFT, due to a more accurate fit of the approximation models used, and is largely universal and suitable for non-stationary signals of many subject areas.

Research Article

Received: 15 November 2024
Accepted: 15 April 2025
Published: 23 May 2025

Functional models of non-stationary signals with parametric functions, including geomagnetic pulsation signals, are represented by the following relation

$$Y(T_i) = Y_0(p(T_i), T_i) + W(T_i) = \sum_{l=1}^{L} A_l(T_i) \cos \varphi_{0l}(T_i) + E(T_i) + W(T_i), \tag{1}$$

where $A_l(T_i)$, $f_{0l}(T_i) = \dot{\varphi}_{0l}(T_i)$, $E(T_i)$ are amplitude, frequency and trend functions, $W(T_i)$ is model noise, L is the number of possible frequency components, T is a sampling step. $Y_0(p(T_i), T_i)$ for (1) depends on the vector of parametric functions $p(T_i)$, composed of $p_j(T_i)$, $j = 1, \ldots, j_0$, where j_0 is the number of components in the model

$$p(T_i)^T = (p_1(T_i), p_2(T_i), \ldots, p_{j_0}(T_i)). \tag{2}$$

In particular, if the components are the amplitude, frequency and trend functions, then $j_0 = 3$. Expressions (1), (2) serve as the basis for constructing local approximation models.

2. The problem of two-stage approximation estimation of non-stationary signal parameters based on local models

At the first stage of the approximation, we represent the observations as $Y(T_i)$, $1 \leq i \leq N_f$. Let us introduce local intervals $N_{1s} \leq i \leq N_{2s}$ dimensions N, $N \ll N_f$ with the boundaries $N_{1s} = 1N(s-1)$, $N_{2s} = Ns$, $s = 1, \ldots, s_0$, where $s_0 = \mathrm{ent}(N_f/N)$, $1 \leq i \leq N_{f1}$ and $N_{f1} = N_{s0}$, $N_{f1} \leq N_f$. Let $Y_s(T_i)$ be observations for $Y(T_i)$ on local intervals. We define local approximation models [*Getmanov et al.*, 2016; 2018] by known functions $y_M(c_s, T_i)$, that depend on the parameter vectors c_s, $s = 1, \ldots, s_0$, and local functionals $F(c_s, Y_s)$, and formulate the problems of local approximation of estimation c_s^0

$$F(c_s, Y_s) = \sum_{i=N_{1s}}^{N_{2s}} (Y_s(T_i) - y_M(c_s, T_i))^2, \; c_s^0 = \arg\{\min_{c_s}(F(c_s, Y_s))\}, \; s = 1, \ldots, s_0. \tag{3}$$

We obtain local functions on local intervals based on c_s^0 (3) local functions $p_{j,s}^0(c_s^0, T_i)$ taking into account (1), (2) for $i < N_{1s}, N_{2s>i}$ and $p_{j,s}^0(c_s^0, T_i) = 0$, $j = 1, \ldots, j_0$. Let us estimate the parametric function with number j

$$p_j^0(c^0, T_i) = \sum_{s=1}^{s_0} p_{j,s}^0(c_s^0, T_i), \; j = 1, \ldots, j_0. \tag{4}$$

We take the functions $p_j^0(c^0, T_i)$, $j = 1, \ldots, j_0$ from (4) as the first approximations to the parametric functions $p_j(T_i)$.

At the second stage of approximation, we introduce local intervals of size equal to N_0 , sliding with a step N_d, with the boundaries $N_{1r} \leq i \leq N_{2r}$, where $N_{1r} = 1 + N_d(r-1)$, $N_{2r} = N_{1r} + N_0 - 1$, $r = 1, \ldots, r_0$, $r_0 = \mathrm{ent}((N_{f1} - N_0)/N_d)$ and $N_{f2} = N_d(r_0 - 1) + N_0$, $N_{f2 \leq N_{f1}}$. Let us define sliding models $y_M(d_{j,r}, T_i)$, and parameter vectors $d_{j,r}$. Let us form local functionals $G(d_{j,r}, p_j^0(c^0, T_i))$ and calculate estimates $d_{j,r}^0$

$$G(c^0, d_{jr}, p_j^0) = \sum_{i=N_{1r}}^{N_{2r}} (p_j^0(c^0, T_i) - y_M(d_{j,r}, T_i))^2, \; d_{j,r}^0 = \arg\{\min_{d_{j,r}}(G(d_{j,r}, p_j^0))\}. \tag{5}$$

