TOKAMAKI

EQUILIBRIUM OF PLASMA WITH INTERNAL SEPARATRIXIN TOKAMAKS

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Abstract. Some plasma equilibria with internal (current and magnetic) separatrices in a tokamak are considered. Comparison of calculated and available experimental data for such equilibria is carried out. It is shown that with a small change in the value of the internal inductance of the plasma in a tokamak, a qualitative change in the equilibrium plasma configuration is possible. It is noted that for some types of flux functions in the Grad-Shafranov equation, an equilibrium solution is possible only if the plasma density at its boundary is not equal to zero. It is established that the formation of a natural poloidal divertor is determined not only by the value β_p, but also by the type of flux functions of the Grad-Shafranov equation. The possibility of existence of equilibrium configurations of plasma with magnetic axes located one above the other and the possibility of existence of equilibrium plasma systems with many magnetic axes are shown.

Keywords: *tokamak, internal separatrix, natural poloidal divertor, equilibrium, internal inductance*

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1. INTRODUCTION

Using the definition [1], we will consider the separatrix in the poloidal section of an axisymmetric tokamak to be a curve separating regions with different properties, for example, with a different sign of the poloidal magnetic flux or toroidal current density. Thus, in a tokamak there can be both an external separatrix surrounding the region occupied by plasma, and internal ones - a magnetic separatrix (the separatrix of the poloidal magnetic flux) and a separatrix of the toroidal current density.

Equilibrium, stability, and some other phenomena in tokamak plasma critically depend on the structure of internal separatrices [2]. It is generally believed that the possibility of internal separatrix emergence in plasma when changing the value of β_p was demonstrated by V.S. Mukhovatov and V.D. Shafranov [3], although they themselves refer to some previous works [4,5]. Here β_p is the ratio of gas-kinetic plasma pressure to the pressure of the poloidal magnetic field. It is stated that the separatrix enters the plasma volume from the strong field side when $\beta_p > 0.5A$, (*and* is the aspect ratio of the tokamak) and as β_p increases, it shifts toward the plasma axis. This statement is valid for plasma with circular cross-section. Increasing plasma elongation increases the boundary value of β_p (in theory). It should be noted that usually the current separatrix enters the plasma first, and at $\beta_p > A$ - the magnetic separatrix [4], forming a so-called natural poloidal divertor [6]. Below, we will show that these statements regarding the value of β_p are not absolute but relate only to certain types of equilibrium.

This paper examines various equilibrium configurations of plasma with internal separatrices and compares them with available experimental data. In all figures, the geometric axis of symmetry of the tokamak is located to the left of the displayed cross-section.

All considered equilibria were obtained by solving the Grad-Shafranov equation using the TOREQ code for plasma with a fixed boundary [7].

This work does not consider equilibria where the current density in the center of the plasma column is close to zero or where the current flows in the opposite direction to the current in the rest of the column [8-10]. Stability and the possibility of practical implementation of the described configurations are not considered in this work.

2. MODEL

Plasma equilibrium in a magnetic field is described by solutions of the Grad-Shafranov (GS) equation, whose dimensionless "canonical" representation in Cartesian coordinates has the form

$$\Delta^*\psi = -\lambda(h^2\beta_0 A(\psi) + (1-\beta_0)B(\psi)), \tag{1}$$

$$\Delta^*\psi = h\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial\psi}{\partial x}\right) + \frac{\partial^2\psi}{\partial y^2}. \tag{2}$$

Here ψ is the poloidal magnetic field flux function, calculated per one radian, $h = 1 + x/A$, $A = R_0/a$ is the aspect ratio, R_0 and a are the major and minor radii of the tokamak, λ is a parameter related to the internal inductance of the plasma l_i, $A(\psi)$ and $B(\psi)$ are arbitrary flux functions, the specification of which determines the structure of the equilibrium configuration, β_0 is a parameter which, along with λ, R_0 and others included in the functions $A(\psi)$ and $B(\psi)$ parameters, determines the value of the total toroidal current and the plasma internal inductance coefficient. In paper [11] it is shown that $\beta_0 \approx \beta_p$. In (1) and (2), all quantities with the dimension of length are normalized to the minor radius of the tokamak, $x = (R - R_0)/a$, y=Z/a , where *R* and *Z are* dimensional cylindrical coordinates "tied" to the geometric center of the tokamak (the line *R* = 0 is the geometric axis of the tokamak, and the plane *Z* = 0 is its symmetry plane).

The GS equation is solved with the following boundary conditions: $\psi = 0$ at the plasma boundary and $\psi = \psi_{\max}$ (maximum value) on its magnetic axis.

The boundary surface is chosen to be symmetric in the form [12]

$$y = \pm \frac{K \cdot (1-\delta^2)^{0.2}}{(1+\delta \cdot x)^{0.7}} \sqrt{1-x^2} \tag{3}$$

where $K = \max|y|$ is the elongation, and δ is the triangularity (the value *x/a,* corresponding to the location of the point $|y(x/a)| = K$) of the plasma column. Formula (3) is valid for $-0.85 \le \delta \le 0.85$. This formula is approximate, so if in calculations it is necessary to precisely maintain the values of elongation and triangularity, then it is necessary to adjust the values *K* and δ included in the formula.

3. CURRENT SEPARATRIX

The poloidal cross-section of the tokamak with constant level lines of the poloidal flux obtained by solving the GS equation (1) with flux functions of the form

$$\begin{aligned} A(\psi) &= e^{\psi} - 1 \\ B(\psi) &= e^{0.8\psi} - 1 \end{aligned} , \tag{4}$$

is presented in Fig.1. The calculations were performed for plasma with elongation 2, triangularity 0.5, aspect ratio 1.5, and $\lambda =$ 0.8. The internal inductance in this case is 1.751. From the figure, it can be seen that in the poloidal plane, the curves ψ =const, corresponding to the constant value of the poloidal flux, are closed lines located around the axis of the plasma column. In the case under consideration, ψ >0 everywhere.

