

Waveform of the Reflected Impulse at the Oblique Sounding of the Sea Surface

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Abstract: The height of sea waves is one of the most important characteristics describing the wave climate of the ocean. At the present, the main radar for remote measurement of wave heights is an altimeter. Measurements are performed at the vertical sounding (incidence angle equal to zero). The Brown model was developed to describe the waveform of the reflected impulse at the vertical sounding. There is no theoretical model for the case of oblique sounding. In the Kirchhoff approximation, the theoretical task about waveform of the reflected impulse at oblique sounding was considered. In the result of the investigation, the analytical formula for the waveform of the reflected impulse for oblique sounding at the small incidence angles (< ¹²◦) for a microwave radar with a narrow antenna beam was obtained. The waveform of the reflected impulse depends on the width of antenna beam, incidence angle, impulse duration, significant wave height (SWH), altitude of the radar, mean square slopes of large-scale, in comparison with radar wavelength, sea waves. It is shown that possibility exist to retrieve SWH using waveform the reflected impulse at the oblique sounding.

Keywords: altimeter, waveform of the reflected impulse, oblique sounding, significant wave height

Citation: Karaev, V. Yu., Yu. A. Titchenko, M. A. Panfilova, E. M. Meshkov, and D. A. Kovaldov (2024), Waveform of the Reflected Impulse at the Oblique Sounding of the Sea Surface, *Russian Journal of Earth Sciences*, 24, ES3004, EDN: [RZDKCC,](https://elibrary.ru/rzdkcc) <https://doi.org/10.2205/2024es000910>

1. Introduction

Over a long time, regular measurements at small incidence angles have been performed by two types of radars: 1) altimeters, that operate during nadir sounding (incidence angle ∼ 0 degrees) [*[Fu and Cazenave](#page-7-0)*, [2000;](#page-7-0) *[Zieger et al.](#page-8-0)*, [1991\]](#page-8-0) and 2) dual-frequency precipitation radar of the GPM satellite [*[Japan Aerospace Exploration Agency](#page-8-1)*, [2014\]](#page-8-1) (Global Precipitation Measurement), which operates in scanning mode (incidence angle ∼ ±18 degrees). In 2018, the Chinese-French satellite (CFOSAT) was launched into orbit, on board of which a SWIM radar was installed. SWIM is performing measurements at the small incidence angles (0–10 degrees) [*[Hauser et al.](#page-7-1)*, [2001,](#page-7-1) [2017\]](#page-7-2). Its main task is to measure the spatial spectrum of sea waves. In 2022, the SWOT satellite was launched into orbit, which also performs measurements at low incidence angles (< 10 degrees) [*[Fu et al.](#page-7-3)*, [2009;](#page-7-3) *[NASA](#page-8-2)*, [2024\]](#page-8-2). SWOT performs a global survey of the Earth's surface water, collecting detailed measurements of how water bodies change over time.

It is usually assumed that backscattering at small incidence angles is quasi-specular and occurs in facets of the wave profile oriented perpendicular to the incident electromagnetic waves. In this case, the Kirchhoff method is used to describe the backscattered field [*[Barrick](#page-7-4)*, [1968;](#page-7-4) *[Garnaker'yan](#page-7-5)*, [1978;](#page-7-5) *[Isakovich](#page-8-3)*, [1952;](#page-8-3) *[Valenzuela](#page-8-4)*, [1978\]](#page-8-4). In the microwave range, a two-scale model (TSM) is used to describe the sea surface and the wave spectrum is divided into two parts: large-scale sea waves and small-scale sea waves compared to the radar wavelength [*[Fuks](#page-7-6)*, [1966;](#page-7-6) *[Kurjanov](#page-8-5)*, [1962;](#page-8-5) *[Wright](#page-8-6)*, [1968\]](#page-8-6).

