ON THE ASSESSMENT OF PERTURBATIONS IN THE ROTATIONAL DYNAMICS OF SMALL ASTEROIDS DURING CLOSE APPROACHES TO EARTH

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Through numerical experiments, the influence of rotation rate, rotation axis orientation, and shape parameters of asteroids on the magnitude of perturbations in their rotational dynamics during close approaches to Earth has been studied. The dynamics of three asteroids have been considered: (99942) Apophis, (367943) Duende, and 2012 TC4. It has been established that asteroids with relatively slow rotation (period $P > 5$ h) are characterized by significant perturbations: in the case of Apophis ($P \approx 30$ h) during its 2029 close approach to Earth, changes in rotation period may reach tens of hours, and deviations in the rotation axis orientation – up to ten degrees. In the case of Duende ($P \approx 8$ h) during its 2013 close approach to Earth, the change in P did not exceed several hours, while deviations in the rotation axis orientation could amount to tens of degrees. For asteroids with fast rotation ($P < 1$ h), perturbations are negligibly small: in the case of asteroid 2012 TC4 ($P \approx 12$ min.) during its 2017 close approach to Earth, changes in P did not exceed 10^{-5} min., while rotation axis deviations were less than 0.01°. It has been shown that for slowly rotating asteroids, errors in determining asteroid shape parameters can lead to noticeable inaccuracies in the assessment of perturbation magnitudes. Conversely, uncertainty in knowing the shape of a fast-rotating asteroid does not affect the assessment of perturbations in its rotational dynamics. In the case of Apophis, perturbations in the rotational motion during the upcoming 2029 close approach to Earth may lead to a decrease in the value of the A_2 parameter, which characterizes the Yarkovsky effect, to -2.4×10^{-14} AU/day^2 or to an increase to -3.2×10^{-14} au/day^2. Perturbations in the rotational dynamics of Duende during its close approach to Earth in 2013 and asteroid 2012 TC4 during its close approach to Earth in 2017 did not have a significant impact on their values A_2.

Keywords: near-Earth asteroids, (99942) Apophis, (367943) Duende, 2012 TC4, rotational dynamics, Yarkovsky effect

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INTRODUCTION

In this paper, we investigated the dependence of the magnitude of perturbations occurring in the rotational dynamics of asteroids during close approaches to Earth on the rotation rate, orientation of the rotation axis, and parameters characterizing the asteroid's shape. As various studies have shown (see, for example, Scheeres et al., 2000; 2004; Boldrin et al., 2020; Melnikov, 2022), a close approach to a planet can significantly affect the rotational dynamics of an asteroid.

The empirical relationship between an asteroid's rotation rate and its size (average diameter D), presented, for example, in (Hestroffer et al., 2019; Hu et al., 2021; Fenucci et al., 2024), shows that as size decreases, rotation speed increases. The rotation periods P of asteroids with $D < 100$ m typically range from several hours to a few tens of minutes. Minute-long periods are found in asteroids with $D \sim 10$ m, which likely represent monolithic bodies. Asteroids hundreds of meters in size typically rotate with $P > 5$ h. Since most near-Earth asteroids (NEAs) are small bodies, it is important to study in detail how close approaches affect their rotational dynamics. One of the objectives of this study was to identify differences in how Earth encounters affect the rotational dynamics of slowly rotating asteroids (D = 100-1000 m, P = 5-100 h) versus very small, fast-rotating asteroids ($D \sim 10$ m, $P < 1$ h). Additionally, for most asteroids, only an estimate of D is available, while the magnitude of perturbations arising during close Earth approaches is significantly affected by the asteroid's shape (which determines the moments of inertia), which is usually unknown. Another objective of this work was to study how uncertainty in knowledge of an asteroid's shape affects estimates of perturbations in the asteroid's rotational dynamics obtained through numerical modeling of its close approach to Earth.

Non-gravitational effects: solar radiation pressure, the Yarkovsky effect (YE) and the YORP effect (Yarkovsky–O'Keefe–Radzievskii–Paddack) play a significant role in the long-term dynamics of small asteroids. Taking these effects into account is especially important when studying the dynamics of NEAs. In particular, as the study of the dynamics of asteroid (99942) Apophis has shown, the secular change in its orbit caused by the YE can increase the risk of a catastrophic collision with Earth (Chesley, 2006; Giorgini et al., 2008; Sokolov et al., 2012; Shor et al., 2012; Farnocchia et al., 2013a; 2013b). As the size of an asteroid decreases, the significance of non-gravitational effects increases. The influence of close approaches to planets for a number of asteroids (including those with $D < 100$ m) on their rotational dynamics and the magnitude of the YE was previously examined by us in (Melnikov, 2022; Martyusheva, Melnikov, 2023). It was shown that close approaches of small asteroids to planets and the resulting perturbations in the rotational motion of asteroids can lead to changes in the magnitude of the YE for them by almost a factor of two. Therefore, it is important to assess the impact of perturbations in the rotational dynamics of an asteroid on the magnitude of the YE.

The paper has the following structure. The first section presents the problem statement and a brief characterization of the studied asteroids. The second section contains a brief description of the YE theory and the definition of one of its characterizing parameters A_2. Based on the outlined methodology for determining the YE and the values of A_2 known from the analysis of astrometric observations, the rotation parameters of several studied asteroids have been refined. The third section describes the results of numerical experiments on modeling close approaches of asteroids to Earth. The fourth section examines the influence of uncertainty in knowledge of the asteroid's shape on the assessment of perturbations arising during approaches to Earth. The fifth section considers the influence of perturbations on the value of A_2. The main results are presented at the end of the paper.

PROBLEM STATEMENT AND PARAMETERS OF THE STUDIED ASTEROIDS

We examined and compared the rotational dynamics of three NEAs during their close approach to Earth: the relatively large ($D \approx 340$ m) asteroid (99942) Apophis with slow rotation ($P \approx 30$ h), a very small ($D \sim 10$ m) asteroid 2012 TC4 with rapid rotation ($P \approx 12$ min), and asteroid (367943) Duende ($P \approx 8$ h, $D \approx 30$ m), representing an intermediate case. For Apophis, we modeled its upcoming approach to Earth in 2029, for Duende – the approach that took place in 2013, and for 2012 TC4, we considered the approach that occurred in 2017.

The potentially hazardous asteroid Apophis in terms of collision with Earth (see details, for example, in Chesley, 2006; Giorgini et al., 2008; Sokolov et al., 2008; 2012; Shor et al., 2012; Farnocchia et al., 2013b) was discovered on June 19, 2004. In 2029, the next close approach of Apophis to Earth will occur at a distance of about 38000 km from the geocenter, which should cause significant perturbations in the asteroid's motion. The rotational dynamics of Apophis has previously been considered by various researchers (Scheeres et al., 2005; Pravec et al., 2014; Souchay et al., 2014; 2018; Benson et al., 2023). In our previous works (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024), we studied the perturbations that will occur in the rotational motion of Apophis during its approach to Earth in 2029 through numerical modeling. It was established that the rotation period of Apophis and the orientation of its rotation axis may undergo significant changes, which, in turn, will lead to a change in the magnitude of the Yarkovsky effect.

Rotational dynamics of the asteroid Duende during its close (distance about 27700 km) approach to Earth in 2013 was considered in papers (Devyatkin et al., 2016; Benson et al., 2020; Moskovitz et al., 2020; Melnikov, 2022). Analysis of available observations showed that Duende's rotation rate due to perturbations caused by the approach could have changed by almost 10%. Note that Duende's rotational state before the approach is unknown. Results of numerical modeling (Benson et al., 2020; Melnikov, 2022) indicate that perturbations in Duende's rotation period could have reached 30%.

Dynamics of asteroid 2012 TC4 during its approaches to Earth that occurred in 2012 (distance about 95000 km) and 2017 (distance about 50000 km) years, was studied in detail in the paper (Lee et al., 2021). Through analysis and modeling of light curves, it was established that the asteroid's rotation periods in 2012 and 2017 differed noticeably. As one of the possible explanations in (Lee et al., 2021), an assumption was made about the action of the YORP effect.