Fig. 2 shows curves of equal values of the normalized toroidal current density. The line corresponding to $j=0$ is the current separatrix. For clarity, the curves in Figures 1 and 2 are calculated for $\beta_p = 6$.

Fig. 3 shows the distributions of relative values of magnetic flux ψ , toroidal current density j and plasma pressure p along the radius in the equatorial plane. The figure shows that the plasma pressure is greater than zero throughout the plasma column, while the current density in the region between the separatrix and the plasma boundary on the side of the strong magnetic field is negative. In the rest of the plasma column, the current density is positive. It is also evident that the relative distributions of magnetic flux and plasma pressure coincide. From Figures 2 and 3, it follows that the magnitude of current flowing in the positive direction significantly exceeds the magnitude of current flowing in the negative direction.

Calculations show that for the specified functions $A(\psi)$ and $B(\psi)$, aspect ratios A =1.5 and A =3, plasma with circular and D -shaped cross-section $(K=2,\ \delta=0.5)$ the current separatrix enters the volume occupied by plasma at one value of $\beta_p \approx 0.8A$. Similar results were obtained, for example, in work [13].

The considered current separatrix exists in plasma with a single magnetic axis. Experiments conducted on the TFTR facility (USA) at $\beta_p \approx 6$ [19] showed that in this case, the current separatrix reaches 1/3 of the minor radius.

It should be noted that with the chosen parameter values and flux functions $A(\psi)$ and $B(\psi)$ the magnetic separatrix does not enter the plasma.

4. EQUILIBRIUM FOR THE CASE WHEN THE TOTAL TOROIDAL CURRENT VALUE IS CLOSE TO ZERO.

In a tokamak, equilibrium configurations are possible where the total toroidal current value is close to zero. Such a situation arises when two current filaments form in the plasma, with currents of approximately equal magnitude flowing in opposite directions [14-19].

Let us consider equilibrium distributions for plasma with circular cross-section and for

$$A(\psi) = 1, \;\; B(\psi) = 2 + 10\psi \;\; \text{и} \;\; \beta_p = 0.01 \tag{5}$$

The plasma pressure equals

$$p(\psi) = \int A(\psi)d\psi - p_b = \psi - p_b \tag{6}$$

where

$$p_b = -\min(\psi) \tag{7}$$

If $p_b = 0$, then the plasma pressure at the boundary equals zero [17,18].

Fig.4 shows the lines of constant poloidal magnetic flux calculated for an equilibrium with β_p =0.01 and $\lambda =$ 1.541 (l_i = 1.1840). The figure shows that the lines ψ = const are symmetrically positioned relative to the vertical dashed line, which is the magnetic separatrix. On the strong field side $\psi > 0$, and on the weak field side . $\psi < 0$.

The typical distribution of isocurves of current density in the poloidal plane for this equilibrium is shown in Fig.5. Similar to the value ψ the density of toroidal current is positive on the strong magnetic field side and negative on the weak side. If the value of λ is changed to 1.542 (l_i = 1.1875), then the signs of ψ and j will change to the opposite.

Fig.6 shows the distributions of toroidal current density and plasma pressure along the radius in the equatorial plane. Curves *1* correspond to l_i = 1.1840, and curves *2* l l_i = 1.1875. It is seen that a small change in the internal inductance coefficient l_i leads to an abrupt change in the equilibrium distribution of plasma parameters.

For functions $A(\psi)$ and $B(\psi)$, chosen in the form (5), equilibrium distributions exist only when the pressure at the plasma boundary is non-zero.

Let's consider another type of functions $A(\psi)$ and $B(\psi)$ in the GS equation

$$A(\psi) = \psi, \;\; B(\psi) = 2 + 10\psi \;\; \text{и} \;\; \beta_p = 0.01 \quad . \tag{8}$$

The plasma pressure in this case is equal to $p(\psi) = 0.5\psi^2 - p_b$. For these functions, an equilibrium solution exists even when the pressure at the boundary is zero (p_b=0).

Qualitatively, the equilibrium solutions in this case coincide with the equilibrium solutions shown in Fig. 4, 5. As in the previous case, the transition from one solution to another occurs with a very small change in inductance from l_i = 1.1875 (λ =1.539) to l_i = 1.1864 (λ =1.540).

Figure 7 shows the same dependencies as in Figure 6, but unlike the previous case, the plasma pressure does not change when the equilibrium distribution changes.

Calculations of the equilibrium distribution of the poloidal magnetic flux, describing experimental data obtained at the HT-7 facility (China), are shown in Figure 8 [15]. The solid line in the figure represents the plasma pressure in the equatorial plane, the dashed line - the current density. It can be seen that the current density dependence coincides with the similar dependence in Figure 5, and the "experimental" pressure coincides with the pressure corresponding to $p_b \neq 0$.

Figure 9 shows the experimental plasma density profile n in this HT-7 facility and the calculated values of plasma pressure p and current density j , corresponding to the data in Figure 8. It can be seen that in this case, the radial dependence of pressure qualitatively coincides with the plasma density dependence n . From Figures 8 and 9, it follows that in this case, the current and magnetic separatrices pass through the center of the plasma column.

The data presented in Figures 4-9 show that for the selected parameters and flux functions, there are two magnetic axes in the plasma.

As can be seen from Figure 4, in the cases under consideration ($\beta_p = 0.01 << A$), there is a magnetic separatrix marked by a vertical dashed line in the figure. This result contradicts the theoretical conclusions that the magnetic separatrix enters the plasma from the strong magnetic field side only when $\beta_p > A$ [3,4].

