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Density Model Creation Based on Separating Gravity Field by Depth

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Abstract: As a rule, it is known from a priori data that the studied field anomalies are caused by geological structures located at a certain depth below the day surface. Separation of anomalies of the observed potential field by depth and their connection with deep objects can form the basis of interpretation schemes for modeling problems. The classical method of field separation includes spectral filtering with subsequent analytical continuation of the separated anomalies. We propose an original method of height-based transformations of potential fields based on solving the inverse problem of analytical continuation of harmonic functions from a plane to the "inner" half-space. This problem is reduced to solving the Fredholm integral equation of the first kind for the Poisson integral, which can be used to represent a harmonic function in the "outer" half-space by its boundary values on the plane. The parallel algorithm for solving the integral equation is implemented on graphics accelerators using the NVidia CUDA and AMD ROCm libraries in the application software. The results of the method application are shown on the example of separation of the vertical component of the gravity field in the Bouguer reduction for the Sarginskaya area (Urals, Russia). For this territory, a detailed 3D density model was created by solving the linear inverse problem of gravimetry.

Keywords: analytical continuation of potential fields, linear inverse problem of gravimetry, 3D density model.

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Research Article

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1. Introduction

Volumetric seismic density models of the Earth's crust provide new information about the internal structure of the Earth and significantly expand the possibilities of forecasting and prospecting for mineral deposits, especially hydrocarbon deposits. Various methods for constructing density models are considered in a number of works. The article [*Nurmukhamedov et al.*, 2020] presents a geological and geophysical model of the structure of the Earth's crust and upper mantle along the regional profile of the settlement of Apacha — Mutnaya Bay in the south of Kamchatka. The two-dimensional model was created based on a comprehensive interpretation of materials from the method of converted waves from remote earthquakes, gravity exploration and magnetotelluric sounding. The work [*Sidorov et al.*, 2022] presents a methodology for creating a density volumetric model of the Earth's crust and upper mantle for the southern part of the Kamchatka Peninsula. The model was created based on the results of interpreting Bouguer anomalies and deep soundings using seismic exploration and electrical exploration methods. In the article [*Sharov et al.*, 2020], a model of the structure of the crystalline part of the Earth's crust of the White Sea region is compiled based on the analysis of geological and geophysical data. Data on deep seismic sounding (DSS) profiles are used, and an analysis of petrophysical, geothermal, and gravimetric data is performed.

Note that, unlike the works of other researchers, the method of constructing three-dimensional models of the Earth's crust used by us is based on the author's algorithms for solving direct and inverse gravimetry problems and is implemented in automated computer technology using graphic accelerators, which allows a thousandfold reduction in the time of calculating the density values in the grid nodes. For the Ural region and adjacent territories of the East European platform and the West Siberian plate, a volumetric model of density distribution (with a discretization step of 500 m) in the layers of the Earth's crust and upper mantle to a depth of 80 km was previously created [*Byzov et al.*, 2024]. In this paper, the regional model will be refined and detailed (up to 50 m) within the Sarginskaya area to a depth of 5 km. An important preliminary stage for this is the separation of local gravity field anomalies by depth.

Method of separating fields by depth

To separate the gravity field anomalies by depth, the idea of height-based transformations is used, first proposed in the work [*Martyshko et al.*, 2003], but significantly improved by the Authors in terms of the application methodology and implementation in the form of an effective numerical algorithm [*Martyshko et al.*, 2021]. We will describe the recalculation methodology. We assume that the initial separable field $g(x,y,0)$ is specified on the plane $z = 0$. The general scheme of the method for isolating the effect of sources in the layer from the Earth's surface to a certain depth $z = -H$ consists of three stages:

1. The field is analytically continued upward to the level $z = H$: $g(x,y,0) \overset{\mathrm{up}(H)}{\Rightarrow} g(x,y,H)$, while we assume that the influence of local near-surface sources (to a depth $z = -H$), if not completely eliminated, then significantly weakens.
2. In order to “get rid” of the influence of local sources located in the horizontal layer from the daytime surface $z = 0$ to the depth $z = -H$, the field $g(x,y,H)$ recalculated upwards is then analytically continued downwards to the depth $z = -H$: $g(x,y,H) \overset{\mathrm{down}(2H,\kappa)}{\Rightarrow} u(x,y,-H \mid (-\infty,-H])$. The resulting field can be considered as a field from sources located below the boundary $z = -H$, defined on the plane $z = -H$.
3. At the last step, the field $u(x,y,-H \mid (-\infty,-H])$ is recalculated again upwards to the level of the daytime surface $z = 0$: $u(x,y,-H \mid (-\infty,-H]) \overset{up(H)}{\Rightarrow} u(x,y,0 \mid (-\infty,-H])$.