This model for describing the scattering of electromagnetic waves at small incidence angles made it possible to obtain an analytical formula for the waveform of the reflected impulse (Brown model) during nadir sounding [*[Brown](#page-7-7)*, [1977\]](#page-7-7). The leading edge of the

RESEARCH ARTICLE

Received: 25 January 2024 Accepted: 8 May 2024 Published: 17 July 2024

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reflected impulse contains information about the significant wave height (SWH) and the developed algorithms make it possible to determine it [*[Fu and Cazenave](#page-7-0)*, [2000\]](#page-7-0). As a result, the orbital altimeter is the only orbital radar that measures the SWH in the World Ocean on a regular basis. If an altimeter with a wide antenna beam is used, then the mean square slopes (*mss*) of sea waves can be determined from the trailing edge of the reflected impulse [*[Karaev, V. Yu. et al.](#page-8-7)*, [2014;](#page-8-7) *[Titchenko, Yu. et al.](#page-8-8)*, [2024\]](#page-8-8). The dual-frequency altimeter can measure the height of low intensity waves [*[Ka and Baskakov](#page-8-9)*, [2015;](#page-8-9) *[Ka et al.](#page-8-10)*, [2016\]](#page-8-10). A special place is occupied by the "delay/Doppler" altimeter which represents a new class of altimeter [*[Raney](#page-8-11)*, [1998,](#page-8-11) [2012\]](#page-8-12). Thus, measurements at small incidence angles are being actively developed.

The disadvantage of the altimeter is that measurements are performed during nadir sounding, and a small deviation from the vertical leads to a rapid deterioration in precision. This problem first appeared on the Jason series satellites, where deviations from the vertical of more than 0.5° were observed. This required improving the Brown model, which was done by taking into account the following term in the series expansion in the Amarouch model [*[Amarouche et al.](#page-7-8)*, [2004\]](#page-7-8).

However, the improved model for the waveform of the reflected impulse is only valid for small deviations from the vertical (< 1[°]). Currently there is no model for the waveform of the reflected impulse at small incidence angles $(3^{\circ}-12^{\circ})$. The advantage of measurements at small incidence angles is that measurements can be performed not only along the track, but also in a swath, the width of which is determined by the selected interval of incidence angles.

In this research, within the framework of a two-scale model (TSM) of a scattering surface and the Kirchhoff method, a formula for the waveform of a reflected impulse for a microwave radar with a narrow antenna beam $(\delta_x, \delta_y < 1^\circ)$ is obtained for the first time.

Figure 1. Scheme of measurement: H_0 – altitude of radar, θ_0 – incidence angle, *R*0 – distance to *XY* plane, *R*1 distance until scattering point and $\varsigma(\vec{r},t)$ – height of scattering surface in the point of the backscattering.

2. Initial problem

Let's consider at the measurement scheme in [Fig](#page-1-0)[ure 1.](#page-1-0) The radar is located at an altitude H_0 from the scattering surface. Measurements are taken at the incidence angle θ_0 , the distance to the *XY* plane is R_0 and the distance to the reflection point is R_1 . The scattering surface is described by a random function $\zeta(\vec{r},t)$ with Gaussian function of distribution of heights.

To describe the reflection of microwave signal by the sea surface, the concept of a two-scale model (TSM) is introduced. The scattering surface appears as a large-scale, compared to the radar wavelength, sea surface covered with short ripples. Within the framework of the TSM, sea waves are divided into large-scale waves and small-scale waves (ripples) in comparison with the radar wavelength. It is suggested, that at the small incidence angles $(< 12^{\circ})$, backscattering mechanism is quasi-specular and occurs on the facets of the wave profile oriented perpendicular to the incident electromagnetic waves. The statistics of such facets is determined by the mean square slopes (*mss*) of large-scale waves.

The backscattering electromagnetic field near the receiving antenna is given by the following formula [*[Bass and Fuks](#page-7-9)*, [1979;](#page-7-9) *[Zubkovich](#page-8-13)*, [1968\]](#page-8-13)

$$
E_{\text{scat}} = \frac{R_{\text{eff}} \cdot E_0 k}{2\pi R_0^2 \cos \theta_0 \cdot i} \int\limits_{S} \exp[-2kR \cdot i] \cdot G^2(\vec{r}) d\vec{r},
$$

where R_{eff} is the effective reflection coefficient which introduced instead of Fresnel coefficient to take into account the influence of the ripple on the amplitude of the reflection

signal; E_0 is the amplitude of the emitted field near antenna and S – square of antenna footprint.

To simplify subsequent mathematical transformations, it is assumed that sounding is performed along the *X* axis and the antenna beam in the Cartesian coordinate system can be represented by the following formula [*[Zubkovich](#page-8-13)*, [1968\]](#page-8-13)

$$
G(\vec{r}) = \exp\left[-1.38\left(\frac{\cos^4 \theta_0}{H_0^2 \delta_x^2}(x - x_0)^2 + \frac{y^2 \cos^2 \theta_0}{H_0^2 \delta_y^2}\right)\right],
$$

where δ_x and δ_y are the width of the antenna beam at the 0.5 power level in the elevation and azimuth planes, respectively.

