Supplementary Material to the article "Exact theory of edge diffraction and launching of transverse electric plasmons at two-dimensional junctions"

I. DERIVATION OF THE ELECTROMAGNETIC SCATTERING EQUATION

We proceed to derive the equation governing the edge diffraction, Eq. (1) of the main text. The derivation will proceed in Gaussian units, and a universal form independent of unit system will be finally written down. The diffracted field at a linear junction of 2DES is the superposition of incident field $\mathbf{E}_{\rm inc}(\mathbf{r})$ and field induced by charges and currents in 2DES $\mathbf{E}_{\rm ind}(\mathbf{r})$:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\rm inc}(\mathbf{r}) + \mathbf{E}_{\rm ind}(\mathbf{r}). \tag{S1}$$

We now switch to a particular case of the s-polarization, where the electric field has only the y-component. An explicit form of the field is

$$E_{{\rm inc},y}(\mathbf{r}) = E_0 e^{ikx\cos\theta - ikz\sin\theta}. \tag{S2}$$

The induced field $\mathbf{E}_{\rm ind}(\mathbf{r})$ emerges due to currents with sheet density $\mathbf{j}(\mathbf{r})$ in 2DES. These currents are directed along the junction (y-axis) and have zero divergence. It implies the absence of induced charge density in 2DES and absence of scalar potential in the considered problem (in the Lorentz gauge). Consideration of vector potential $\mathbf{A}_{\rm ind}(\mathbf{r})$ is sufficient for the formulation of the scattering equation. The vector potential $\mathbf{A}_{\rm ind}(\mathbf{r})$ created by current distribution $\mathbf{j}(\mathbf{r})$ is

$$\mathbf{A}_{\rm ind}(\mathbf{r}) = \frac{1}{c}\int \frac{\mathbf{j}(x')e^{ik|\Delta\mathbf{r}|}}{|\Delta\mathbf{r}|} dx'dy', \tag{S3}$$

In the above equations, the integration is performed over all the area of the conductor, $\Delta\mathbf{r} = \mathbf{r'} - \mathbf{r}$ is the distance between observation point and location of the current element, $|\Delta\mathbf{r}| = [(x-x')^2 + (y-y')^2 + z^2]^{1/2}$, $k = \omega/c$ is the wave number. The term $e^{ik|\Delta\mathbf{r}|}$ is responsible for the retarded nature of electromagnetic potentials.

Further simplification is possible taking into account the absence of y-dependence for currents, which is guaranteed by the extended character of the edge. Integration along the edge, i.e. over dy' in the infinite limits, is performed analytically using an identity

$$\int\limits_{-\infty}^{\infty} \frac{dy' e^{ik[(x-x')^2+z^2+y'^2]^{1/2}}}{[(x-x')^2+z^2+y'^2]^{1/2}} = i\pi H_0\left(k|\Delta\mathbf{r}_\perp|\right), \tag{S4}$$

where $|\Delta\mathbf{r}_\perp| = [(x-x')^2+z^2]^{1/2}$ is the transverse distance and H_0 is the Hankel function. Formulation of scattering equation requires the knowledge of currents and fields only in the 2DES plane. For this reason, we shall set $z=0$ in the argument of the Hankel function hitherto.

The final equation is obtained by linking the vector potential and electric field

$$\mathbf{E}_{\rm ind} = \frac{i\omega}{c}\mathbf{A}_{\rm ind}. \tag{S5}$$

Combination of (S1-S5) leads us to the final scattering equation:

$$E(x) = E_0 e^{ik_x x} - \frac{\pi k_0}{c}\int\limits_{-\infty}^{+\infty} H_0\left(k_0|x-x'|\right) j(x')\, dx'. \tag{S6}$$

Expressing the speed of light via free-space impedance in the prefactor of integral in (S6) as $Z_0 = 4\pi/c$, we get a form of equation independent of the unit system

$$E(x) = E_0 e^{ik_x x} - \frac{Z_0 k_0}{4}\int\limits_{-\infty}^{+\infty} H_0\left(k_0|x-x'|\right) j(x')\, dx', \tag{S7}$$

The latter form is also applicable in SI units, where $Z_0 = \sqrt{\mu_0/\varepsilon_0} \approx 377$ Ohm.