Using $d_{j,r}^0$ (5) we find the sliding models $p_{j,r}^0(c^0, d_r^0, T_i)$, $N_{1r \leq i \leq N_{2r}}$, $p_{j,r}^0(c^0, d_r^0, T_i) = 0$, $i < N_{1r}$, $N_{2r} > i$, $r = 1, \ldots, r_0$, $j = 1, \ldots, j_0$ and sum them up

$$p_{0j}^0(c^0, d_j^0, T_i) = \sum_{r=1}^{r_0} p_j^0(c^0, d_{r,j}^0, T_i), \; j = 1, \ldots, j_0. \tag{6}$$

The function $p^0_{0j}(c^0, d^0_j, T_i)$ is implemented as a sum with overlaps from sliding, the result (6) is averaged with weighting, as proposed in [*Getmanov et al.*, 2015]. Let us introduce unit functions, find their sum, calculate the weight function $W(T_i)$ and estimates of parametric functions

$$\begin{aligned} &W_{0r}(T_i) = 1,\ N_{1r} \le i \le N_{2r},\\ &W_{0r}(T_i) = 0,\ 1 \le i \le N_{1r}-1,\ N_{2r}+1 \le i \le N_{f2},\\ &W_{0r}(T_i) = \sum_{r=1}^{r_0} W_{0r}(T_i),\ W(T_i) = 1/W_0(T_i). \end{aligned} \tag{7}$$

$$p^0_j(c^0, d^0, T_i) = W(T_i) p^0_{0j}(c^0, d^0, T_i),\ j = 1,\ ldots, j_0. \tag{8}$$

We accept functions $p^0_j(c^0, d^0, T_i)$ based on (7), (8) as second approximations to $p_j(T_i)$ (2).

Let us make a choice of parameters N, N_0, N_d for approximation procedures. We write down the funtional $S_0(N, N_0, N_d, Y)$ and implement its optimization

$$\begin{aligned} &S_0(N, N_0, N_d, Y) = (1/N_{f2}) \sum_{i=1}^{N_{f2}} (Y(T_i) - Y(p^0(c^0(N), d^0(N, N_0, N_d), T_i))^2)\\ &(N^0, N^0_0, N^0_d) = \arg\{\min_{N,N_0,N_d} S_0(N, N_0, N_d, Y)\}. \end{aligned}$$

3. Application of local approximation piecewise sinusoidal models with piecewise linear additive trends for estimating non-stationary parameters of geomagnetic pulsations

In practice, geomagnetic pulsations can be adequately represented by local approximate piecewise sinusoidal models with linear additive trends.

For the first stage, we adopted local models $y_M(c_s, T_i)$ and functionals $F(c_s, Y_s)$

$$y_M(c_s, T_i) = a_s \cos \omega_s T_i + b_s \sin \omega_s T_i + g_{1s} + g_{2s} T_i,\ c^T_s = (a_s, b_s, g_{1s}, g_{2s}, \omega_s), \tag{9}$$

$$F(c_s, Y_s) = \sum_{i=N_{1s}}^{N_{2s}} (Y_s(T_i) - y_M(c_s, T_i))^2,\ s = 1, \dots, s_0. \tag{10}$$

For model (9) frequency boundaries were fixed ω_n, $\omega_{\min} \le \omega_n \le \omega_{\max}$, $\omega_{\min}$, $\omega_{\max}$ partially optimal parameters were calculated $a^0_s(\omega_n)$, $b^0_s(\omega_n)$, $g^0_{1s}(\omega_n)$, $g^0_{2s}(\omega_n)$ and partially optimal sums $F(c^0_s(\omega_n), Y_s)$ (10), minimization was performed by enumeration for $\omega_n = \omega_{\min} + \Delta\omega(n-1)$, $n = 1, \dots, n_f$, $\Delta\omega = (\omega_{\max} - \omega_{\min})/n_f$.

$$\omega^0 = \arg\{\min_{\omega_{\min} \le \omega_n \le \omega_{\max}} F(c^0_s(\omega_n), Y_s)\}. \tag{11}$$