In our previous works (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024), devoted to studying Apophis dynamics, a software complex based on the DOP853 integrator implementing an explicit 8 - th order Runge-Kutta method was developed for conducting numerical experiments on modeling the asteroid's rotational dynamics. The concept and capabilities of the integrator are described in detail in (Hairer et al., 1993). The following assumptions were used: the asteroid is represented as a rigid body, its figure is approximated by a triaxial ellipsoid. Earth was considered as a gravitating point. At the initial moment, the asteroid's rotation around one axis coinciding with the axis of maximum moment of inertia was set. The orientation of the rotation axis relative to the normal to the orbit plane was determined by the angle γ, taking values from 0° to 180°. Through numerical integration of the equations of motion (see details in (Lobanova, Melnikov, 2024)), the evolution of the asteroid's proper rotation period P and angle γ was investigated.

The dynamics of the asteroid was considered on a segment of its orbit limited by a geocentric sphere with a radius of 100 R_E. The chosen size of the region in which the asteroid dynamics is studied is typical for the problem under consideration (see, for example, Araujo, Winter, 2014; Richardson et al., 1998; Boldrin et al., 2020; Melnikov, 2022). The analysis of Apophis orbital dynamics conducted in (Lobanova, Melnikov, 2024) showed that its motion in the vicinity of the close approach point with Earth can be approximated by an unperturbed hyperbolic geocentric orbit. The paper (Lobanova, Melnikov, 2024) provides a methodology for constructing such an orbit based on NASA JPL ephemerides (https://ssd.jpl.nasa.gov/horizons/). For Duende and 2012 TC4, orbits of the same kind can be constructed, and the parameters of geocentric orbits for all studied asteroids in the considered events have similar values. Only the parameters characterizing the rotation of NEAs and their shapes differ significantly.

Table 1. Orbital and physical parameters of the studied asteroids adopted for modeling

Asteroid	d/R_E	e	A/C	B/C	P, h	γ, deg
(99942) Apophis	5.96	4.26	0.73	0.95	30.60	140
(367943) Duende	5.34	4.22	0.25	0.85	8.72	27 (160)
2012 TC4	7.86	6.36	0.42	0.81	0.20	105 (160)

Note: Data on the moments of inertia, rotation period and axis tilt of Apophis are taken from (Pravec et al., 2014), Duende – from (Benson et al., 2020), asteroid 2012 TC4 – from (Lee et al., 2021). Values d and e were obtained based on NASA JPL ephemerides (https://ssd.jpl.nasa.gov/horizons/)

The orbit parameters, inertial parameters of asteroids, and rotation parameters determined based on the analysis of observations that were adopted for modeling are shown in the table. 1, namely: the eccentricity value e and the minimum approach distance $d = a (e - 1)$ (pericentric distance), expressed in mean Earth radii, $R_E = 6371$ km; ratios of the principal central moments of inertia A/C and B/C ($A < B < C$); the asteroid's rotation period P and the angle γ between the normal to the orbital plane and the rotation axis. Note that in the case of Apophis, the orbital parameters indicated in Table 1 differ from those given in (Lobanova, Melnikov, 2024), since in this work we used updated data from the NASA JPL ephemeris.

The orientation of the asteroid's angular momentum vector is usually determined with a large error. Therefore, specifying the actual value of the angle γ may be difficult. When conducting numerical experiments and analyzing their results in the case of asteroids 2012 TC4 and Duende, we considered two possible values of γ, and for Apophis - one value of γ. An explanation for this approach will be given below. Note that the angle γ can be estimated based on the value of the Yarkovsky effect, if it is known from the analysis of astrometric observations of the asteroid. In this regard, we provide below the theory of the Yarkovsky effect, necessary for determining γ. This theory is also necessary for the subsequent assessment of the influence of perturbations in the rotational dynamics of the asteroid on the magnitude of the Yarkovsky effect after the close approach to Earth.

THE YARKOVSKY EFFECT AND PARAMETER A_2

The Yarkovsky effect (Yarkovsky, 1901; Radzievsky, 1952; Rubincam, 1995; 1998; 2000; Farinella et al., 1998; Vokrouhlický, 1999; Vokrouhlický et al., 2000; 2015a) plays a significant role in the secular orbital dynamics of small asteroids. The essence of the Yarkovsky effect consists in the emergence of non-gravitational acceleration in orbital motion, caused by anisotropic re-emission of solar radiation from the surface of a rotating asteroid. One of the manifestations of the Yarkovsky effect – is the secular change in the semi-major axis of an asteroid and the corresponding change in the value of the mean motion of the asteroid. Below, based on the work of (Farinella et al., 1998), we present a brief theory that we used to estimate the magnitude of the Yarkovsky effect.

To calculate the rate of change of the asteroid's semi-major axis under the influence of the Yarkovsky effect, it is necessary to know the tangential component of the perturbing force f_Y.

Assuming that the orbit is close to circular, the formula for the average rate will have the form

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2f_{\mathrm{Y}}}{n}, \tag{1}$$

where n – the mean motion of the asteroid. The total magnitude of the Yarkovsky effect consists of *diurnal* and *seasonal* components. Let us consider how f_{Y} is calculated for the seasonal and diurnal Yarkovsky effect. In both cases, we will deal with an expression of the form (Burns et al., 1979)

$$f_{\mathrm{Y}} = \frac{2}{\rho R}\frac{\varepsilon\sigma T^4}{c}\frac{\Delta T_\nu}{T}\tilde{f}(\gamma), \tag{2}$$

where ρ – the density of the asteroid, R – the radius of a homogeneous sphere with a volume equal to the volume of the asteroid, ε – the emissivity of the asteroid's surface, σ – the Stefan-Boltzmann constant, c – the speed of light, ΔT_ν – the temperature difference between the most and least heated parts of the asteroid's surface, $\tilde{f}(\gamma)$ – some function of the angle γ between the asteroid's rotation axis and the perpendicular to its orbital plane, T – the average temperature of the asteroid, calculated by the formula $T = (\alpha S/(4\,\varepsilon\sigma))^{1/4}$, where α – the absorption coefficient on the asteroid's surface, $S = 1370\ \mathrm{W/m^2}\,(a_{\mathrm{E}}/a)^2$ – the solar energy flux for an asteroid with semi-major axis a, a_{E} – the semi-major axis of Earth's orbit.

The Daily Yarkovsky Effect

Following (Peterson, 1976), in the work (Farinella et al., 1998) it is assumed $\tilde{f}(\gamma) = cos\gamma$ and the following formula for the temperature factor is given:

$$\frac{\Delta T_\omega}{T} = 0.667\frac{\Theta_\omega}{1 + 2.03\,\Theta_\omega + 2.04\,\Theta_\omega^2}, \tag{3}$$

where the parameter

$$\Theta_\omega = \frac{\sqrt{\rho C K \omega}}{2\pi\varepsilon\sigma T^3} \tag{4}$$

characterizes the ratio of the temperature relaxation time to the considered time period (in the case of the daily Yarkovsky effect, the rotation period of the asteroid P is taken); C – specific heat capacity of the asteroid, K – thermal conductivity coefficient, $\omega = 2\pi/P$ – rotation frequency of the asteroid. Thus, the daily Yarkovsky effect will lead to $\mathrm{d}a/\mathrm{d}t > 0$ for rotation axis inclinations $0° <$

$\gamma < 90°$ (prograde motion: the directions of rotation and orbital motion of the asteroid coincide) and $da/dt < 0$ for $90° < \gamma < 180°$ (retrograde motion). Note that for the parameter values $\Theta_\omega >> 1$ (small values of P) the following relation is valid

$$\frac{\Delta T_\omega}{T} \sim \Theta_\omega^{-1} \sim \sqrt{P},$$

and the perturbing force tends to zero.