Thus, the appearance of the internal magnetic separatrix is determined not only by the value of β_p , but also by the type of flux functions $A(\psi)$ and $B(\psi)$, which define the pressure and current profiles.

5. POLOIDAL DIVERTOR.

In the literature [3,4] it is indicated that if $\beta_p \geq A$, then a magnetic separatrix is formed in the plasma, and a second magnetic axis may emerge. As β_p increases, the separatrix moves toward the center of the plasma. Experimentally, the formation of a magnetic separatrix was observed at the TFTR (USA) [19] and QUEST (Japan) [6] facilities.

Hereafter, unless specifically noted, we will consider plasma with circular or D -shaped cross-section with $K = 2,\ \delta = 0.5$.

Fig.10 shows isolines of the poloidal flux for

$$
\begin{aligned}
A(\psi) &= \psi - 1 \\
B(\psi) &= 10 + \psi
\end{aligned}
\tag{9}
$$

and β_p =4.9. The figure shows that at this value of β_p the magnetic separatrix (marked with dots) has already entered the plasma. As β_p increases, the separatrix moves toward the center of the plasma, and a second magnetic axis appears in the device. Fig.11 shows isolines of the poloidal flux for plasma with β_p =6. In this figure, dots mark the separatrix, and asterisks indicate the positions of the magnetic axes.

A divertor is defined by the existence of an X-point in the magnetic configuration, where the poloidal magnetic field magnitude equals zero. In the considered case of "up-down" symmetry, this point is usually located in the poloidal cross-section on the side of the strong magnetic field, although opposite cases are also known (see, for example, [20]). To find the X -point, it is necessary to solve the equation for plasma with a free boundary, taking into account external magnetic fields. In this case, an X-point appears to the left of the plasma boundary [3, 6, 19], forming a poloidal divertor configuration.

Such a divertor configuration is qualitatively shown in Fig.12 [21]. Solid lines describe the isolines of the poloidal magnetic flux for

$$
\begin{aligned}
A(\psi) &= e^{\psi} \\
B(\psi) &= 0.9e^{0.3\psi}
\end{aligned}
\tag{10}
$$

Equilibrium calculations for dependencies (10) were performed at β_p =6, / =0.2 (l_i=0.774). The vertical dashed line marks the position of the magnetic separatrix. Fig.13 shows the distribution of toroidal current density in the equatorial plane. Curve *1* represents calculation results from the present work, curve *2* shows data from [21]. In [21], such a distribution is called "quasi-constant."

The figure shows that the density distributions of both currents qualitatively coincide with each other. In [19], it is noted that when the magnetic separatrix enters the plasma, i.e., when a natural poloidal divertor forms, the plasma becomes "flattened," meaning that hot plasma is considered absent in the region to the left of the separatrix.

6. SOME OTHER EQUILIBRIA

Above, we considered equilibria with two magnetic axes located in the symmetry plane of the tokamak $y = 0$.

Here, we will consider a configuration with two magnetic axes located symmetrically with respect to this plane.

Let us choose in the GS equation

$$A(\psi) = B(\psi) = \sqrt{1+\psi} - 1 \quad . \tag{11}$$

Calculations were performed for A =3, $K^{+} = 3$, $K^{-} = -3.3$, $\delta = 0$, $\lambda = 9$. Here, plus and minus signs denote the upper and lower halves of the torus.

The equilibrium solution for functions of the form (11) and values β_p =0.585 (l_i = 1.0467) is shown in Fig.14. The figure shows that the isolines of the toroidal magnetic flux ψ are located symmetrically with respect to the magnetic separatrix, indicated in the figure by a horizontal dashed line. In the upper part of the plasma column, ψ and j are positive, and in the lower part they are negative. A slight change in β_p to 0.586 (l_i = 1.0511) leads to a change in the sign of the poloidal flux and current density to the opposite in both parts of the plasma.

In a tokamak, equilibria with a different number of magnetic axes are also possible. As an example, let's select the flux functions in the form (9). Then for $A = 3,\ K = 2,\ \delta = 0.5,\ \beta_p = 6,\ \lambda = 25\ (l_i = 2.113)$ the equilibrium magnetic configuration has 6 magnetic axes (see Fig.15).

7. CONCLUSION

Some equilibrium configurations of "up-down" symmetric tokamaks with internal separatrices of toroidal current and poloidal magnetic flux have been calculated, and a comparison of calculated and available experimental data has been made.

It is shown that qualitatively different plasma equilibria are possible in a tokamak, which transition into each other with a very small change in the internal inductance value.

It is established that for certain types of flux functions in the Grad-Shafranov equation, which determine the pressure and current profiles in the plasma, and the typical boundary shape, an equilibrium solution exists only if the plasma density at its boundary is not equal to zero.

As a result of the performed calculations, it has been demonstrated that the appearance of a magnetic separatrix in the plasma is determined not only by the relative pressure of the poloidal magnetic field β_p , but also by the form of the flux functions $A(\psi)$ and $B(\psi)$ in the Grad-Shafranov equation.

The possibility of the existence of equilibrium configurations with magnetic axes arranged one above the other, as well as configurations with many magnetic axes, is shown.

The possibility of practical implementation of the considered configurations, as well as the issues of plasma stability, and their advantages and disadvantages require special consideration.

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9. FIGURE CAPTIONS

Fig.1. Equilibrium distribution of poloidal magnetic flux for plasma in a tokamak with elongation 2, triangularity 0.5, aspect ratio 1.5 with $\lambda =$ 0.8, β_p =6 The internal inductance is equal to 1.751.