We calculated optimal local parameters $\omega^0_s = \omega^0$, $a^0_s = a^0_s(\omega^0)$, $b^0_s = b^0_s(\omega^0)$, $g^0_{1s}(\omega^0)$, $g^0_{2s}(\omega^0)$ for (11) and evaluation of parameter functions $p^0_1(c^0, T_i) = A^0_L(T_i)$, $p^0_2(c^0, T_i) = f^0_{0L}(T_i)$, $p^0_3(c^0, T_i) = E^0_L(T_i)$, $1 \le i \le N_{f1}$,

$$\begin{aligned} &A^0_L(c^0, T_i) = \sum_{s=1}^{s_0} A^0_s(c^0_s, T_i),\ A^0_s(c^0_s, T_i) = ({a^0_s}^2 + {b^0_s}^2)^{1/2},\ A^0_s(c^0_s, T_i) = 0,\ i < N_{1s},\ i > N_{2s},\\ &f^0_{0L}(c^0, T_i) = \sum_{s=1}^{s_0} f^0_{0s}(c^0_s, T_i),\ f^0_{0s}(c^0_s, T_i) = \omega^0_{0s}/2\pi,\ f^0_{0s}(c^0_s, T_i) = 0,\ i < N_{1s},\ i > N_{2s},\\ &E^0_L(c^0, T_i) = \sum_{s=1}^{s_0} E^0_s(c^0_s, T_i),\ E^0_s(c^0_s, T_i) = (d^0_{1s} + d^0_{2s} T_i),\ E^0_s(c^0_s, T_i) = 0,\ i < N_{1s},\ i > N_{2s}. \end{aligned} \tag{12}$$

At the second stage we used local models $y_{Mj}(d_{r,j}, T_i)$ and functionals $G_j(d_r, Y_r)$ and calculated weighted parameter estimations $p_j^0(c^0, d^0, T_i)$, $j = 1, \dots, j_0$

$$\begin{aligned}
&y_{Mj}(d_{r,j}, T_i) = d_{1r,j} + d_{2r,j}T_i,\ d_{r,j}^T = (d_{1r,j}, d_{2r,j}),\\
&G_j(d_{r,j}, p_j^0(c^0)) = \sum_{i=N_{1r}}^{N_{2r}} (p_j^0(c^0, T_i) - d_{1r,j} - d_{2r,jT_i})^2,\ r = 1, \dots, r_0,\ d_{r,j}^0 = \arg\{\min_{d_{r,j}} G(d_{r,j}, Y_r)\},\\
&p_{0j}^0(c^0, d^0, T_i) = \sum_{r=1}^{r_0} p_j(c^0, d_{r,j}^0, T_i),\ p_j^0(c^0, d^0, T_i) = W(T_i)p_{0j}^0(c^0, d_{r,j}^0, T_i),\\
&p_1^0(c^0, d^0, T_i) = A_W^0(T_i),\ A_W^0(T_i),\ p_2^0(c^0, d^0, T_i) = f_{0W}^0(T_i),\ p_3^0(c^0, d^0, T_i) = E_W^0(T_i).
\end{aligned} \tag{13}$$

4. Estimation of non-stationary parameters for an experimental signal with geomagnetic pulsations

We received the observations $Y(T_i)$ nT, $i = 1, \dots, N_f$ of the experimental signal with pulsations Pc1, station PG4 (Antarctica), 17.12.2016, 14.00–16.00, $N_f = 69144$, $T = 0.1$c, $N_f T = 1.920$ h. Figure 1 shows an image $Y(T_i)$, its non-stationarity for amplitude, trend and frequency is obvious.

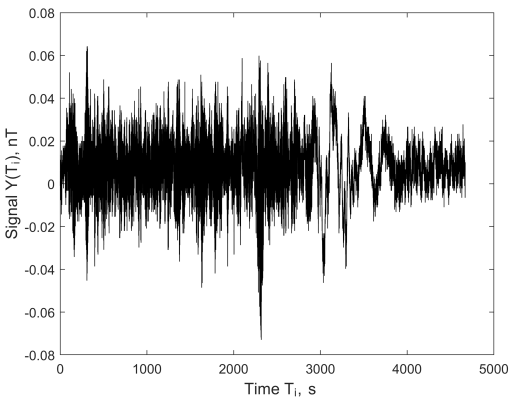

Figure 1. Experimental signal $Y(T_i)$ with pulsations Pc1; station PG4 (Antarctica), 17.12.2016, 14.00–16.00 UT.

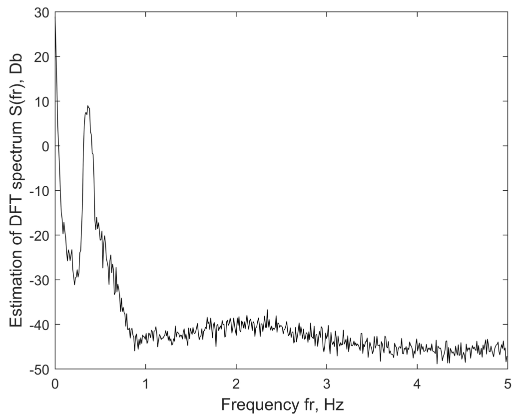

Figure 2. DFT-spectrum $S(f_k)$ for $Y(T_i)$, average frequency $\bar{f}_0 \approx 0.3$ Hz.