Seasonal Yarkovsky Effect

According to (Rubincam, 1995), $\grave{f}(\gamma) = -sin^2\gamma$ for the seasonal effect $da/dt \leq 0$ for any value of γ. The temperature factor in the work (Farinella et al., 1998) is derived based on papers (Rubincam, 1987; 1998; Afonso et al., 1995). It has the form

$$\frac{\Delta T_n}{T} = \frac{1}{3}\frac{1}{1-\tau}A_n \sin\delta_n\,, \tag{5}$$

where $\tau = \pi\, l_s \Theta_n/(2\,R)$, parameter Θ_n is calculated using formula (4) with replacement of frequency ω by the asteroid's mean motion n, and the value $l_s = \sqrt{K/\rho C n}$ represents the characteristic depth of thermal wave penetration. The factor $A_n \sin\delta_n$, where A_n – amplitude, δ_n – phase of the harmonic corresponding to frequency $\nu = n$ from the Fourier series expansion of the heat conduction equation solution, is found by the formula

$$A_n e^{i\delta_n} = \left(1 + \frac{\tau}{1+\tau}\psi(z)\right)^{-1}, \quad \text{where} \quad \psi(z) = \frac{(z^2-3)\sin z + 3z\cos z}{\sin z - z\cos z}\cdot z = \frac{R}{l_s}\sqrt{i}.$$

The total value of the Yarkovsky Effect is calculated using the formula

$$\frac{da}{dt} = \frac{2}{n}\left(f_Y^{(d)} + f_Y^{(s)}\right), \tag{6}$$

where $f_Y{}^{(d)}$ and $f_Y{}^{(s)}$ are determined using equation (2) taking into account expressions (3) and (5) for the temperature factor in the cases of diurnal and seasonal Yarkovsky effects, respectively.

Parameter A_2

The magnitude of acceleration caused by the Yarkovsky effect for asteroids is often given in notation from the work of (Marsden et al., 1973), namely: the transverse component of acceleration A_2 is indicated, as it can be estimated based on the analysis of a series of astrometric measurements (ground-based, space, and radar) within the orbit improvement procedure. Next, we will transition from the value da/dt to the parameter A_2 according to the formula (Farnocchia et al., 2013a):

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a\sqrt{1-e^2}}{nr} A_2\, g(r), \tag{7}$$

where $g(r)$ – is some function of the heliocentric distance r. In the work (Farnocchia et al., 2013a) they assume $g(r) = (r_0 / r)^d$, where $r_0 = 1$ AU – is a normalizing parameter, and the value of the exponent d for most NEAs is between 2 and 3 and has little effect on the value of da/dt . Usually (see, for example, Farnocchia et al., 2013b) $d = 2$ is assumed. We will do the same. In the case of a circular orbit, formula (7) takes the form

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{n} A_2\, g(r),$$

and taking into account (1) we get

$$A_2 = \frac{f_Y}{g(r)} = f_Y \times \left(\frac{a}{1\ \text{a.e.}}\right)^2. \tag{8}$$

Note that the parameter A_2 is a function of the physical parameters of the asteroid (see more details in (Farnocchia et al., 2013b)) and does not depend on the degree of orbital eccentricity. The value of A_2 for an asteroid, if determined, is usually provided on the NASA JPL website. Next, based on the known values of A_2 for the asteroids we are studying, we will try to estimate the value of γ. Then, after estimating the magnitude of perturbations in the rotational dynamics of asteroids during their close approaches to Earth, we will consider the influence of perturbations on A_2.

Orientation of the rotation axis of the studied asteroids

For Apophis, based on radar (Brozović et al., 2018) and photometric (Pravec et al., 2014) observations, estimates of the orientation of the angular momentum vector in ecliptic coordinates at epoch J2000 were obtained. Based on the estimates of longitude λ and latitude β, we calculated the inclination angle γ of Apophis' rotation axis relative to the normal to its orbital plane. The results are presented in Table 2. In the case of asteroid 2012 TC4, in the work (Lee et al., 2021) based on photometric observations, it was established: λ = 103° and
β = –88.5°, which, according to our calculations, corresponds to γ = 160°.

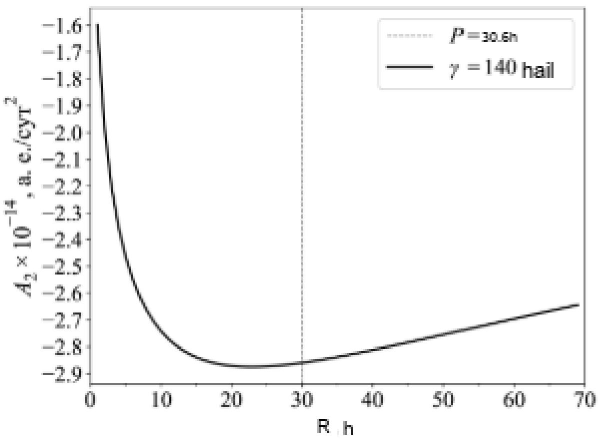
Table 2. Estimation of the inclination angle of the rotation axis γ of asteroid Apophis relative to the normal to its orbital plane based on observation results (Brozovic et al., 2018; Pravec et al., 2014)

Observations	λ, deg	β, deg	γ, deg
photometric	250	–75	148
radar	247	–9	132

The value γ can also be estimated based on the value A_2, by constructing a theoretical dependence $A_2(\gamma)$ and using it to determine the value γ corresponding to the value A_2, known from the analysis of observational data (provided on the NASA JPL website).

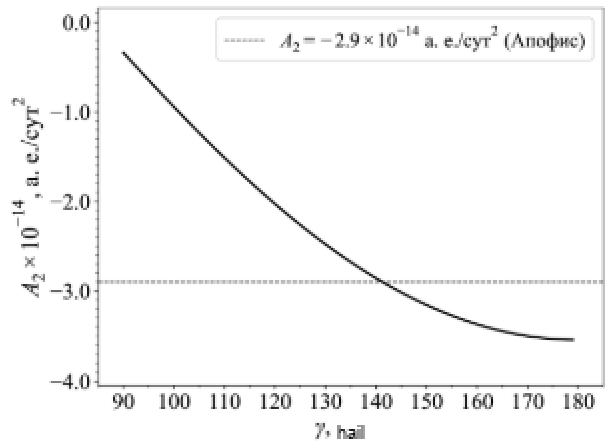
In Fig. 1 presents the dependencies $A_2(P)$ and $A_2(\gamma)$ that we constructed for the asteroids Apophis, 2012 TC4 and Duende. In the case of Apophis, similar dependencies for the value da/dt are presented in the paper (Lobanova, Melnikov, 2023). The value A_2 was determined using the formulas provided in the previous section. The values of the parameters α, ε, C and K, included in equations (2) and (4), as well as density and diameter for Apophis were taken from (Vokrouhlický et al., 2015b). For 2012 TC4 and Duende, average values for NEAs of the specified parameters were used, as provided in (Fenucci et al., 2024). For 2012 TC4, the density and diameter were taken from (Lee et al., 2021). The density value for Duende was taken from (Devyatkin et al., 2016) under the assumption that the asteroid belongs to the spectral class L, and the diameter was taken from (Moskovitz et al., 2020). The values of the orbital period and semi-major axis for all asteroids were obtained from NASA JPL.

As can be seen from Fig. 1b, for Apophis, the value of A_2, determined based on observational data and presented on the NASA JPL website, corresponds to a value of $\gamma \approx 140°$. This value of γ falls within the interval between the point estimates provided in Table 2. Due to the large uncertainty in the knowledge of the angular momentum vector orientation, we further assumed $\gamma = 140°$ for Apophis. In the case of asteroid 2012 TC4, the accepted value of $A_2 = -26.8\times10^{-14}$ au/day^2 corresponds, according to Fig. 1d, to $\gamma = 105°$. For the value $\gamma = 160°$, determined by (Lee et al., 2021) based on photometric observations of 2012 TC4, Fig. 1c shows $A_2 = -70\times10^{-14}$ au/day^2. Due to such a significant difference in the estimates of γ, we further considered both possible options for asteroid 2012 TC4. For Duende, apparently, there are no astrometric estimates of A_2. However, in (Benson et al., 2020), two rotation models of Duende during its approach to Earth in 2013 were constructed, based on which we determined $\gamma = 27°$ *and* $\gamma = 160°$. Theoretical dependencies $A_2(P)$ and $A_2(\gamma)$ for Duende are shown in Fig. 1. In the first case for Duende, we have $A_2 = 23\times10^{-14}$ au/day^2, in the second – $A_2 = -24\times10^{-14}$ au/day^2. Let us proceed to estimating the magnitudes of perturbations in the rotational dynamics of asteroids during their approaches to Earth.