Fig.2. Curves of equal values of normalized toroidal current density for tokamak plasma with parameters listed in Fig.1. The line corresponding to $j=0$ is the current separatrix.

Fig.3. Distribution of relative values of magnetic flux ψ , toroidal current density j and plasma pressure p by radius in the equatorial plane of the tokamak.

Fig.4. Equilibrium distribution of poloidal magnetic flux calculated for β_p =0.01 and $\lambda =$ 1.541 (l_i = 1.1840).

Fig.5. Distribution of current density isocurves in the poloidal plane for plasma equilibrium with parameters presented in Fig.4.

Fig.6. Distributions of toroidal current density and plasma pressure by radius in the equatorial plane of the tokamak. Curves *1* correspond to l_i = 1.1840, and curves *2* 1 l_i = 1.1875.

Fig.7. Shows the same dependencies as in Fig.6, but unlike the previous case, the plasma pressure does not change when the equilibrium distribution changes.

Fig.8. Equilibrium distribution of the poloidal magnetic flux describing experimental data [15]. The solid line in the figure is pressure, the dashed line is current density.

Fig.9. Experimental value of plasma density *n* and calculated values of plasma pressure and toroidal current density corresponding to the data presented in Fig.8.

Fig.10. Isolines of poloidal flux in tokamak plasma for β_p =4.9. The magnetic separatrix is marked with dots.

Fig.11. Isolines of poloidal flux for plasma with β_p =6. The separatrix is marked with dots, and the positions of magnetic axes are marked with asterisks.

Fig.12. Qualitative image of divertor configuration [21]. Solid curves are isolines of equilibrium poloidal flux from [21]. The dashed line is the magnetic separatrix.

Fig.13. Distribution of toroidal current density in the equatorial plane. Curve *1* is the calculation result in this work, curve *2* is the data from work [21].

Fig.14. Isolines of poloidal magnetic flux in plasma with vertical arrangement of magnetic axes.

Fig.15. Distribution of magnetic flux isolines in plasma with 5 magnetic axes.

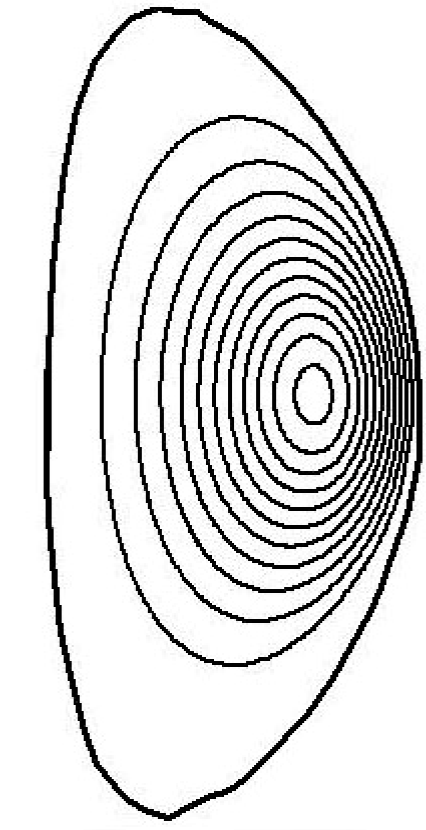
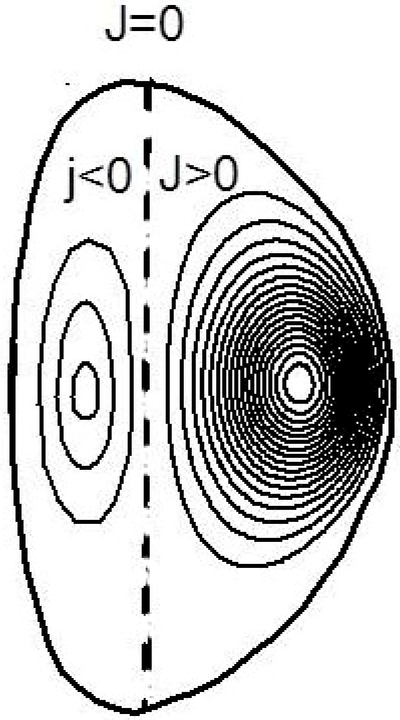


Fig.1



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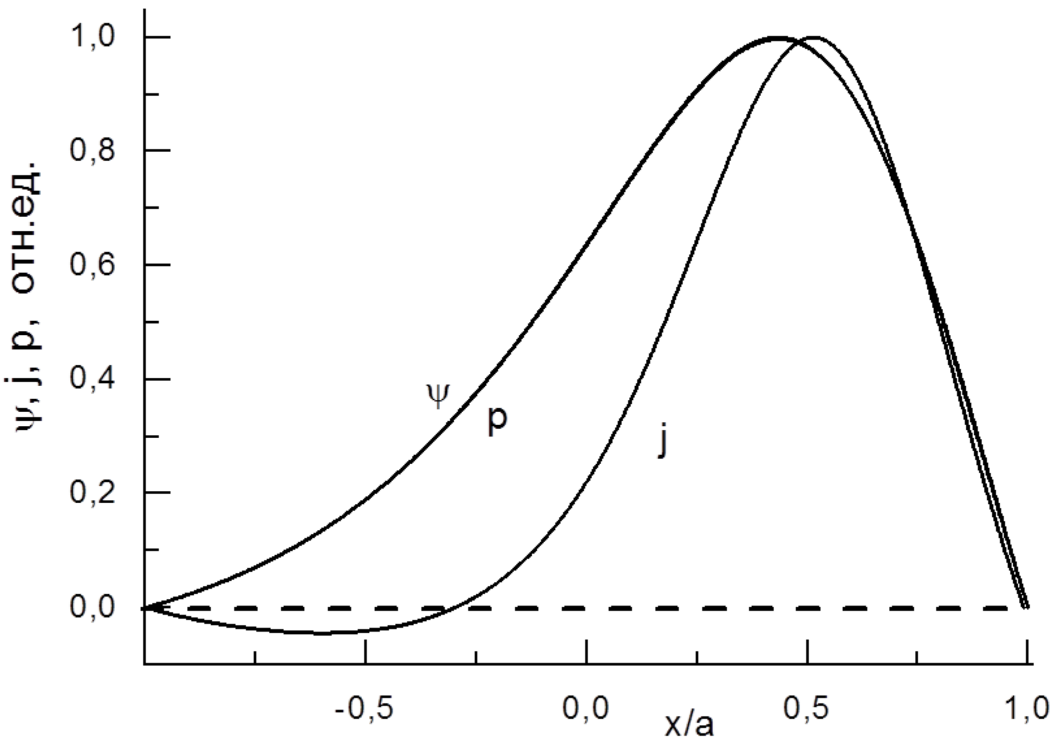