To calculate the dependence the power of the reflected signal on the time (waveform of the backscattered impulse), the following formula are used

$$
P(t) = \left\langle E_{\text{scat}}(t) E_{\text{scat}}^*(t) \right\rangle,
$$

where brackets denote statistical averaging over the sea waves and hence

$$
P(t) \sim \left\langle \int G^2(\vec{r}_1) G^2(\vec{r}_2) \exp \left[-2i\vec{k}\vec{R}_2\right] \exp \left[2i\vec{k}\vec{R}_1\right] d\vec{r}_1 d\vec{r}_2 \right\rangle,
$$

where the distances from radar to the reflection points are R_1 and R_2 .

In this formula and in further transformations, terms will be omitted that are important for calculating the power of the reflected signal (a backscattering radar cross section – RCS), but do not affect the waveform of the reflected impulse.

Let us expand R_1 and R_2 into a series relative to the center of the antenna footprint R_0 and saving only significant terms, obtain the following formula [*[Bass and Fuks](#page-7-9)*, [1979\]](#page-7-9)

$$
P(t) \sim \int G^{4}(\vec{r})
$$

$$
\times \exp\left[-2ik\left(\sin\theta_{0}\rho_{x} + \frac{\cos^{3}\theta_{0}}{H_{0}}(x_{1} - x_{0})\rho_{x} + \frac{\cos\theta_{0}}{H_{0}}y_{1}\rho_{y}\right)\right]
$$

$$
\times \left\langle \exp\left[2ik\cos\theta_{0}(z_{2} - z_{1})\right] \right\rangle d\vec{r}d\vec{\rho},
$$

where was used the following assumption: $\vec{r}_2 = \vec{r}_1 + \vec{\rho} = \vec{r} + \vec{\rho}$. The distribution function of sea wave heights is Gaussian, so averaging over a statistical ensemble is easily performed [*[Bass and Fuks](#page-7-9)*, [1979;](#page-7-9) *[Tikhonov](#page-8-14)*, [1982\]](#page-8-14), and the following formula was obtained

$$
\left\langle \exp\left[2ik\cos\theta_0(\varsigma_2-\varsigma_1)\right]\right\rangle = \exp\left[-2k^2\cos^2\theta_0\left(ms_{xx}\rho_x^2 + mss_{yy}\rho_y^2\right)\right],
$$

where *mssxx* and *mssyy* are *mss* of large-scale waves along *X* and *Y* axis respectively. As a result, the formula for the reflected signal power has the following form

$$
P(t) \sim \int \exp\left[-5.52\left(\frac{\cos^4 \theta_0}{H_0^2 \delta_x^2}(x - x_0)^2 + \frac{\cos^2 \theta_0}{H_0^2 \delta_y^2}y^2\right)\right]
$$

$$
\times \exp\left[-2ik\left(\sin \theta_0 \rho_x + \frac{\cos^3 \theta_0}{H_0}(x - x_0)\rho_x + \frac{\cos \theta_0}{H_0}y\rho_y\right)\right]
$$

$$
\times \exp\left[-2k^2 \cos^2 \theta_0 \left(mss_{xx}\rho_x^2 + mss_{yy}\rho_y^2\right)\right]dxdy d\rho_x d\rho_y.
$$

Dependence of the reflected power on the time describes the waveform of the reflected impulse. After integration over $d\vec{\rho}$, the formula for the waveform of the reflected impulse has the following kind

$$
P(t) \sim \int_{S} \exp\left[-5.52\left(\frac{\cos^{4}\theta_{0}}{H_{0}^{2}\delta_{x}^{2}}(x-x_{0})^{2} + \frac{\cos^{2}\theta_{0}}{H_{0}^{2}\delta_{y}^{2}}y^{2}\right)\right]
$$

$$
\times \exp\left[-\frac{1}{\cos^{2}\theta_{0}mss_{xx}}\left(\frac{\sin\theta_{0}\cos^{3}\theta_{0}}{H_{0}}(x-x_{0}) + \frac{\cos^{6}\theta_{0}}{2H_{0}^{2}}(x-x_{0})^{2}\right)\right]
$$

$$
\times \exp\left[-\frac{y^{2}}{2H_{0}^{2}mss_{yy}}\right]dxdy.
$$

Thus, it is necessary to integrate the following formula

$$
I_{xy} = \int_{S} \exp\left[-5.52\left(\frac{\cos^{4}\theta_{0}}{H_{0}^{2}\delta_{x}^{2}}(x-x_{0})^{2} + \frac{\cos^{2}\theta_{0}}{H_{0}^{2}\delta_{y}^{2}}y^{2}\right)\right]
$$