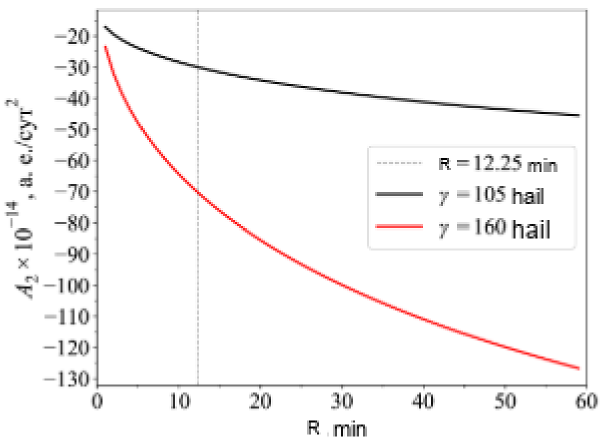
P = 30.6h
γ = 140 hail
$A_2 \times 10^{-14}$, а. е./сут^2
R h

(a)



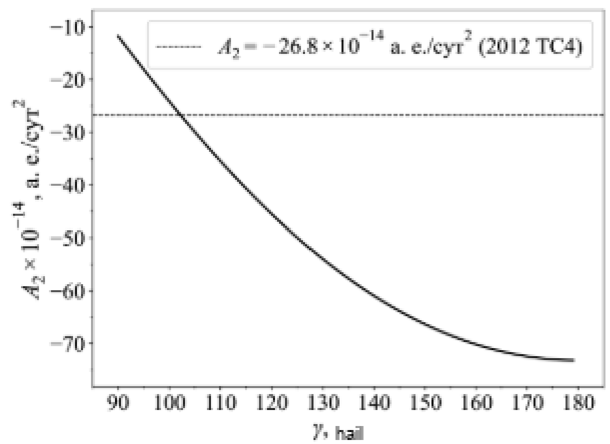
$A_2 = -2.9 \times 10^{-14}$ а. е./сут^2 (Апофис)
$A_2 \times 10^{-14}$, а. е./сут^2
γ, hail

(b)



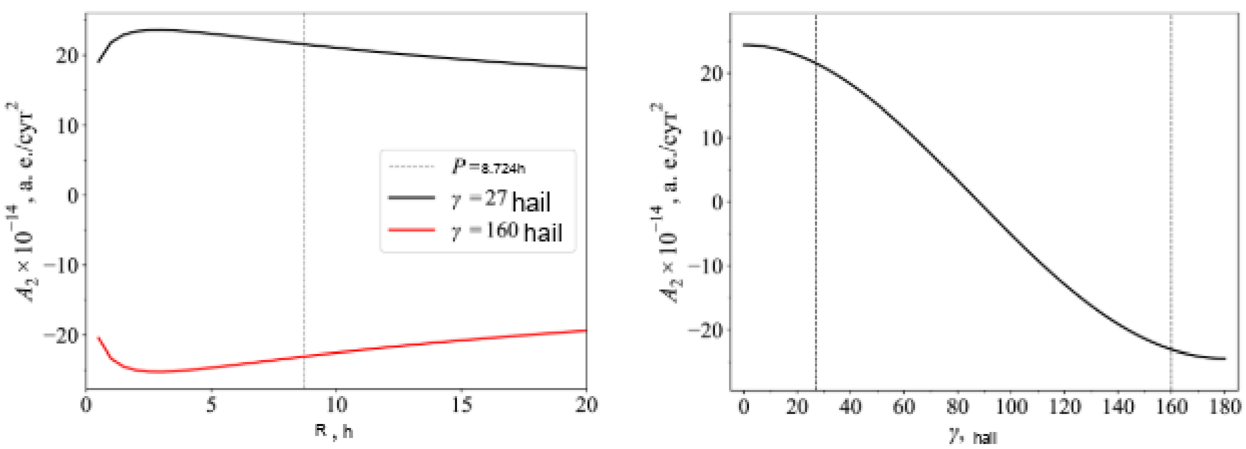
R = 12.25 min
γ = 105 hail
γ = 160 hail
$A_2 \times 10^{-14}$, а. е./сут^2
R min

(c)



$A_2 = -26.8 \times 10^{-14}$ а. е./сут^2 (2012 TC4)
$A_2 \times 10^{-14}$, а. е./сут^2
γ, hail

(d)



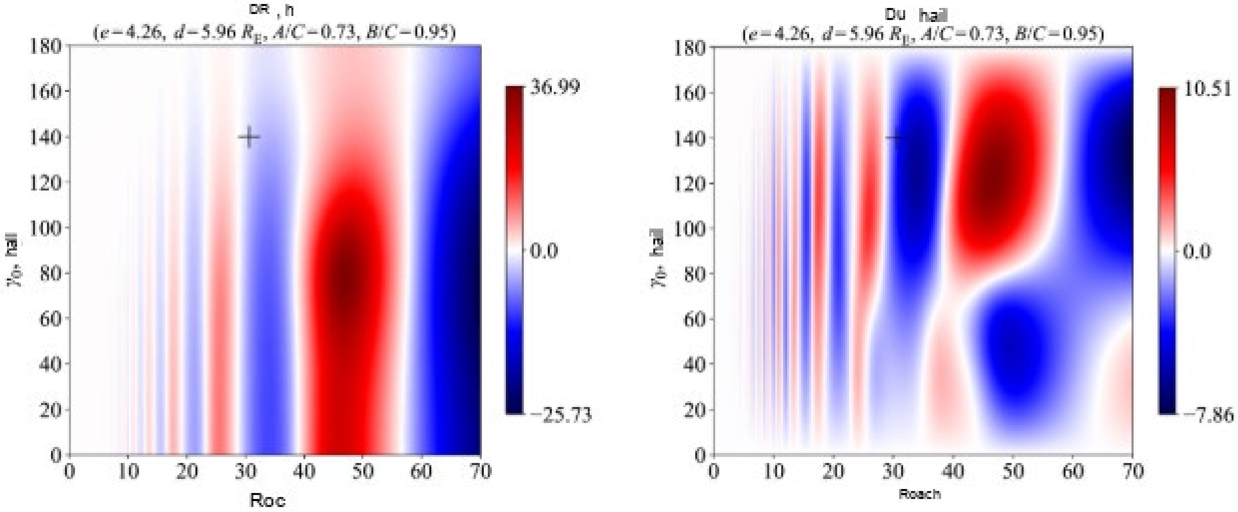
(e) (f)

Fig. 1. Dependence of the value A_2 on the rotation period P and the inclination angle of the rotation axis γ: (a) – for Apophis at $\gamma = 140°$; (b) – for Apophis at $P = 30.6$ h; (c) – for asteroid 2012 TC4 at $\gamma = 105°$ and 160°; (d) – for asteroid 2012 TC4 at $P = 12.25$ min; (e) – for Duende at $\gamma = 27°$ and 160°; (f) – for Duende at $P = 8.724$ h. Dashed vertical lines (panels (a), (c) and (e)) correspond to data (Pravec et al., 2014; Lee et al., 2021; Benson et al., 2020), on panel (f) – to the accepted values of γ for Duende. Dashed horizontal lines on panels (b) and (d) correspond to the values of A_2 indicated on the NASA JPL website.