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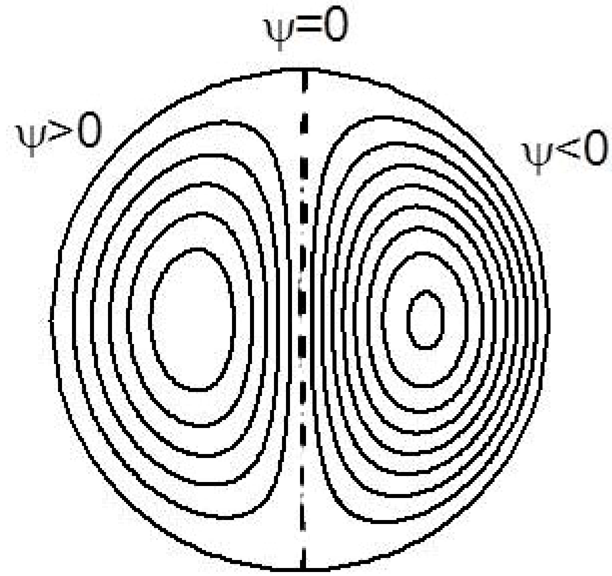

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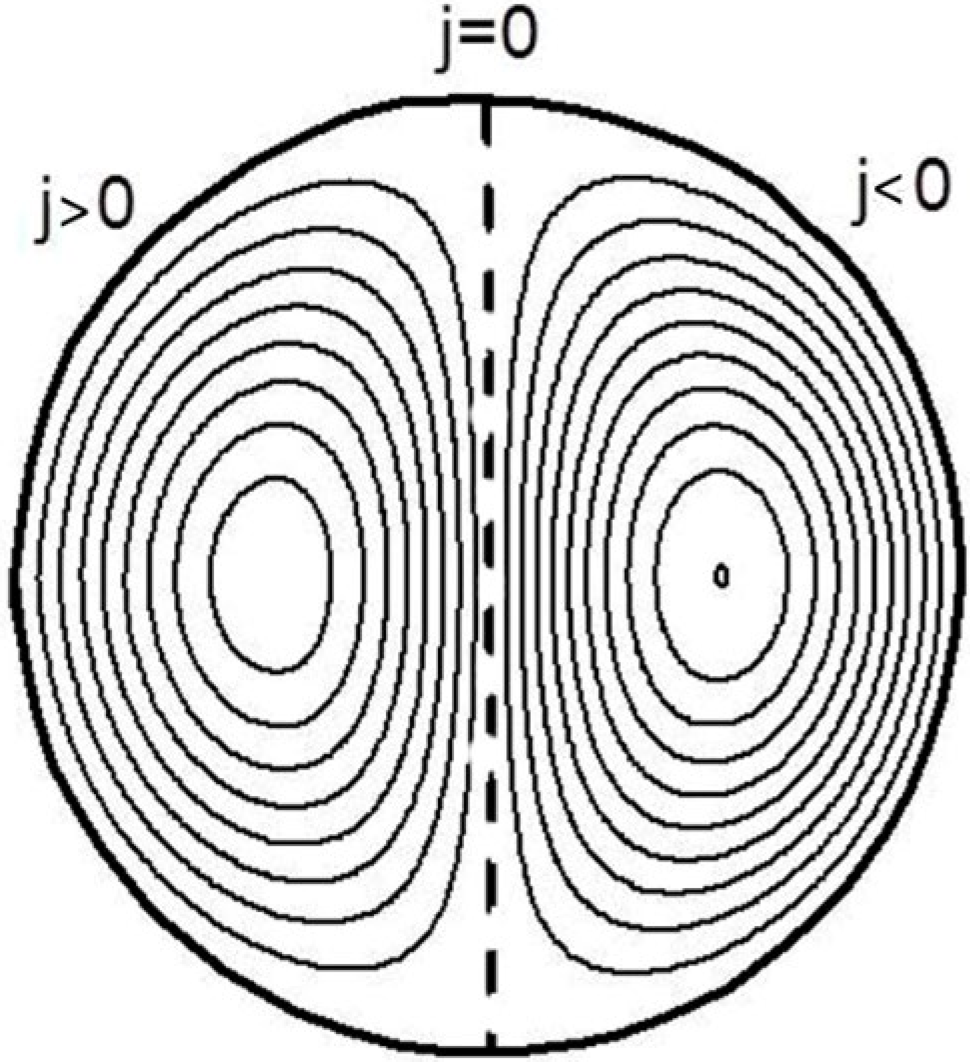