× $\exp\left[-\frac{1}{\cos^{2}\theta_{0}mss_{xx}}\left(\frac{\sin\theta_{0}\cos^{3}\theta_{0}}{H_{0}}(x-x_{0}) + \frac{\cos^{6}\theta_{0}}{2H_{0}^{2}}(x-x_{0})^{2}\right)\right]$ (1)
× $\exp\left[-\frac{y^{2}}{2H_{0}^{2}ms_{yy}}\right]dxdy.$

The problem of integration of formula [\(1\)](#page-3-0) over a footprint is not simple. However, with oblique sounding and short impulse duration, the task is simplified.

In a conventional altimeter (incidence angle equal to zero), the incident spherical wave reaches the surface at a point directly below the radar and illuminates a circle, which eventually turns into a running ring.

Figure 2. Scheme of measurement.

During oblique sounding (the incidence angle is nonzero), the incident wave also illuminates a circle on the reflecting surface, the size of which increases with time. The maximum size of the circle is determined by the duration of the emitted impulse. After this, the circle turns into a running ring, the radius of which increases over time. The antenna beam forms an footprint on the surface and at some point of time the running ring reaches the footprint (see [Figure 2\)](#page-3-1). At this moment, the receiving antenna receives the reflected signal.

When integrating over *dx* and *dy* it is necessary to take into account the features of oblique sounding. The boundaries of the ring are specified by *R*left and *R*right and its width depends on the impulse duration *τ*imp, because $R_{\text{right}} - R_{\text{left}} = c\tau_{\text{imp}}/2$ (see [Figure 2\)](#page-3-1), where *c* – speed of light.

On the other hand, they can be expressed in terms of the incidence angles for the right and left boundaries of the ring, i.e.

$$
H_0 = R_{\text{right}} \cos \theta_{\text{right}}, \qquad H_0 = R_{\text{left}} \cos \theta_{\text{left}}.
$$

Thus, it is possible to calculate the difference in incidence angles between the left and right boundaries of the ring, i.e. determine the width of the ring in degrees. This it is necessary for comparison with the antenna beam.

Figure 3. Dependence of the width of ring on the incidence angle: *H*₀ = 10,000 m and τ_{imp} = 3.2 ns.

[Figure 3](#page-4-0) shows the dependence of the ring width in degrees on the current incidence angle for an altitude of 10,000 m (airplane) and impulse duration 3.2 ns. Since the current incidence angle depends on time, this dependence can also be interpreted as the dependence of the difference in angles on the propagation time. As the incidence angle increases, the size of the ring "narrows" in degrees.

The second factor affecting the width of the ring is the impulse duration. The shorter the impulse, the narrower the ring.

Let a radar (altimeter) has a narrow antenna beam, for example 0.2°–0.5°. For a short impulse used in altimetry, the width of the ring is much smaller than the radar footprint. This fact simplifies further transformations. In this case it is possible to assume that the integrands with respect to *X* are "slow" and practically do not change over the width of the ring. This allows to take them out from under the sign of the integral over *X* and take the integral over *Y* over infinite limits. The integral over *X* will give us a slowly varying function, which can be considered a constant in radar footprint.

After integration [\(1\)](#page-3-0), the formula for the dependence of the power of the reflected signal on time (the waveform of the reflected impulse) was obtained

$$
P(t) \sim \exp[-(x - x_0)^2 A_x] \exp[-A_{xx}(x - x_0)], \qquad (2)
$$

where
$$
A_x = 5.52 \frac{\cos^4 \theta_0}{H_0^2 \delta_x^2} + \frac{\cos^4 \theta_0}{2H_0^2 m s_{xx}}, A_{xx} = \frac{\sin \theta_0 \cos \theta_0}{H_0 m s_{xx}}.
$$

A comparison of the waveform of reflected impulses for vertical (left) [*[Karaev, V. Yu.](#page-8-7) [et al.](#page-8-7)*, [2014\]](#page-8-7) and oblique (right) sounding is shown in [Figure 4.](#page-4-1) Calculations were done for $\tau_{\text{imp}} = 3.2 \text{ ns}, H_0 = 10,000 \text{ m}, \delta = 0.5^\circ, \theta = 8^\circ.$

Figure 4. Dependence of the power of the reflected signal on two-way time for nadir sounding (top) and oblique sounding (8° – bottom): impulse duration 3.2 ns, width of antenna beam 0.5°, distance 10000 m.