II. DETAILS OF THE WIENER-HOPF METHOD

In this section, we comment on some mathematical details of the Wiener-Hopf method omitted in the main text. The Fourier transformed scattering equation has the form:

$$\varepsilon_L(q)\,E_L(q)+\varepsilon_R(q)\,E_R(q)=E_0\left[\frac{-i}{q-(k_x+i\epsilon)}+\frac{i}{q-(k_x-i\epsilon)}\right], \tag{S8}$$

We apply the factorization procedure to the dielectric functions

$$\varepsilon_i(q)=\varepsilon_{i+}(q)\varepsilon_{i-}(q), \tag{S9}$$

where the subscript $i=\{L,R\}$ distinguishes the left and right half-spaces, the plus function is analytic in the upper half-plane (UHP) of the q-variable, the minus function is analytic in the lower half-plane (LHP) of the q-variable. Dividing the scattering equation by $\varepsilon_{R+}(q)\varepsilon_{L-}(q)$, we get

$$\frac{\varepsilon_{L+}(q)}{\varepsilon_{R+}(q)}E_L(q)+\frac{\varepsilon_{R-}(q)}{\varepsilon_{L-}(q)}E_R(q)=\frac{E_0}{\varepsilon_{R+}(q)\,\varepsilon_{L-}(q)}\left[\frac{i}{q-(k_x-i\epsilon)}+\frac{-i}{q-(k_x+i\epsilon)}\right]. \tag{S10}$$

The left hand side of Eq. S10 is factorized completely, as the first term is analytic in the UHP, the second term is analytic in the LHP. The right-hand side still does not have well-defined analytic properties, as so does the product $\varepsilon_{R+}(q)\,\varepsilon_{L-}(q)$. The situation is resolved by the fact that this poorly behaved function is multiplied by the Fourier transform of the incident field. The latter tends to a delta-function as $\epsilon\to 0$:

$$\left[\frac{i}{q-(k_x-i\epsilon)}+\frac{-i}{q-(k_x+i\epsilon)}\right]\to 2\pi\delta(q-k_x). \tag{S11}$$

Keeping in mind taking the limit $\epsilon\to 0$ in the final result, we can safely set $q=k_x$ in the prefactor of Fourier transformed incident field $\varepsilon_{R+}(q)\,\varepsilon_{L-}(q)\to\varepsilon_{R+}(k_x)\,\varepsilon_{L-}(k_x)$. With this trick, the right-hand side is now fully factorized. We have verified that the trick with replacement $q\to k_x$ in the prefactor of delta-like function provides exactly the same result as a more complex method based on the substraction of 'most divergent' fields at infinity. The latter was implemented in a prior work [1].

We further collect the functions analytic in the UHP and in the LHP in the left and right-hand sides of the scattering equation:

$$\frac{\varepsilon_{L+}(q)}{\varepsilon_{R+}(q)}E_L(q)-\frac{E_0}{\varepsilon_{R+}(k_x)\,\varepsilon_{L-}(k_x)}\frac{i}{q-(k_x-i\epsilon)}=-\frac{\varepsilon_{R-}(q)}{\varepsilon_{L-}(q)}E_R(q)-\frac{E_0}{\varepsilon_{R+}(k_x)\,\varepsilon_{L-}(k_x)}\frac{i}{q-(k_x+i\epsilon)}. \tag{S12}$$

The two functions analytic in the upper and lower half-planes, respectively, and having a common stripe of analyticity, define a function $F(q)$ that is entire in the all complex q-plane. The function $F(q)$ therefore reduces to a polynomial,

$$F(q)=c_0+c_1q+c_2q^2+.... \tag{S13}$$

It is possible to show that retaining even the coefficients before the lowest powers of q in $F(q)$ would lead to the divergent field in the real space. Therefore, the function $F(q)$ should be identically zero. To show this explicitly, we write the contribution of $F(q)$ to the total field