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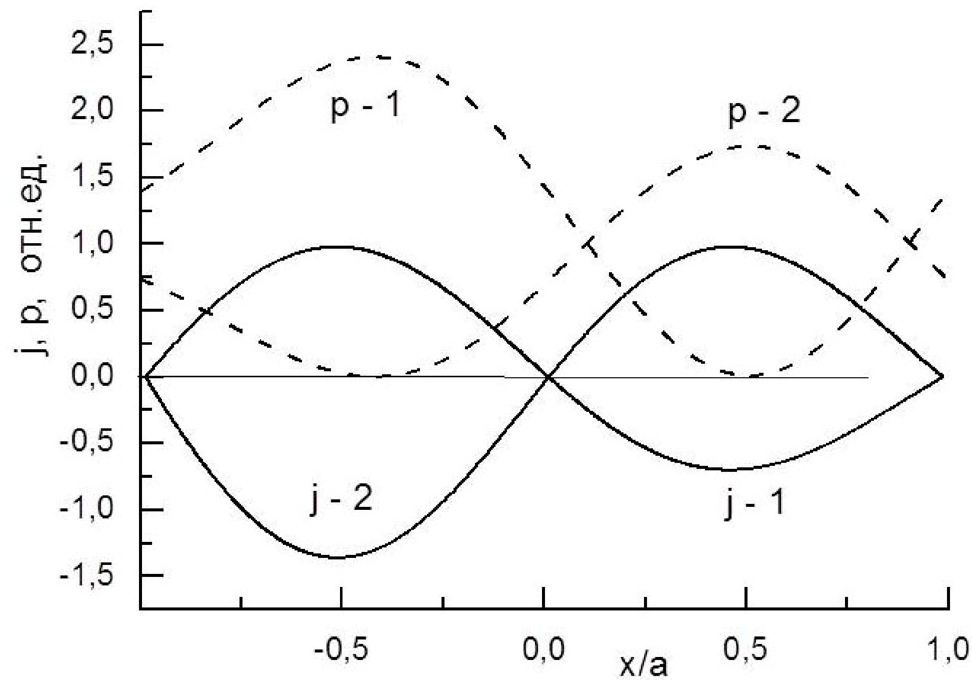

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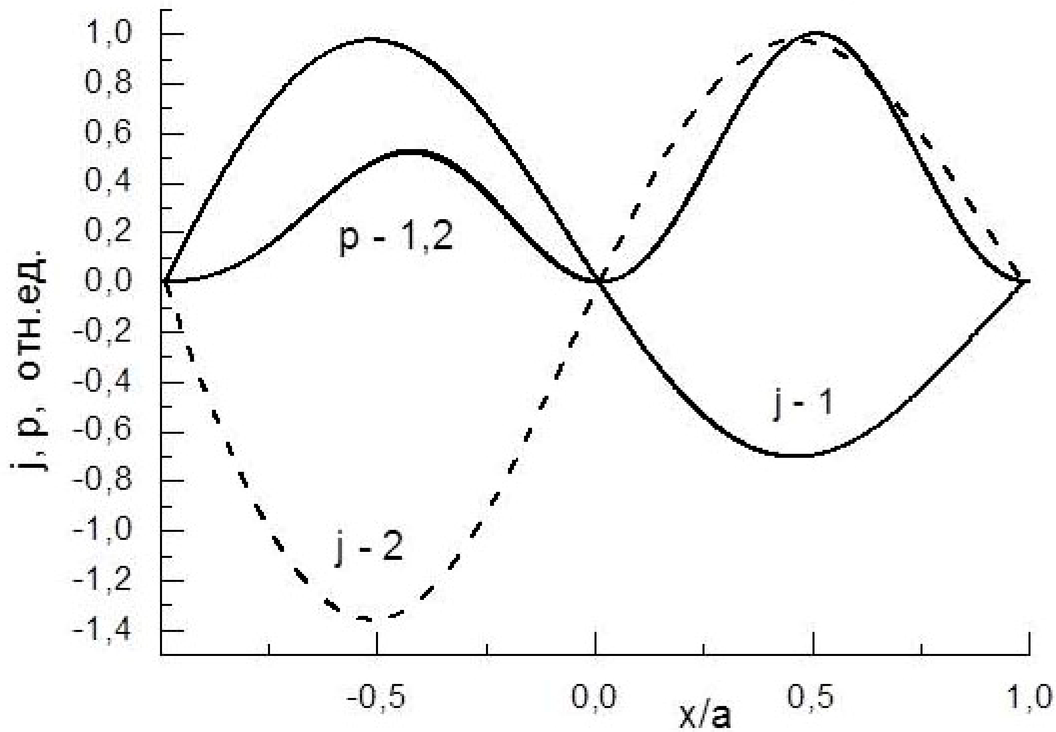