The resulting field can be considered as a field from sources located below the boundary $z = -H$, defined on the plane $z = 0$. Next, we subtract this field from the original one and obtain the field from the layer $z \in (-H, 0]$: $u(x,y,0 \mid (-H,0]) = g(x,y,0) - u(x,y,0 \mid (-\infty,-H])$.

Let us assume that the gravitating masses are located in a layer below the horizontal plane with the applicate z. On this plane, we denote the gravitational field by $u(\cdot,\cdot,z)$ and take it as the boundary function of the Dirichlet problem for the Laplace equation over a semi-infinite domain. From the values on the boundary, the solution $u(\cdot,\cdot,\zeta)$ of this problem restores the harmonic function of the field everywhere above z. Thus, for points of the upper half-space $\zeta \geq z$, the solution $u(\cdot,\cdot,\zeta)$ of the problem can be written in terms of the Poisson integral [*Blakely*, 1995]:

$$u(\xi,\eta,\zeta) = \frac{\zeta - z}{2\pi} \iint_{-\infty}^{+\infty} \frac{u(x,y,z)dxdy}{((x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2)^{\frac{3}{2}}}. \tag{1}$$

The operation $u(\cdot,\cdot,z) \overset{\mathrm{up}(H)}{\Rightarrow} u(\cdot,\cdot,\zeta)$ of recalculation upwards by the difference in heights (depths) $H = \zeta - z \geq 0$ is a direct taking of the integral in formula (1). The operation $u(\cdot,\cdot,\zeta) \overset{\mathrm{down}(H,\kappa)}{\Rightarrow} u(\cdot,\cdot,z)$ of recalculation downwards by the difference in heights (depths) $H = \zeta - z \geq 0$ is a solution to the Fredholm integral equation of the first kind (1), where the values of the field $u(\cdot,\cdot,\zeta)$ are considered given, and it is necessary to find the values of $u(\cdot,\cdot,z)$ under the integral sign. In the general case, finding a solution to this equation is an ill-posed problem requiring regularization. Since the kernel of integral (1) is symmetric and positive definite, we can apply M. M. Lavrentiev's regularization [*Lavrentiev*, 1967]. The regularized equation has the form

$$u(\xi,\eta,\zeta) = \kappa u(\xi,\eta,z) + \frac{\zeta - z}{2\pi} \iint_{-\infty}^{+\infty} \frac{u(x,y,z)dxdy}{((x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2)^{\frac{3}{2}}},$$

where ζ and z are fixed, $\kappa > 0$ is the regularization parameter. This equation, for any left-hand side from L_2, necessarily has exactly one solution from L_2 and it depends continuously on κ [*Lavrentiev*, 1967].

Let us consider the proposed method for extracting from the general observed field $g(x,y,0)$ the gravitational effect $u(x,y,0 \mid (-\infty,-H])$ of sources in the half-space below some depth $z = -H < 0$: $g(x,y,0) \overset{\mathrm{up}(H)}{\Rightarrow} g(x,y,H) \overset{\mathrm{down}(2H,\kappa)}{\Rightarrow} u(x,y,-H \mid (-\infty,-H]) \overset{\mathrm{up}(H)}{\Rightarrow} u(x,y,0 \mid (-\infty,-H])$. We assume that $g(x,y,0)$ is specified exactly by its piecewise constant representation. If the operations $\mathrm{up}(H)$ and $\mathrm{down}(H,0)$ were performed analytically exactly, then, of course, in As a result, we would get $u(x,y,0 \mid (-\infty,-H]) \equiv g(x,y,0)$. If we strive for the greatest accuracy in implementing the recalculation scheme using the analytical continuation of harmonic functions, then no separation of fields will be obtained. In fact, formal regularization was introduced into the $\mathrm{down}(H,\kappa)$ operation for the purposes of separation.

