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Acoustic metric and Planck constants

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Based on Akama–Diakonov (AD) theory of emergent tetrads in quantum gravity [1–5] it was suggested [6] that one can introduce two Planck constants, $\hbar$ and $\not{h}$, which are the parameters of the corresponding components of Minkowski metric, $g^{\mu\nu}_{\rm Mink} = {\rm diag}(-\hbar^2, \not{h}^2, \not{h}^2, \not{h}^2)$. In the AD theory, the interval ds is dimensionless, as all the diffeomorphism invariant quantities (we call this "the dimenionless physics"). The metric elements and thus the Planck constants are not diffeomprphism invariant and have nonzero dimensions. The Planck constant $\hbar$ has dimension of time, and the second Planck constant has dimension of length. It is natural to compare $\not{h}$ with the Planck length $l_{\rm P}$. However, this connection remains an open question, because the microscopic (trans-Planckian) physics of the quantum vacuum is not known. Dimensionless physics emerges also in some other approaches. This includes the BF-theories of gravity [7–12] and the model of superplastic vacuum [13] described in terms of the so-called elasticity tetrads [14]. Here we study this question using the effective gravity emerging for sound wave quanta (phonons) in superfluid Bose liquid, where the microscopic physics is known [15, 16]. The interval of the effective acoustic metric is also dimensionless [17, 18].

For massive particles in the non-relativistic limit the wave equation has the following form:

$$i\sqrt{-g^{00}_{\rm Mink}}\partial_t\psi = -\frac{1}{2M}g^{ik}_{\rm Mink}\nabla_i\nabla_k\psi. \qquad (1)$$

Here the mass M is the rest energy, which is diffeomorphism invariant and is dimensionless in the AD approach. Eq. (1) looks as the Schrödinger wave equation:

$$i\hbar\partial_t\psi = -\frac{\not{h}^2}{2M}\nabla^2\psi, \qquad (2)$$

which contains two Planck constants:

$$\sqrt{-g^{00}_{\rm Mink}} = \hbar,\ g^{ik}_{\rm Mink} = \not{h}^2\delta^{ik}. \qquad (3)$$

The Planck constants $\hbar$ and $\not{h}$ enter correspondingly the time derivative and space derivative terms in Schrödinger equation, and have different dimensions. Since M is dimensionless, the Planck constant $\hbar$ has dimension of time, $[\hbar] = [t]$, while the second Planck constant $\not{h}$ has dimension of length, $[\not{h}] = [L]$.

In dimensionless physics the Newton constant has the dimension of length, $[G] = [L]$, i.e. the same dimension as the spacelike Planck constant $\not{h}$. There are two quantities, which can be constructed by combination of the Newton constant G and $\not{h}$. One of them is the Planck mass $M_{\rm P} = \sqrt{\not{h}/G}$. It enters the Einstein equations as the diffeomorphism invariant quantity, and thus is dimensionless, $[M_{\rm P}]^2 = [\not{h}]/[G] = 1$. Another quantity is the Planck length

$$l_{\rm P} = \frac{\not{h}}{M_{\rm P}} = \sqrt{\not{h}G}\,, \qquad (4)$$

with the natural dimension of length, $[l_{\rm P}] = [L]$. Since the Planck length $l_{\rm P}$ and the Planck constant $\not{h}$ have the same dimension, the natural suggestion arises: maybe they are the equivalent quantities. This connection remains an open question, because the microscopic (trans-Planckian) physics of the quantum vacuum is not known. However we can study this problem using the acoustic metric of Bose liquid, where the microscopic physics is well known: it is atomic physics. The corresponding acoustic interval for phonons propagating in moving liquid is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \\ = \frac{\hbar n}{ms}[-s^2dt^2 + (dx^i - v^i dt)\delta_{ij}(dx^j - v^j dt)]. \qquad (5)$$

Here n is the density of bosonic particles; m is the particle mass; s is the speed of sound; and $\mathbf{v}$ is the superfluid velocity (the velocity of the "superfluid vacuum"), which is the shift function in the Arnowitt–Deser–Misner approach. The analog of the Minkowski metric corresponds to the zero value of the shift function, $\mathbf{v} = 0$, and thus

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$g^{0i} = 0$. Then the effective acoustic Minkowski metric experienced by the propagating phonons is:

$$g_{00} = \frac{\hbar ns}{m},\ g_{ik} = \frac{\hbar n}{ms}\delta_{ik},\ \sqrt{-g} = \frac{\hbar^2 n^2}{m^2 s}, \tag{6}$$

with dimensions

$$[g_{00}] = \frac{1}{[t]^2}\ ,\ [g_{ik}] = \frac{1}{[L]^2}\ ,\ [\sqrt{-g}] = \frac{1}{[t][L]^3}\,. \tag{7}$$

The acoustic interval (5) is dimensionless, $[ds] = 1$, which demonstrates that the interval ds describes the dynamics of phonons in the "superfluid vacuum", rather than the distances and time intervals. The same is valid for the interval in general relativity, where it describes the dynamics of a point particle in the relativistic quantum vacuum.

One can also introduce the acoustic Planck constants $\hbar_{\rm ac}$ and $\hslash_{\rm ac}$ obeying the corresponding Eq. (3). Then one obtains that for phonons in liquid helium, the length of the acoustic Planck constant $\hslash_{\rm ac}$ is on the order of the interatomic distance $a = n^{-1/3}$. This suggests that in quantum vacuum, the Planck constant $\hslash$ is on the order of the Planck length $l_{\rm P}$. If this is so, then the Planck mass $M_{\rm P} = \sqrt{\hslash/G}$, which enters the Einstein–Hilbert action and which is dimensionless as all the masses in the AD quantum gravity, is on the order of unity, $M_{\rm P} \sim 1$. That is why the Planck mass becomes the natural choice for the unit of mass.

We considered liquid helium in its self-sustained vacuum state, i.e. in the ground state at $T = 0$ and in the absence of external pressure, $P = 0$. In applied pressure $P \neq 0$, the parameters of the quantum vacuum s and a deviate from their vacuum values. For pressure small compared with the ultraviolet (UV) scale, $P \ll nms^2$, the relative change of these parameters is small, $\Delta s/s \sim \sim \Delta a/a \sim P/nms^2 \ll 1$. The same is valid for the relative change of the acoustic Planck constants:

$$\frac{\Delta\hslash_{\rm ac}}{\hslash_{\rm ac}} \sim \frac{\Delta\hbar_{\rm ac}}{\hbar_{\rm ac}} \sim \frac{P}{nms^2} \ll 1. \tag{8}$$

In relativistic quantum vacuum, the non-zero vacuum pressure gives rise to the de Sitter expansion. Then, applying Eq. (8) with the corresponding Planck scale parameters, one may suggest that the Planck constants acquire the following corrections in the expanding Universe:

$$\frac{\Delta\hslash}{\hslash} \sim \frac{\Delta\hbar}{\hbar} \sim \hbar^2 H^2 \sim T_{\rm GH}^2 \ll 1. \tag{9}$$

Here H is the Hubble parameter and $T_{\rm GH}$ is the Gibbons–Hawking temperature, which is dimensionless in the AD quantum gravity.

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