ESTIMATION OF PERTURBATIONS IN ROTATIONAL DYNAMICS

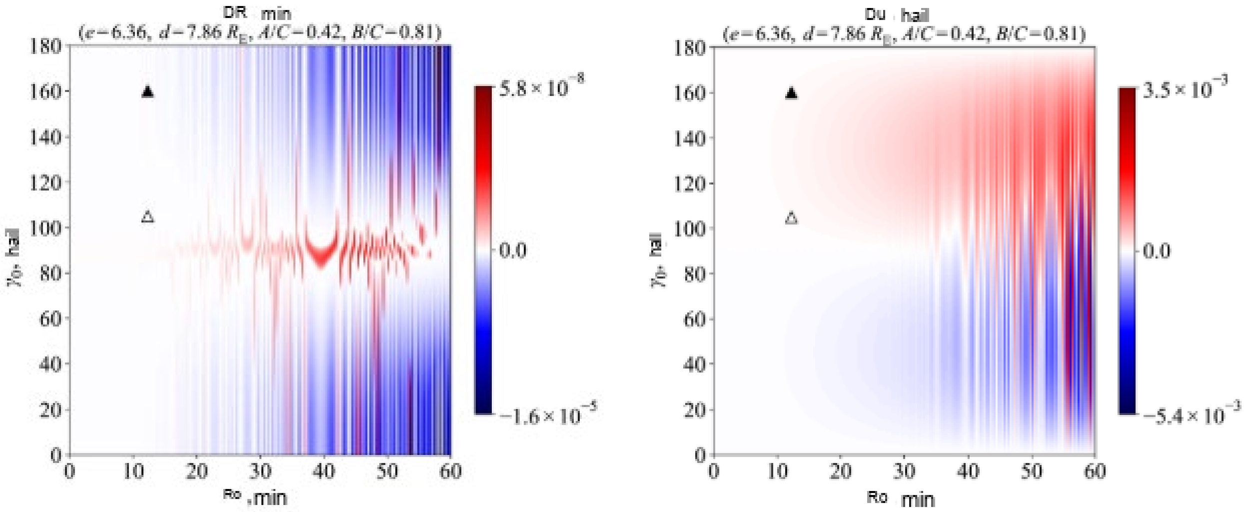
For all studied asteroids on the set (P_0, γ_0) of possible initial (before the approach) values of P and γ the following quantities were determined: $\Delta P = P_{fin} - P_0$ and $\Delta\gamma = \gamma_{fin} - \gamma_0$, where the lower index "fin" corresponds to the values after the approach (when the asteroid moves away from the geocenter to a distance of 100 R_E). The values P_0 and γ_0 were set on a uniform grid, defined as follows: 1) for Apophis 15 min. $\leq P_0 \leq$ 70 h with a step of 15 min., $0° \leq \gamma_0 \leq 180°$ with a step of 0.5°; 2) for 2012 TC4 15 s $\leq P_0 \leq$ 60 min. with a step of 15 s, $0° \leq \gamma_0 \leq 180°$ with a step of 1.0°; 3) for Duende 1 min. $\leq P_0 \leq$ 20 h with a step of 1 min., $0° \leq \gamma_0 \leq 180°$ with a step of 1.0°.

In Fig. 2–4, the calculated changes in the period and orientation of the rotation axis of Apophis, 2012 TC4 and Duende due to their approaches to Earth are presented. Diagrams for the case of Apophis's approach to Earth in 2029 were obtained earlier in (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024). For Duende, the dependence ΔP (P_0, γ_0) was constructed in (Melnikov, 2022). In the present work, all diagrams are constructed with significantly higher resolution and for orbits based on current data from the NASA JPL website. On all diagrams in accordance with Table. 1 shows the positions of the asteroids. For all asteroids on the diagrams, there are alternating regions when changing P_0, where there is a slowdown ($\Delta P > 0$), or acceleration ($\Delta P < 0$) of the asteroid's rotation, and changes in γ occur. Discussion of the details identified in the diagrams was conducted in (Lobanova, Melnikov, 2024), where it is indicated that the positions of local maxima and minima of ΔP and $\Delta\gamma$ values are determined by orbit parameters, while the inertial parameters of the asteroid determine the amplitude of extreme perturbation values.



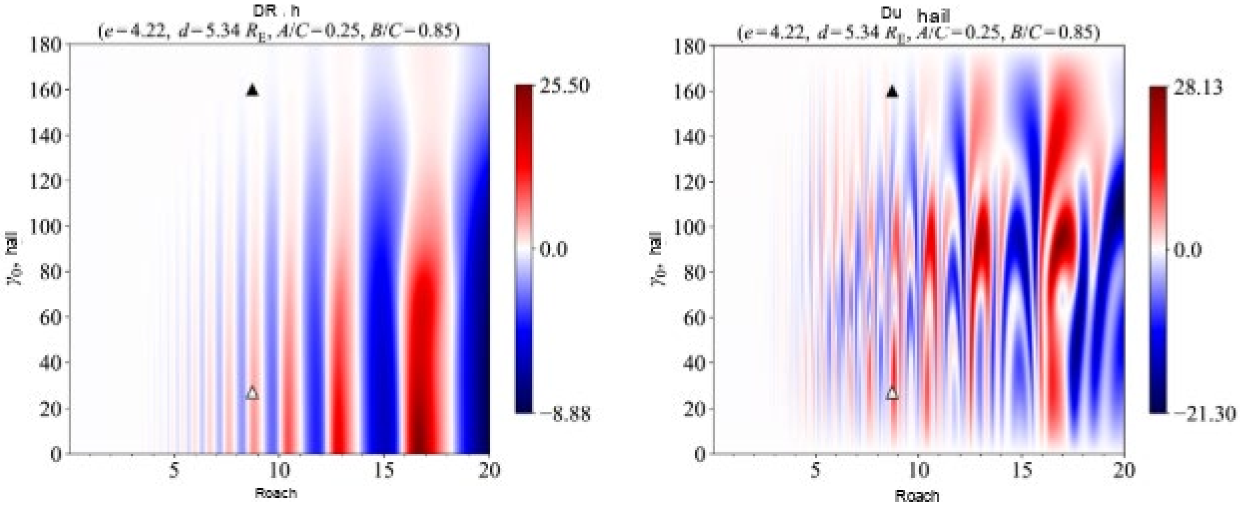
(a) (b)

Fig. 2. Change in rotational state of Apophis due to the close approach to Earth in 2029 depending on P_0 and γ_0: (a) – change in ΔP rotation period; (b) – change in $\Delta\gamma$ angle characterizing the deviation of Apophis's rotation axis from the normal to the orbital plane. The cross indicates the position of Apophis according to the data from Table 1



(a) (b)

Fig. 3. Change in rotational state of asteroid 2012 TC4 due to the close approach to Earth in 2017 depending on P_0 and γ_0: (a) – change in ΔP rotation period; (b) – change in $\Delta\gamma$ angle characterizing the deviation of the asteroid's rotation axis from the normal to the orbital plane. Triangles indicate possible positions of asteroid 2012 TC4, according to the data from Table 1.



(a) (b)

Fig. 4. Change in rotational state of asteroid Duende due to the close approach to Earth in 2013 depending on P_0 and γ_0: (a) – change in ΔP rotation period; (b) – change in $\Delta\gamma$ angle characterizing the deviation of the asteroid's rotation axis from the normal to the orbital plane. Triangles indicate possible positions of Duende, according to the data from Table 1

According to (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024), Apophis's rotation period due to the 2029 close approach may either decrease by 20 h or increase by 40 h relative to the current value $P = 30.6$ h. The most likely change in P is 10–15 h, which is consistent with results obtained in (Scheeres et al., 2000; 2004; Boldrin et al., 2020; Melnikov, 2022) by modeling close approaches of various asteroids to planets. The results of modeling Apophis's rotational dynamics presented in Fig. 2 for other orbital parameter values and with higher resolution of initial data are consistent with these conclusions. Note that changing the orbital parameters (value of e) led to a decrease in the maximum value of ΔP to 35 h. The amplitude of $\Delta\gamma$ perturbations in the orientation of Apophis's rotation axis, according to Fig. 2, can reach ten degrees. This estimate is consistent with results obtained in (Souchay et al., 2014; 2018; Lobanova, Melnikov, 2023). In some cases (with values of γ_0 close to 90°), a transition from prograde to retrograde rotation and vice versa is observed. Note that such a transition leads to a change in the sign of the diurnal Yarkovsky effect and the total Yarkovsky effect, since the seasonal component of the Yarkovsky effect is always negative and small compared to the diurnal component.

In the case of asteroid 2012 TC4 during its close approach to Earth in 2017, according to Fig. 3, the perturbations in rotational motion were very small: $|\Delta P| < 10^{-5}$ min., $|\Delta\gamma| < 0.01°$. Thus, the close approach to Earth had practically no effect on the asteroid's rotational dynamics. In (Lee et al., 2021), it is indicated that the rotation periods of 2012 TC4 determined during the close approaches in 2012 and 2017 differed by almost 0.04 min. The acceleration of the asteroid's rotation could have been caused by the YORP effect (see discussion therein). Note that although the perturbations in 2012 TC4's rotation due to close approaches to Earth are small, they need to be taken into account as they may affect the correct estimation of the YORP effect.