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1,0
x/a

Fig.7

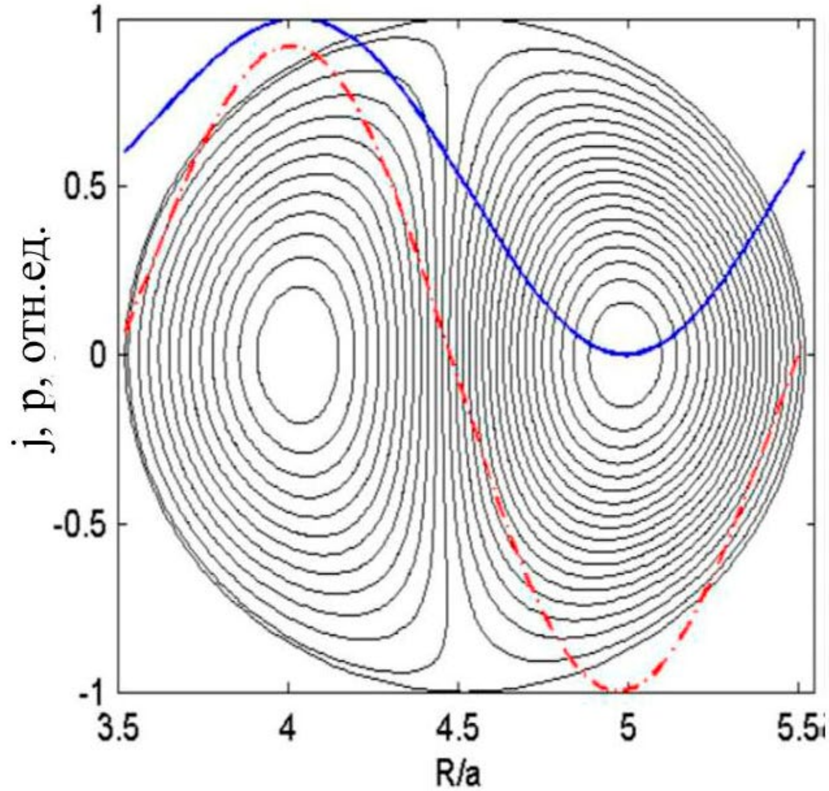


Fig.8

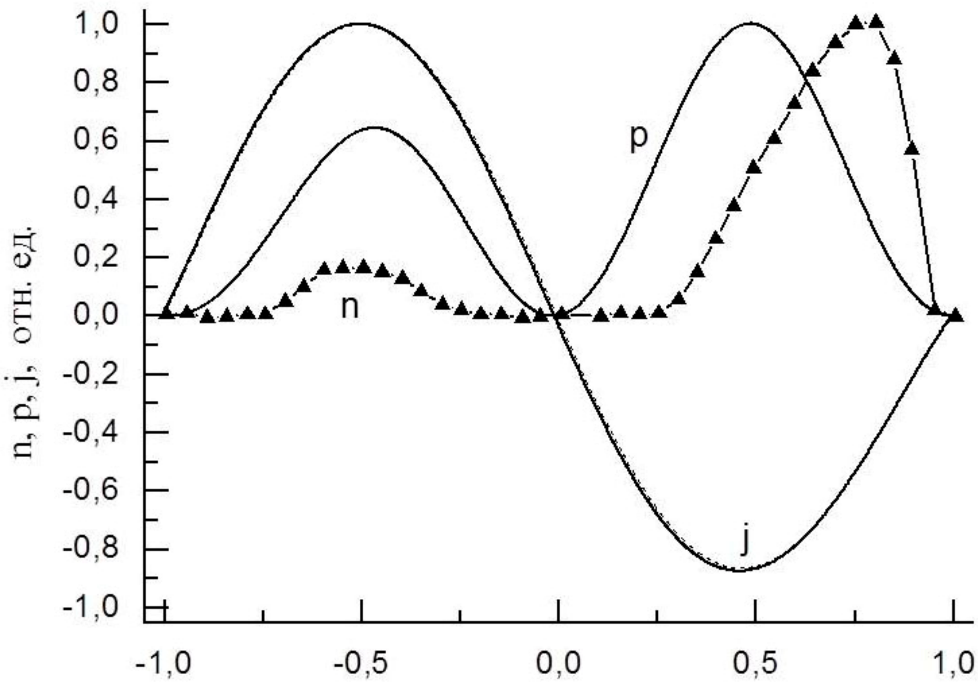


Fig.9

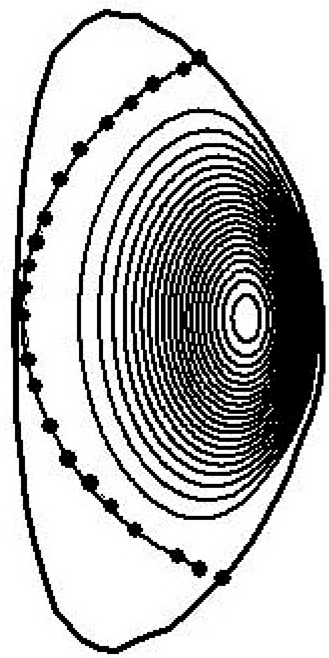


Fig.10

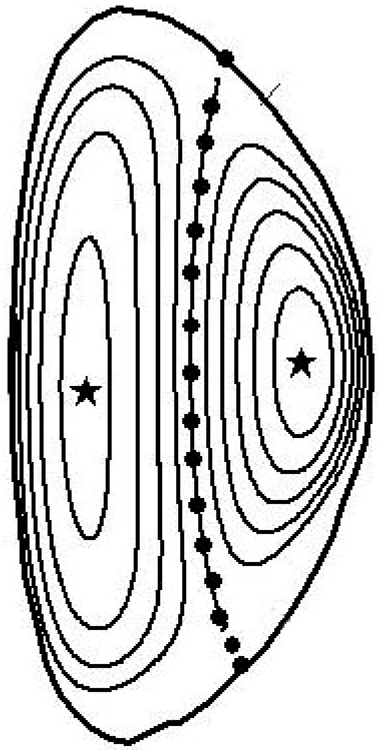
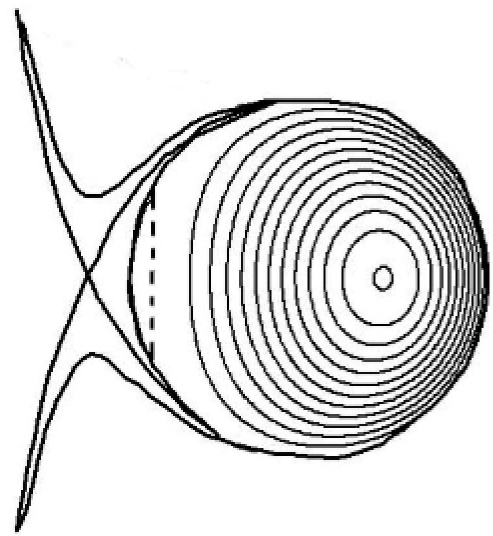