From the comparison it is clear that the reflected impulse during vertical sounding of a flat surface is much narrower than the reflected impulse during oblique sounding. This is explained by the fact that in the first case, the duration of the reflected impulse is largely determined by the duration of the incident impulse. Thanks to this, altimeters accurately measure the distance from the radar to the reflecting surface [*[Fu and Cazenave](#page-7-0)*, [2000\]](#page-7-0).

At the oblique sounding, the width of the reflected impulse strongly depends on the incidence angle and the width of the antenna beam (footprint). The narrow ring "runs" across the antenna beam and repeats its form therefore the reflected impulse is significantly broadened. In result, the precision of measuring the distance from the radar to the reflecting surface with an oblique altimeter will be low.

Now consider backscattering from a sea surface that is not flat. Conventional altimeters measure the SWH with good precision [*[Fu and Cazenave](#page-7-0)*, [2000\]](#page-7-0). To assess the possibility of measuring the SWH using an oblique altimeter, a formula for the waveform of the reflected impulse was obtained.

To move from a flat surface to sea waves, it is necessary to perform averaging over heights of sea waves. As is known, the height distribution function of sea waves is Gaussian. However, for further transformations it is better to move to its representation in the time domain, i.e.

$$
W(\tau) = \frac{\exp\left[-\frac{\tau^2}{2\sigma_t^2}\right]}{\sqrt{2\pi\sigma_t^2}},
$$
\n(3)

where $t = \frac{2\varsigma}{c}$ *c*

.

Then the waveform of the reflected impulse is a convolution of the waveform of the impulse from a flat surface (2) and the distribution function of sea wave heights (3)

Z $R_{\textit{tek}}$ H_{α} x_{tek} \overline{X}

Figure 5. Scheme of sounding.

Both integrands in formula [\(4\)](#page-5-1) depend on time. Consequently, for further transformations it is necessary to move from spatial coordinates to time in the formula [\(2\)](#page-4-2) for waveform of the impulse. [Figure 5](#page-5-2) shows the sounding scheme and the position of the current point x_{tek} is given by the following formula

$$
x_{\text{tek}}^2 = R_{\text{tek}}^2 - H_0^2 \cong H_0 c \tau_{\text{tek}},
$$

where time τ_{tek} is counted from the moment the leading edge of the incident impulse touches the *XY* plane $(\theta_0 = 0^\circ).$

Then the difference $x - x_0$ will take the form

$$
x - x_0 = \sqrt{H_0 c \tau_{\text{tek}}} - \sqrt{H_0 c \tau_0} = (\sqrt{\tau_{\text{tek}}} - \sqrt{\tau_0}) \sqrt{H_0 c},
$$

where τ_0 corresponds to the center of the footprint of antenna beam on the *XY* plane. This replacement complicates the integrand in formula [\(4\)](#page-5-1) and obtaining an

analytical expression for the waveform of the reflected impulse becomes impossible. Let us take advantage of the fact that the antenna beam is assumed to be narrow and in the formation of the reflected impulse it will take a surface area near the center of the footprint, i.e. near point x_0 or time τ_0 . Then

$$
x = \sqrt{H_0 c (\tau_0 + \Delta \tau)} \approx \sqrt{H_0 c \tau_0} + \frac{H_0 c}{2 \sqrt{H_0 c \tau_0}} \Delta \tau - \frac{H_0^2 c^2}{4 (H_0 c \tau_0)^{3/2}} \Delta \tau^2.
$$
 (5)

For given parameters of the measurement scheme, the third term of the expansion [\(5\)](#page-5-3) can be neglected. As a result, it is assumed that

$$
x - x_0 \cong \frac{1}{2} \sqrt{\frac{H_0 c}{\tau_0}} \Delta \tau.
$$

Time τ_0 corresponding to the center of the antenna beam can be expressed in terms of the incidence angle and altitude

$$
\tau_0 = \frac{H_0 \, \text{tg}^2 \, \theta_0}{c}.
$$

To find the waveform of the reflected impulse, it is necessary to calculate the following integral

$$
F(t) \sim \int \exp\left[-\frac{\tau^2}{2\sigma_t^2}\right] \exp\left[-\frac{A_x c^2}{4 \text{tg}^2 \theta_0}(\tau - t)^2\right] \exp\left[-\frac{A_{xx}c}{2 \text{tg} \theta_0}(\tau - t)\right] d\tau.
$$