$$E_L(q)=\frac{\varepsilon_{R+}(q)}{\varepsilon_{L+}(q)}F(q)+..., \tag{S14}$$

$$E_R(q)=-\frac{\varepsilon_{L-}(q)}{\varepsilon_{R-}(q)}F(q)+..., \tag{S15}$$

where the omitted terms marked by ... come from the incident field. Noting the limiting behavior of $\varepsilon_{i\pm}(q)$ at infinity $\varepsilon_{i\pm}(q)\to 1$, we see that $E_{L/R}(q)$ would have no well-defined Fourier transform if we retain $c_0\neq 0$. Indeed, the real-space field would contain the poorly-defined terms

$$E_L(x)\sim\frac{c_0}{\pi}\int\limits_{-\infty}^{+\infty}e^{iqx}dq. \tag{S16}$$

The absence of inverse Fourier transform for $E_{L/R}(q)$ becomes even more dramatic if we retain higher-order terms c_1, c_2, etc. It is also pronounced for the case of metal-contacted 2DES, where

$$\varepsilon_{L\pm}^{\leftrightarrow} = \frac{\sqrt{k_0}}{\sqrt{k_0 \pm q}}. \tag{S17}$$

Retaining the c_0-term in $F(q)$ would lead to the real-space field given by a divergent integral

$$E_L^{\leftrightarrow}(x) \sim \frac{c_0}{\pi} \int_{-\infty}^{+\infty} \sqrt{k_0 + q} e^{iqx} dq. \tag{S18}$$

At the same time, physical constraints on zero value of electric field in metals require $E_L(x) \equiv 0$ at $x < 0$.

Therefore, physical constraints on finiteness of real-space field constrain $F(q)$ to be identically zero. This leads us to the Wiener-Hopf solutions

$$\frac{\varepsilon_{L+}(q)}{\varepsilon_{R+}(q)} E_L(q) = \frac{E_0}{\varepsilon_{R+}(k_x)\varepsilon_{L-}(k_x)} \frac{i}{q - (k_x - i\epsilon)}, \tag{S19}$$

$$-\frac{\varepsilon_{R-}(q)}{\varepsilon_{L-}(q)} E_R(q) = \frac{E_0}{\varepsilon_{R+}(k_x)\varepsilon_{L-}(k_x)} \frac{i}{q - (k_x + i\epsilon)}. \tag{S20}$$

To reach the form (6) of the main text, we perform a replacement in Eq. S19:

$$\varepsilon_{L-}(k_x) = \frac{\varepsilon_L(k_x)}{\varepsilon_{L+}(k_x)} = \frac{1 + \eta_L/\sin\theta}{\varepsilon_{L+}(k_x)}. \tag{S21}$$

The first equality here follows from the definition of factorized function, the second equality is the result of dielectric function evaluation at $q = k_x$. A similar line of transforms is made in Eq. S20 to reach the form (7) of the main text:

$$\varepsilon_{R+}(k_x) = \frac{\varepsilon_R(k_x)}{\varepsilon_{R-}(k_x)} = \frac{1 + \eta_R/\sin\theta}{\varepsilon_{R-}(k_x)}. \tag{S22}$$

After these replacements, we reach the final result:

$$E_L(q) = \frac{+iE_0}{1 + \eta_L/\sin\theta} \frac{\varepsilon_{R+}(q)}{\varepsilon_{L+}(q)} \frac{\varepsilon_{L+}(k_x)}{\varepsilon_{R+}(k_x)} \frac{1}{q - (k_x - i\epsilon)}, \tag{S23}$$

$$E_R(q) = \frac{-iE_0}{1 + \eta_R/\sin\theta} \frac{\varepsilon_{L-}(q)}{\varepsilon_{R-}(q)} \frac{\varepsilon_{R-}(k_x)}{\varepsilon_{L-}(k_x)} \frac{1}{q - (k_x + i\epsilon)}. \tag{S24}$$

The rationale beyond these replacements is the symmetry of final results with respect to interchanging the left and right media. Further on, in such form it is explicitly seen that split functions $\varepsilon_{i\pm}(q)$ can be determined up to a multiplier that would drop out of the final results.