Figure 2 shows the DFT-spectrum graph $S(f_k) = \Delta f(k-1)$, $k = 1, \dots, N_S/2$, $N_S = 512$ is the dimension of the DFT. The average frequency of the signal and the trend are $\bar{f}_0 \approx 0.3$ Hz $\bar{f}_E \approx 0.05$ Hz.

We used the local models (9) and functionals (10). The boundaries $\bar{f}_{\min} = 0.005$, $\bar{f}_{\max} = 0.605$ Hz, $n_f = 200$ were established taking into account the DFT spectrum (Figure 2), and the model parameters were found using formulas (12), (13). The optimal parameters of the approximation procedures with ranges $N_0^0 = 2048 \div 4096$, $N_d^0 = 64 \div 128$ were determined using formulas (9), (10).

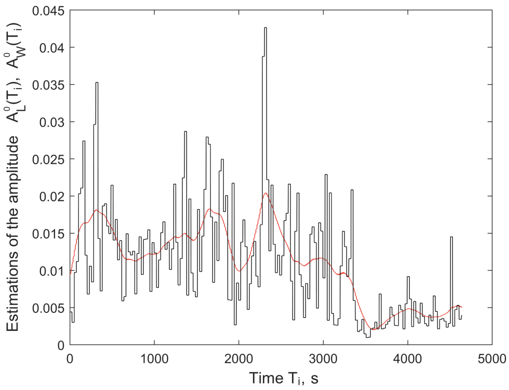

Figure 3. Amplitude estimates for the 1st and 2nd approximation stages.

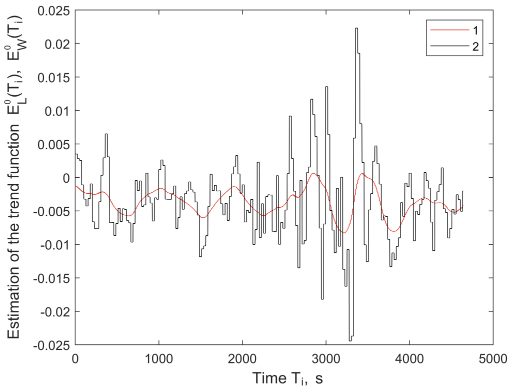

Figure 4. Trend estimates for the 1st and 2nd approximation stages.

Estimates of parametric functions were calculated for $N = 512$, $N_0 = N * 8$, $N_d = N_0/32$. Figure 3 shows amplitude estimations: 1 — $A_L^0(T_i)$, 2 — $A_W^0(T_i)$; Figure 4 shows trend estimates: 1 — $E_L^0(T_i)$, 2 — $E_W^0(T_i)$. Estimates of frequency functions were calculated. Figure 5 (for $N = 512$, $N_0 = N * 8$, $N_d = N_0/32$) and Figure 6 (for $N = 512/2$, $N_0 = N * 16$, $N_d = N_0/8$) show graphs: 1 — $f_{0L}^0(T_i)$, 2 — $f_{0W}^0(T_i)$.

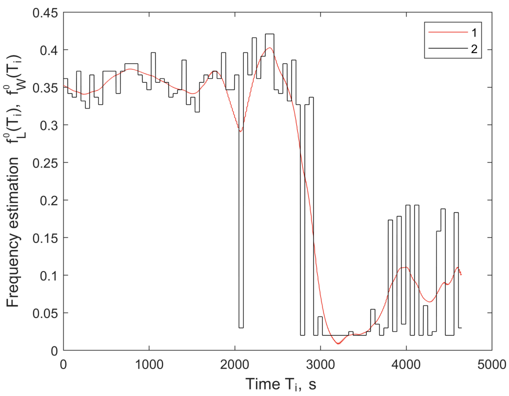

Figure 5. Frequency function estimates for the 1st and 2nd approximation stages, $N = 512$, $N_0 = N * 8$, $N_d = N_0/32$.

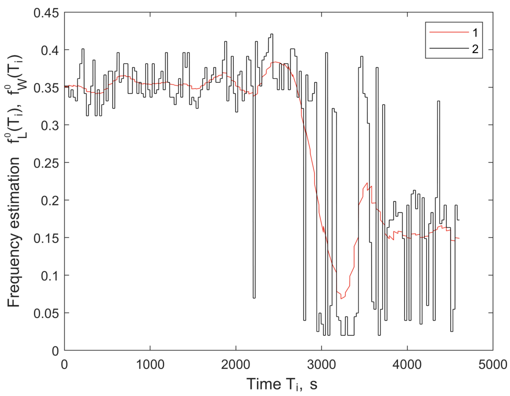

Figure 6. Frequency function estimates for the 1st and 2nd approximation stages, $N = 512/2$, $N_0 = N * 16$, $N_d = N_0/8$.