In addition to 2012 TC4, we also investigated the dynamics of asteroid 2023 BU, which has similar dimensions, during its record-close (distance from geocenter about 9900 km) approach to Earth in 2023 (Martyusheva et al., 2023). The rotation of 2023 BU is even faster ($P < 2$ min) than that of 2012 TC4. The nature of the diagrams obtained for 2023 BU differs little from those shown in Fig. 3 for 2012 TC4. The magnitudes of perturbations in the rotational dynamics of 2023 BU were of the same order as for 2012 TC4, and this is probably typical for such objects.

According to our numerical experiments (see Fig. 4), the perturbations in the case of Duende's approach to Earth in 2013 should have been quite noticeable. For the values indicated in Table 1, we have: with direct rotation of Duende $\Delta P \approx 4.2$ h, $\Delta\gamma \approx 4.2°$, with retrograde rotation -

$\Delta P \approx 0.1$ h, $\Delta\gamma \approx -1.1°$. Attention should be paid to the significant difference in the value of ΔP for different orientations of the rotation axis and the large amplitude of possible variations of γ, significantly exceeding the values of $\Delta\gamma$ obtained for the case of Apophis (see Fig. 2b). In the work (Devyatkin et al., 2016), based on modeling light curves and rotational dynamics of Duende, it was concluded that its rotation period increased due to the approach to Earth by more than 1 h. In works (Benson et al., 2020; Moskovitz et al., 2020), analysis of observational data obtained immediately after the approach revealed a change in Duende's period by almost 0.4 h. Numerical experiments (Melnikov, 2022) on modeling the rotational dynamics of Duende showed that $\Delta P < 2.4$ h. All these works identified a noticeable change in Duende's rotation due to the approach.

INFLUENCE OF ASTEROID SHAPE PARAMETERS ON ΔP AND $\Delta\gamma$

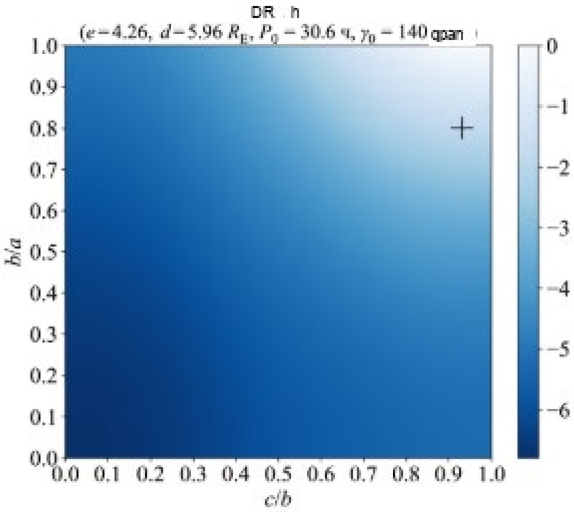
Asteroid shapes are usually known with large uncertainties, and for small asteroids, there are often only diameter estimates based on the absolute stellar magnitude of the asteroid under certain simplifications. For Apophis and 2012 TC4, radar observations are available and approximations of their shapes have been constructed (see Pravec et al., 2014; Lee et al., 2021). For Duende, only shape parameter estimates are available (Devyatkin et al., 2016; Benson et al., 2020; Moskovitz et al., 2020). The moments of inertia (values Moments of inertia (meanings A/C and B/C), knowledge of which is necessary for accurate modeling of the gravitational interaction of an asteroid and a planet during a close approach, are usually determined on the basis of constructed models of figures under the assumption of a uniform density of the asteroid. Therefore, the error in the assessment A/C and B/C can be significant. We studied the influence of asteroid shape, assuming it to be a triaxial ellipsoid with semi-axes $a > b > c$, on estimates of ΔP and $\Delta\gamma$. Specifically, for all studied asteroids, diagrams of $\Delta P(c/b, b/a)$ and $\Delta\gamma(c/b, b/a)$ were constructed and analyzed for values $0 < c/b, b/a \leq 1$ and values P and γ specified in Table 1.

For the case of a triaxial ellipsoid of uniform density, we have the following relationship between its semi-axes and moments of inertia (Kupriyanov, Shevchenko, 2006):

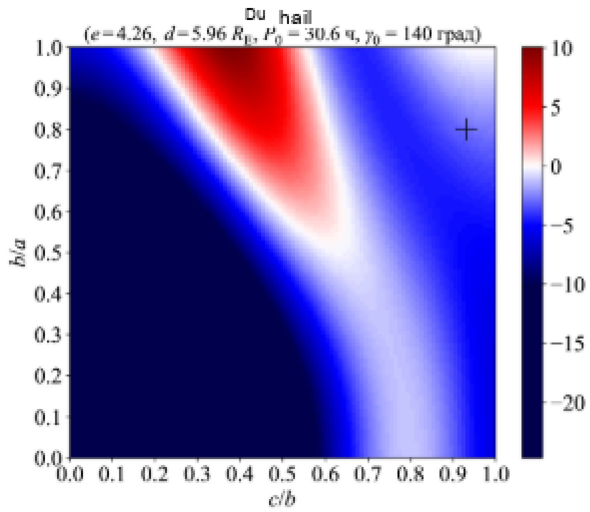
$$\frac{c}{b} = \sqrt{\frac{-1 + A/C + B/C}{1 + A/C - B/C}}, \quad \frac{b}{a} = \sqrt{\frac{1 + A/C - B/C}{1 - A/C + B/C}}.$$

By varying the ratios c/b and b/a from 0 to 1, we consider all possible values of A/C and B/C.

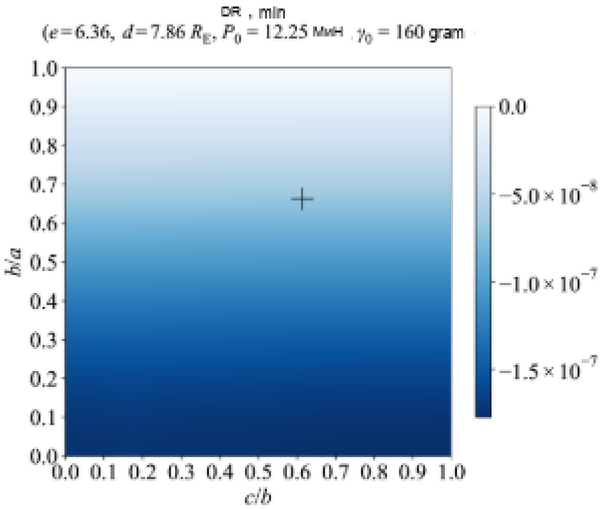
In Fig. 5 and 6 show diagrams of $\Delta P(c/b, b/a)$ and $\Delta\gamma(c/b, b/a)$, constructed for Apophis, 2012 TC4 and Duende. It is evident that in the case of Apophis, uncertainties in the knowledge of the moments of inertia can lead to errors in the estimation of ΔP, amounting to several hours. Note that the diagram $\Delta P(c/b, b/a)$ has no explicit maximums and minimums, while on the diagram for $\Delta\gamma$ they are clearly expressed. The error in estimating $\Delta\gamma$ during numerical modeling of Apophis's close approach to Earth, if its real moments of inertia differ significantly from those determined in (Pravec et al., 2014), can be tens of degrees. For example, with $c/b = 0.5$, $b/a = 0.2$, according to Fig. 5, we have $\Delta\gamma \approx -25°$, while the data from Pravec et al. (2014) corresponds to $\Delta\gamma \approx -5°$. One should expect that behavior similar to that presented in Fig. 5 is inherent to other asteroids with relatively slow rotation (period $P > 5$ h), which is confirmed by the diagrams for Duende shown in Fig. 6. It is particularly noteworthy that the perturbations for Duende are large with direct rotation ($\gamma_0 = 27°$) – ΔP reaches 12 h, $-15° < \Delta\gamma < 20°$ – and noticeably smaller with retrograde rotation ($\gamma_0 = 160°$) – $\Delta P < 0.15$ h, $-2° < \Delta\gamma < 4°$. A similar conclusion was made by us earlier when analyzing the diagrams $\Delta P(P_0, \gamma_0)$ and $\Delta\gamma(P_0, \gamma_0)$ for Duende, presented in Fig. 4.