As a result of further transformations, an analytical formula was obtained for the waveform of the reflected impulse during oblique sounding

$$
F(\tau) \sim \exp\bigg[-\tau^2 \frac{A_x c^2}{4 \text{tg}^2 \theta_0} \bigg(1 - \frac{4A_x \sigma_\varsigma^2}{c_x}\bigg)\bigg] \exp\bigg[-\tau \frac{A_{xx} c}{2 \text{tg}^2 \theta_x} \bigg(1 - \frac{4A_x \sigma_\varsigma^2}{c_x}\bigg)\bigg],\tag{6}
$$

where
$$
c_x = 2 \text{tg}^2 \theta_0 + 4A_x \sigma_\varsigma^2
$$
.

Figure 6. Waveforms of the reflected impulses for three values of SWH: 1 m (black curve), 3 m (red curve) and 5 m (blue curve). Altitude $H_0 = 10,000$ m; width of antenna beam $\delta_x = 0.2^\circ$; incidence angle $\theta_0 = 8^\circ$.

3. Discussion

The waveform of the reflected impulse depends on the following parameters: the width of the antenna beam, the SWH, the incidence angle, the altitude of the radar, the *mss* of large-scale waves.

The slopes *(mss*) has a significant impact on the properties of the backscattered signal for the radar with a wide antenna beam [*[Karaev, V. Yu. et al.](#page-8-15)*, [2002,](#page-8-15) [2005;](#page-8-16) *[Meshkov](#page-8-17) [and Karaev](#page-8-17)*, [2004\]](#page-8-17). When using a narrow antenna beam $($ < 1^o), the *mss* does not affect the waveform of the reflected impulse, but only the power of the reflected signal, which in this case is not analysed.

It is seen from formula [\(6\)](#page-6-0), that increasing the distance to the reflecting surface leads to a broadening of the waveform of the reflected impulse. Increasing the incidence angle leads to broadening of the waveform too.

An increase in the width of antenna beam leads to a decrease in the dependence of the waveform of the reflected impulse on the SWH. Therefore, the choice of optimal width of antenna beam depends on the distance to the reflecting surface and the requirements for the

sensitivity of the waveform of the reflected impulse to the SWH (precision of the retrieval algorithm). [Figure 6](#page-6-1) shows reflected impulses for three values of SWH: 1 m (black curve), 3 m (red curve) and 5 m (blue curve); altitude $H_0 = 10,000$ m; width of antenna beam $\delta_x = 0.2^\circ$; incidence angle $\theta_0 = 8^\circ$.

A transition was made to time relative to the center of the antenna beam, so negative time appeared. It can be seen from the figure that the waveform of the reflected impulse is sensitive to changes in the SWH, and when choosing a sufficiently narrow antenna beam, the SWH can be determined from the waveform of the reflected impulses.

It should be noted that with sounding, the problem of measuring the mean sea level (*msl*) will be solved with a significantly larger error in comparison with nadir sounding.

Thus, with oblique sounding, it is possible to measure the SWH in a wide swath (scanning mode) with high spatial resolution. The distance to the sea surface (altitude of altimeter) will be measured with low precision. In addition, in the scanning mode it will be possible to measure the *mss* of large-scale waves.

4. Conclusions

In the Kirchhoff approximation, the backscattering of the electromagnetic waves of the sea waves was considered. Research was carried out and an analytical formula was obtained for the waveform of the reflected impulse for oblique sounding at the small incidence angles $(< 12°)$ for a microwave radar with a narrow antenna beam.

The waveform of the reflected impulse depends on the width of antenna beam, incidence angle, impulse duration, significant wave height, distance from radar to sea surface, mean square slopes of large-scale sea waves.

It is shown that SWH influences on the waveform of the reflected impulse and hence, may be retrieved. However, it is necessary the use microwave radar with a narrow antenna beam and a short impulse.

To determine the optimal width of antenna beam and impulse duration, it is necessary to use information about the altitude of the altimeter and the incidence angle. Investigation in this direction will be continued.

Acknowledgments. This work was funded by the State assignment FFUF-2024-0033.

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