The results of the calculations in Figure 3–6 showed that the developed technology for estimating non-stationary parameters of geomagnetic pulsations turned out to be efficient and effective.

The mathematical modeling of pulsations and the implemented statistical testing method [*Mixajlov et al.*, 2018] allowed us to conclude that the relative errors in parameter estimates were (3÷4)% for amplitude and trend functions and (4÷5)% for frequency functions. The results obtained made it possible to consider that similar estimations using the proposed technology should also be implemented for experimental signals with pulsations.

The residual sum functional is a multi-extremal function of frequency and the position of its global and local minima depends on the noise in the observations. For large noises, their positions changed places, which led to large errors. The use of weighted averaging significantly reduced the errors in frequency estimates, as can be seen in Figure 5, 6.

5. Conclusion

1. The developed technology for estimating non-stationary parameters of geomagnetic pulsations based on two-stage approximations turned out to be efficient and effective.

2. Mathematical modeling of signals with pulsations allowed us to conclude that the relative errors in parameter estimates were (3÷4)% for amplitude and trend functions and (4÷5)% for frequency functions; the results obtained made it possible to consider that similar estimations using the proposed technology should also be realized for experimental signals with pulsations.
3. It was found that the use of weighted averaging significantly reduced the errors in frequency estimations.
4. The proposed estimation of non-stationary parameters of geomagnetic pulsations based on two-stage approximations has large reserves for improvement and a favorable prospect for use in information problems of geophysics.

Acknowledgments. The work was carried out within the framework of the state assignment for the GC RAS and the IPE RAS, approved by the Ministry of Science and Higher Education of the Russian Federation.

References

Getmanov V. G., Sidorov R. V., Dabagyan R. A. A Method of Filtering Signals Using Local Models and Weighted Averaging Functions // Measurement Techniques. — 2015. — Vol. 58, no. 9. — P. 1029–1036. — DOI: 10.1007/s11018-015-0837-5.

Getmanov V. G., Dabagyan R. A., Sidorov R. V. Studying geomagnetic pulsation characteristics with the local approximation method // Geomagnetism and Aeronomy. — 2016. — Vol. 56, no. 2. — P. 195–202. — DOI: 10.1134/S0016793216020055.

Getmanov V. G., Dovbnya B. V., Kornilov A. S. Estimating the Frequency and Amplitude Parameters of the Serpentine-Emission Type of Geomagnetic Pulsations // Geomagnetism and Aeronomy. — 2018. — Vol. 58, no. 4. — P. 523–532. — DOI: 10.1134/s0016793218040060.

Getmanov V. G. Digital Signal Processing With Applications for Geophysics and Experimental Mechanics. — Moscow : Texnosfera, 2021. — 354 p. — (In Russian).

Guglielmi A. V., Troiczkaya V. A. Geomagnetic Pulsations and Diagnosis of the Magnetosphere. — Moscow : Nauka, 1973. — 208 p. — (In Russian).

Guglielmi A. V., Potapov A. S., Dovbnya B. V. On the origin of frequency modulation of serpentine emission // Solnechno-Zemnaya Fizika. — 2015. — Vol. 1, no. 2. — P. 85–90. — DOI: 10.12737/9617. — (In Russian).

Katkovnik V., Egiazarian K., Astola J. Local Approximation Techniques in Signal and Image Processing. — SPIE, 2006. — 576 p. — DOI: 10.1117/3.660178.

Klejmyonova N. G. Geomagnetic Pulsations // Models of the Space / ed. by M. I. Panasyuk. — Moscow : MGU, 2007. — P. 511–627. — (In Russian).

Le'j E. Digital Signal Processing for Engineers and Technical Specialists. — Moscow : Gruppa IDT, 2007. — 336 p. — (In Russian).

Mixajlov G. A., Vojtishek A. V. Numerical Statistical Modeling. Monte Carlo Method. — 2018. — (In Russian).

Potapov A. S., Polyushkina T. N., Tsegmed B. Morphology and diagnostic potential of the ionospheric Alfvén resonator // Solar-Terrestrial Physics. — 2021. — Vol. 7, no. 3. — P. 36–52. — DOI: 10.12737/stp-73202104.

Time-Frequency Analysis: Concepts and Methods / ed. by F. Hlawatsch, F. Auger. — Wiley, 2008. — 434 p. — DOI: 10.1002/9780470611203.