The result $u(x,y,0 \mid (-\infty,-H])$ of the three-stage recalculation scheme continuously depends on the regularization parameter κ, in addition, the larger κ, the smoother it is. Let us divide the observed field $g(x,y,0)$ into L horizontal layers with depth intervals $((-H_{i+1},-H_i])_{i=0}^{L-1}$ and take as the field of this layer $u(x,y,0 \mid (-H_{i+1},-H_i]) = u(x,y,0 \mid (-\infty,-H_i]) - u(x,y,0 \mid (-\infty,-H_{i+1}])$, while fields $u(x,y,0 \mid (-\infty,-H_i])$ are obtained using the regularizers κ_i, with the exception of $u(x,y,0 \mid (-\infty,-H_0]) = g(x,y,0)$, $H_0 = 0$, $\kappa_0 = 0$. It turns out that to fulfill the condition of “continuous” joining of separated fields $u(x,y,0 \mid (-H_{i+1},-H_i])$ of adjacent layers, we need to choose κ_i in ascending order between $\kappa_0 = 0$ and κ_L without sharp transitions from κ_{i-1} to κ_i. Of course, there are infinitely many variants of “continuous” increasing sequences of κ_i, and these variants can give significantly different recalculations and, accordingly, density models, but this is where the non-uniqueness of the solution of the linear inverse problem of gravimetry is manifested. The choice of one of the variants falls on the shoulders of the interpreter, this is an element of subjectivity. The big plus is that small changes in the sequence κ_i will lead to small changes in the separated fields and the resulting density model.

Thus, κ is used in the recalculations not for regularization, but as a continuous filtering factor (the larger κ, the larger the band of “high frequencies” we filter). For the purposes of separating fields, filters operating in the “frequency” region [*Serbulenko*, 1967] can also be used, but, in the opinion of the Authors, they are less clear in terms of binding to specific depths.

2. Creation of a density model of Sarginskaya area

As a practical example of the application of the method of separating fields by depth, we will consider the construction of a 3D density model of the Sarginskaya area. The digital model of the gravity field in the Bouguer reduction for the specified territory (within the boundaries of sheets O-40-XXIII, XXVIII, XXIX, XXXIV, XXXV) was taken from the electronic version of the report of the Bazhenov Geophysical Expedition [*Sokolova et al.*, 2010]. The grid step along the X and Y coordinates is 50 m, the total area is 3750 km^2, the declared root-mean-square error of the digital model is 60 μGal. As an initial approximation model for interpreting the field, a regional 3D density model with a discretization step of 500 m for the territory of sheets O-40, O-41, previously constructed by the Authors [*Byzov et al.*, 2024], was taken. The field of the initial approximation model was subtracted from the interpreted field. The range of variations of the resulting difference field is ±1.9 mGal, the standard deviation is 0.45 mGal. Then the difference field was divided according to our method into 100 terms corresponding to sources in layers limited by horizontal planes at successive depths from 0 km to 5 km with a step of 50 m. In this case, a restriction was imposed during the division process: for each layer, the 1st and 99th percentiles of the corresponding field should deviate from 0 by approximately 18 μGal. Due to the continuous dependence of the field-result of the division method on the parameter κ, a field satisfying these restrictions can be found for each layer using a simple binary search for κ. Naturally, each layer will have its own κ. Figure 1 (on the left) shows the statistical indicators of a set of fields divided for each layer. The ordinate axis shows the depth to the upper boundary of the layer, the abscissa axis shows the minimum, maximum, 1st and 99th percentiles, and standard deviation corresponding to its field. The limitation of 18 μGal was chosen so that when performing the separation procedure, the difference field would "end" at a depth of up to 5 km. If we take the limitation of 17 μGal, then the difference field will "end" after a depth of 5 km.

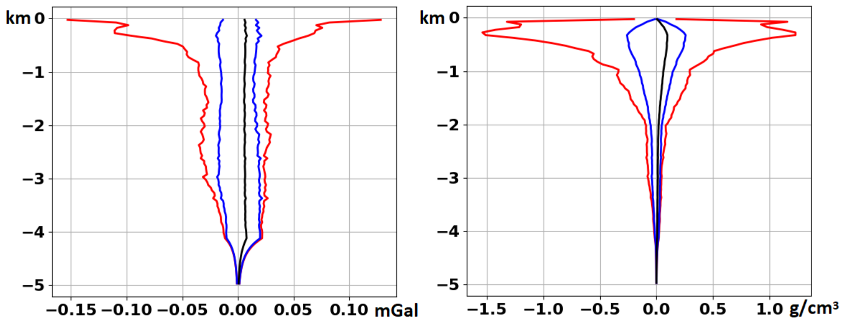

Figure 1. Statistical indicators for horizontal layers at depths plotted along the ordinate axis: on the left – separated fields, on the right – corresponding density distributions. Red lines – minimum and maximum, blue – 1st and 99th percentiles, black – standard deviation.