III. COMPARISON WITH SIMULATIONS

Our analytical solution for electric field in the metal-contacted 2DES, Eq. (9), was compared to the electromagnetic simulations. In these simulations, the local fields $E(x)$ were obtained using the finite-element method applied to the Maxwell's equations. The method was implemented within the commercially available CST Microwave Studio module. The metal and 2DES were treated as 'sheet conductors', moreover, the metal was treated as 'perfect conductor'. To pass from the Fourier spectrum $E_R^{\leftrightarrow}(q)$ given by (9) to the real-space profile $E(x)$, we have performed a numerical inverse Fourier transform with cutoff wave vector $|q_{\max} = 20k_0|$. This cutoff ensures sufficient decay of $E_R^{\leftrightarrow}(q)$ and convergence of the Fourier integrals.

Results of such comparison are shown in Fig. S1 for normal incidence $\theta = \pi/2$ and two values of conductivity, $\eta = 0.1$ (A) and $\eta = -0.8i + 0.01$ (B). The decaying oscillation in (A) is the cylindrical electromagnetic wave radiated by the junction. The oscillation in (B) is the TE plasmon, it has shorter wavelength $\lambda_{pl} < \lambda_0$.

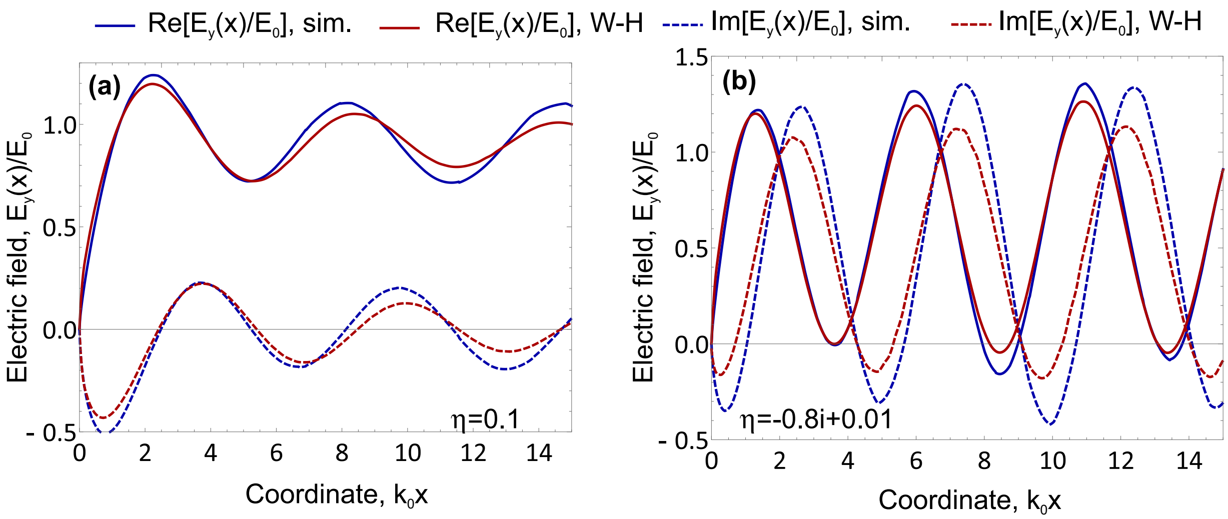

FIG. S1. Comparison of Wiener-Hopf solution of the diffraction problem (red lines, marked as 'W-H') with electromagnetic simulations in the CST Microwave Studio package (blue lines, marked as 'sim.'). Panel (a) is plotted for 2DES conductivity $\eta = 0.1$, panel (b) – for $\eta = -0.8i + 0.01$. In both panels, normal incidence is assumed ($\theta = \pi/2$), the 2DES is contacting a perfectly conducting metal at $x < 0$

The analytical solution agrees fairly well with numerical simulation. A residual discrepancy can be attributed to the sensitivity of the numerical method to the size of simulation box. In the present case, it should largely exceed the wavelength λ_0 due to the slow decay of the scattered fields.

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[1] E. Nikulin, D. Mylnikov, D. Bandurin, and D. Svintsov, Edge diffraction, plasmon launching, and universal absorption enhancement in two-dimensional junctions, Physical Review B **103**, 085306 (2021).