DR h
(e=4.26, d=5.96 R_E, P_0 = 30.6 ч, γ_0 = 140 qpan
b/a
c/b
0
−1
−2
−3
−4
−5
−6

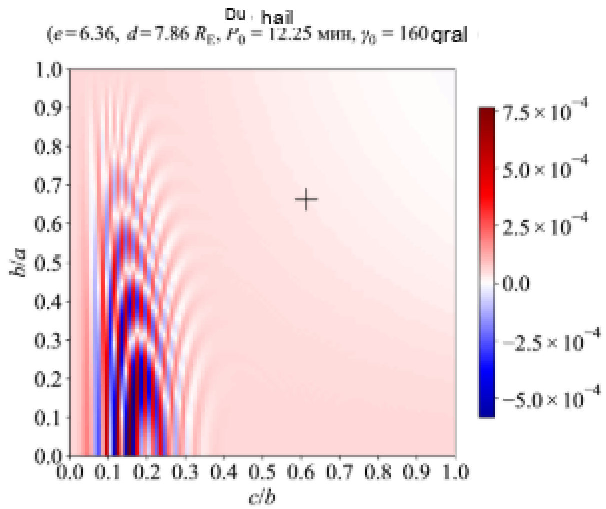
(a)


Du hail
(e=4.26, d=5.96 R_E, P_0 = 30.6 ч, γ_0 = 140 град)
b/a
c/b
10
5
0
−5
−10
−15
−20

(b)

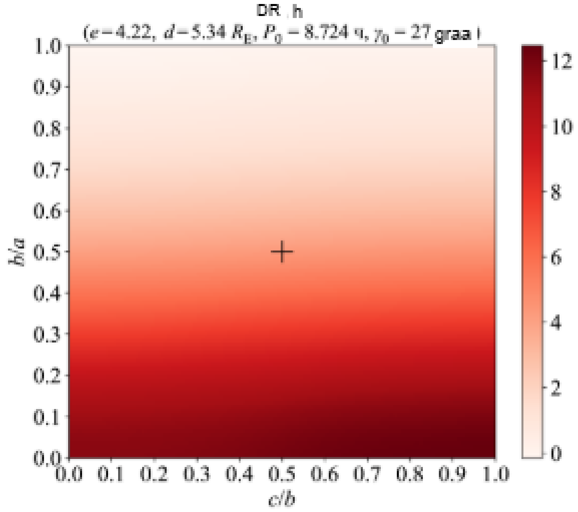

DR , min
(e=6.36, d=7.86 R_E, P_0 = 12.25 мин , γ_0 = 160 gram
b/a
c/b
0.0
-5.0×10^{-8}
-1.0×10^{-7}
-1.5×10^{-7}

(c)

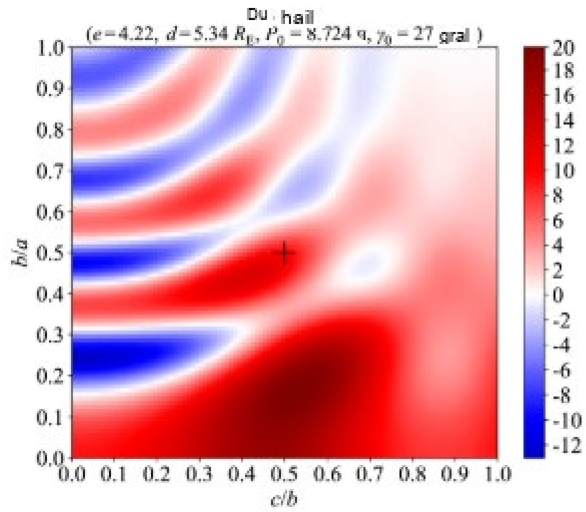

Du hail
(e=6.36, d=7.86 R_E, P_0 = 12.25 мин, γ_0 = 160 qral
b/a
c/b
7.5×10^{-4}
5.0×10^{-4}
2.5×10^{-4}
0.0
-2.5×10^{-4}
-5.0×10^{-4}

(d)

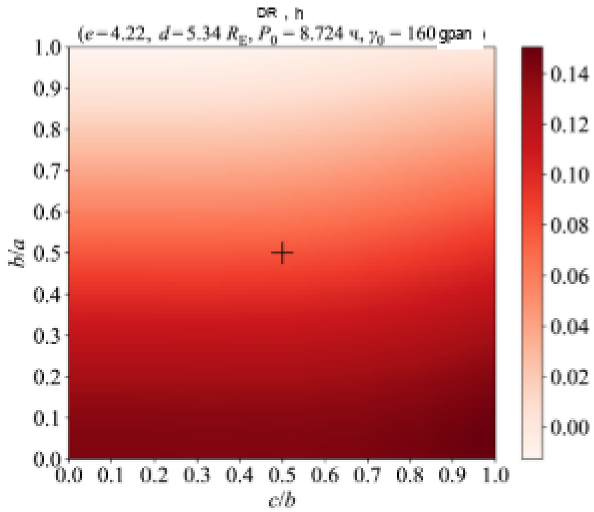
Fig. 5. Change in the rotational state of the asteroid depending on the parameters c/b and b/a , characterizing the asteroid's shape: (a) – change ΔP of the rotation period and (b) – $\Delta\gamma$ of the angle characterizing the deviation of the asteroid's rotation axis from the normal to the orbital plane of Apophis due to the close approach with Earth in 2029; (c) – change ΔP and (d) – $\Delta\gamma$ for asteroid 2012 TC4 due to the close approach with Earth in 2017. The adopted orbital parameters (e, d) and initial rotation parameters of the asteroids (P_0, γ_0) are shown in the figures. The cross indicates the positions of the asteroids according to (Pravec et al., 2014; Lee et al., 2021).


DR h
(e = 4.22, d = 5.34 R_E, P_0 = 8.724 ч, γ_0 = 27 graa
b/a
c/b
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 2 4 6 8 10 12

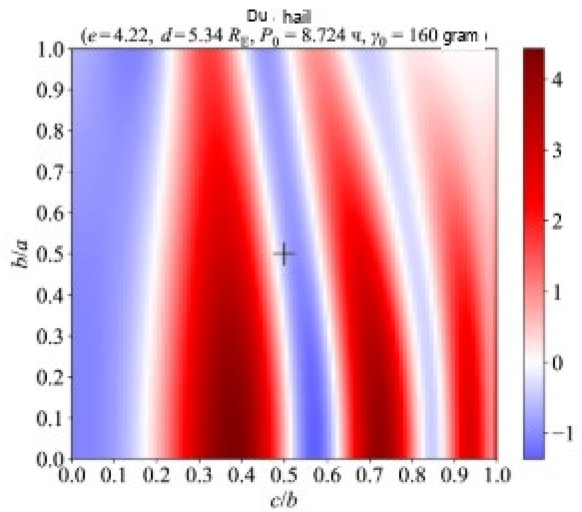
(a)


Du hail
(e = 4.22, d = 5.34 R_E, P_0 = 8.724 ч, γ_0 = 27 gral
b/a
c/b
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
-12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18 20

(b)


DR h
(e = 4.22, d = 5.34 R_E, P_0 = 8.724 ч, γ_0 = 160 gpan
b/a
c/b
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14

(c)


Du hail
(e = 4.22, d = 5.34 R_E, P_0 = 8.724 ч, γ_0 = 160 gram
b/a
c/b
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
−1 0 1 2 3 4

(d)

Fig. 6. Change in the rotational state of asteroid Duende due to the close approach with Earth in 2013 depending on the parameters c/b and b/a , characterizing the asteroid's shape: (a) – change ΔP of the rotation period at $\gamma_0 = 27°$; (b) – change $\Delta\gamma$ of the angle characterizing the deviation of the asteroid's rotation axis from the normal to the orbital plane, at $\gamma_0 = 27°$; (c) – change ΔP at $\gamma_0 = 160°$; (d) – change $\Delta\gamma$ at $\gamma_0 = 160°$. The adopted orbital parameters (e, d) and initial rotation parameters of the asteroid (P_0, γ_0) are shown in the figures. The cross indicates the possible position of Duende (see Melnikov, 2022).