Fig.11



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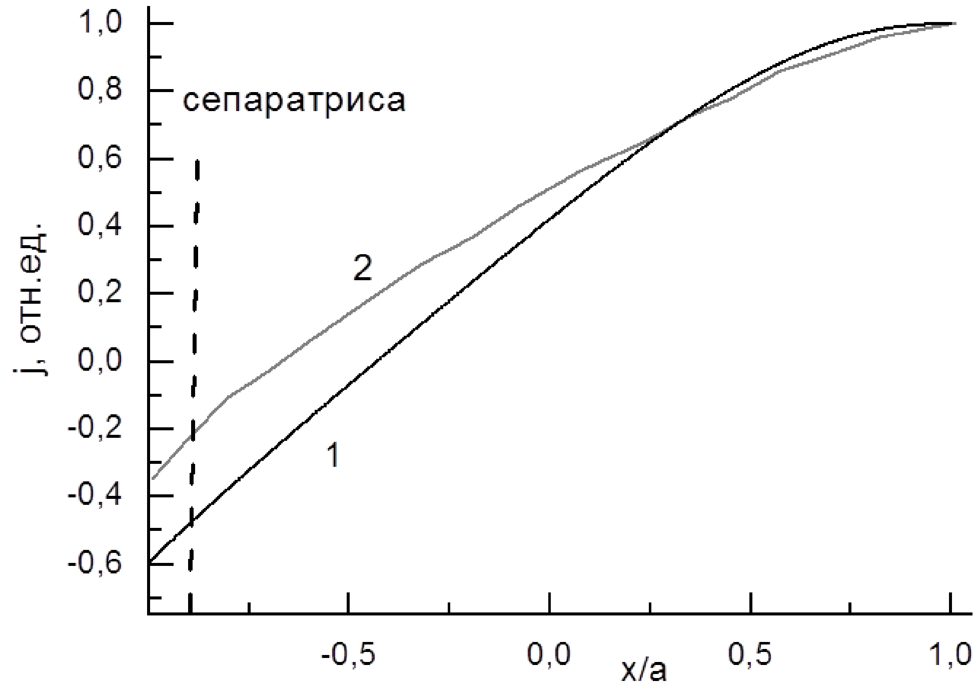

Fig.13

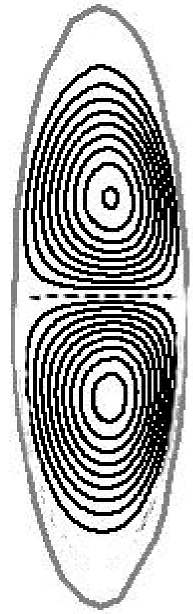

Fig.14

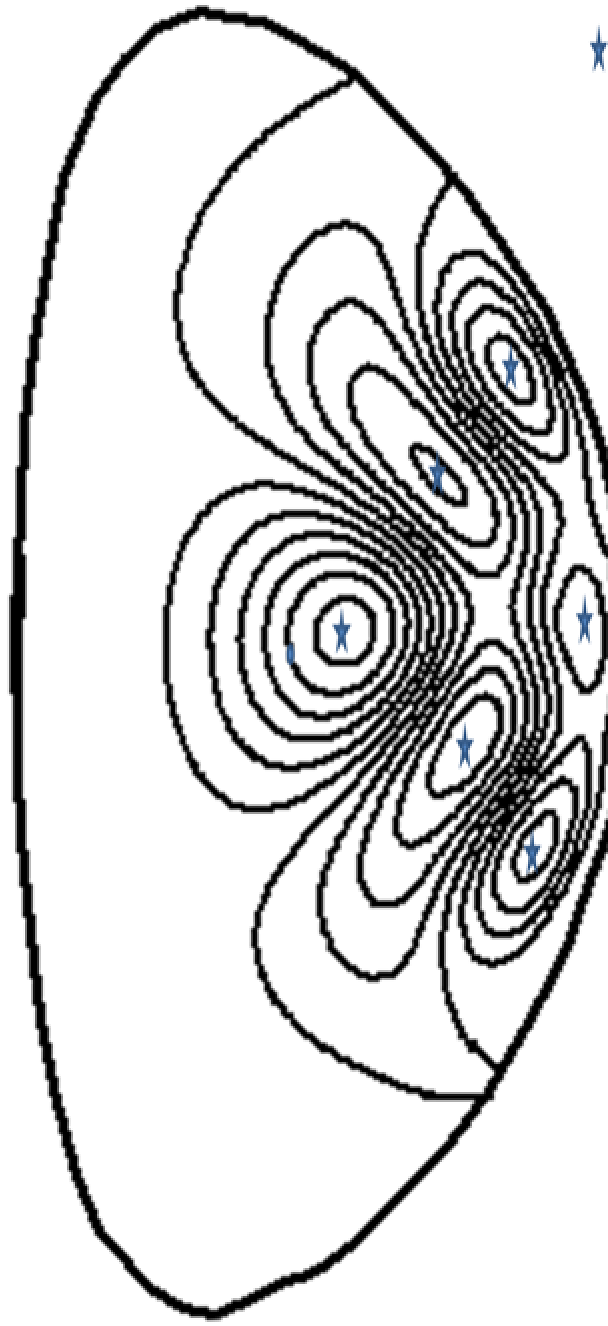

Fig.15