After the separation procedure, a lateral density distribution was selected independently in each horizontal layer by the corresponding field by solving the linear inverse problem of gravity exploration. As is known [*Novoselitskii*, 1965], the problem of finding a laterally variable density in a horizontal layer has a unique solution that continuously depends on the input field. Figure 1 (on the right) shows the statistical indicators of the found density distributions for all layers. It can be seen that the 1st and 99th percentiles at all depths "fit" into $0.2\,\mathrm{g/cm^3}$, but the minimum and maximum in the upper layers reach values of $-1.5\,\mathrm{g/cm^3}$ and $1.1\,\mathrm{g/cm^3}$, respectively. For a difference field, these are fairly large density values, and we will adjust the initial model by these values. Note that there are less than 1% of such "bad" points, and almost all additional masses are concentrated at depths of up to 2 km. It has already been indicated above that the densities in the layer continuously depend on the layer field, and therefore on the parameter κ. Therefore, the restrictions on the 1st and 99th percentiles could be imposed not on the divided field, but directly on the density distribution in the layer, then it would be possible to achieve a more uniform (or even arbitrary, at the interpreter's discretion) distribution of masses by depth. However, this is a task for future research.

After calculating the density distribution by the difference field, it was added to the regional model. And thus, the final 3D density model of the Sarginskaya area was obtained with a discretization step of 50 m to a depth of 5 km (Figure 2). The profile along which vertical sections of the regional and final models were made for comparison is marked with a red line above (Figure 3).

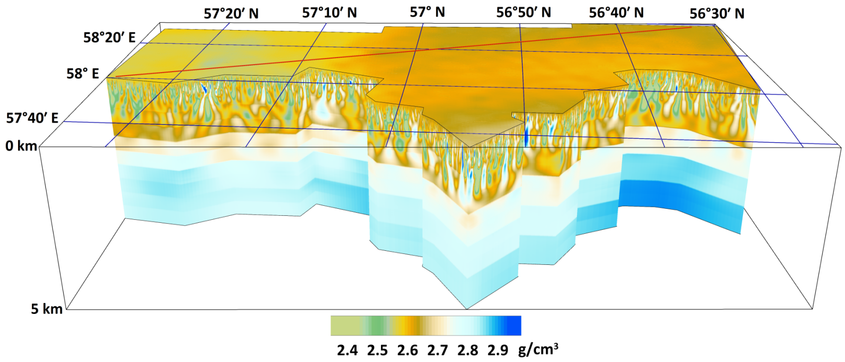

Figure 2. General view of the final 3D density model of Sarginskaya area.

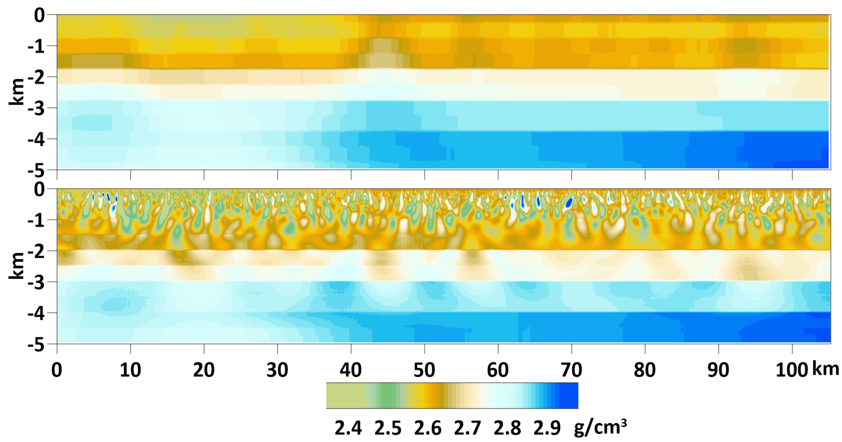

Figure 3. Vertical sections of the regional (top) and final (bottom) 3D density models.

3. Conclusion

A grid density model of the Earth's crust of the Sarginskaya area of the eastern part of the Volga-Ural oil and gas province was constructed to a depth of 5 km with a grid step of 50 m, on which local heterogeneities are clearly visible. The method is based on parallel algorithms for solving the equation of the first kind. A regional density model, previously constructed by the authors based on seismic-gravity modeling, was used as the initial model for solving the inverse gravimetry problem.

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