For asteroids with fast rotation, of which 2012 TC4 is an example, the uncertainty in the knowledge of the asteroid's shape does not significantly affect the estimates of perturbations arising in the rotational dynamics. According to Fig. 5, in the case of 2012 TC4, the acceptable (due to determination errors) variations in c/b and b/a correspond to $|\Delta P < 10^{-7}$ min and $|\Delta\gamma| < 0.001°$. These values are negligibly small compared to the asteroid's rotation period $P = 12.25$ min. and the perturbations in rotation that occur during the Earth approach (see Fig. 3). In the case of 2012 TC4, diagrams are provided only for $\gamma_0 = 160°$, since diagrams constructed for $\gamma_0 = 105°$ have a similar character, and the amplitude of perturbations is of the same order. It should be noted that in our diagrams $\Delta P(c/b,\ b/a)$ and $\Delta\gamma(c/b,\ b/a)$ for the asteroid 2023 TC4 and not shown here, the amplitude of perturbations was also very small. BU, which is similar to 2012

IMPACT OF EARTH APPROACH ON THE MAGNITUDE OF THE YARKOVSKY EFFECT

As noted earlier, perturbations in the rotational motion of asteroids due to close approach to Earth should affect the magnitude of the Yarkovsky effect. Let's estimate the change in the parameter A_2 for the studied asteroids. For this purpose, using the theoretical magnitude of the Yarkovsky effect calculated with equations (2) and (8), we constructed (see Fig. 7) diagrams of A_2 $(P,\ \gamma)$; on them, according to Table 1, the positions of the asteroids are indicated. Analysis of the diagrams shows that the value of A_2 is significantly affected by changes in the inclination of the rotation axis. Perturbations in the period for Apophis and Duende, even if they are large (amounting to several hours), have practically no effect on the magnitude of the Yarkovsky effect. In contrast, in the case of fast rotation, as with 2012 TC4, even small perturbations in P can lead to noticeable changes in A_2. However, as we have shown above, for asteroids with fast rotation, perturbations due to Earth approach are very small.

Perturbations due to Apophis' close approach to Earth in 2029, which correspond to (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024) $\Delta P = 10–15$ h, will lead to changes in A_2 by several percent (see also (Benson et al., 2023)). If we assume (see Fig. 2b) that $|\Delta\gamma| = 10°$, then changes in the current value of $A_2 = -2.9\times10^{-14}$ AU/day^2 will be significant: according to Fig. 7a, it may either decrease to -2.4×10^{-14} AU/day^2, or increase to -3.2×10^{-14} AU/day^2. Since the perturbations in the rotational dynamics of asteroid 2012 TC4 during its close approach to Earth in 2017 were very small (see above), they did not affect the value of the parameter A_2. For Duende, as follows from Fig. 4, significant rotational perturbations in 2013 would have occurred only with direct rotation ($\gamma_0 = 27°$), but since they predominated in rotation speed ($\Delta P \approx 4.2$ h), rather than in γ ($\Delta\gamma \approx 4.2°$), they also did not affect the value of A_2.

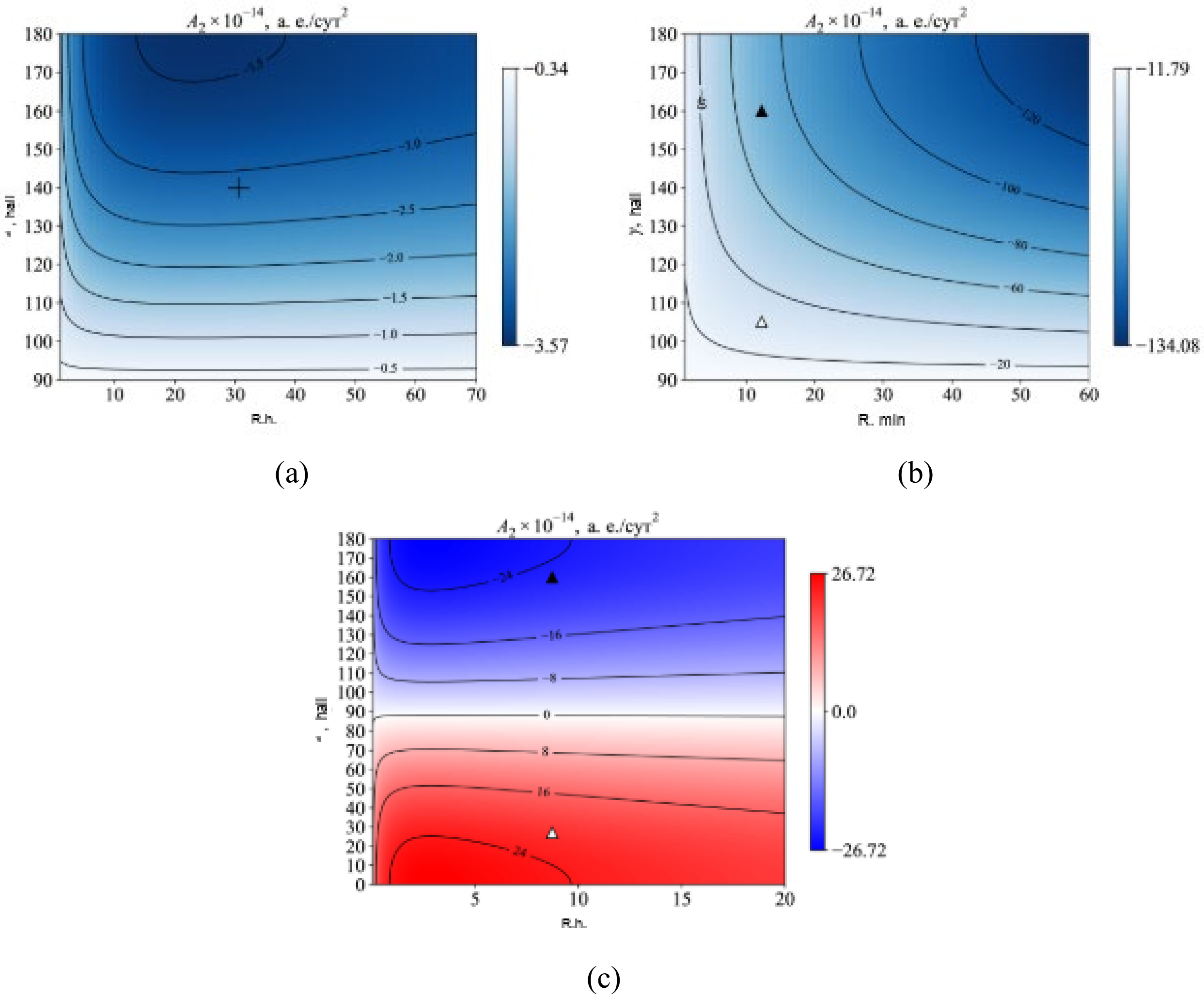


Fig. 7. Dependence of the value of A_2 on P and γ: (a) - for Apophis. The cross indicates the current position of Apophis; (b) - for asteroid 2012 TC4; (c) - for Duende. Triangles indicate possible positions of the asteroids (see discussion in the text). The contour lines correspond to the values of A_2 indicated on them.

CONCLUSIONS

In this paper, the influence of the asteroid's rotation rate and shape parameters on the magnitude of perturbations in its rotational dynamics that occur during a close approach to Earth, and the effect of these perturbations on the Yarkovsky effect, has been studied through numerical experiments. For this purpose, the dynamics of asteroids (99942) Apophis, (367943) Duende, and 2012 TC4 were considered, which have different sizes (diameters D = 10–340 m) and significantly different rotation periods (P = 12 min. – 30 h).

It was found that in the case of Apophis ($P \approx 30$ h) during its close approach to Earth in 2029, changes in the rotation period ΔP can reach tens of hours, and deviations $\Delta\gamma$ in the orientation of the rotation *axis* can reach ten degrees. Similar results were obtained by us earlier (Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024) for other orbital parameters of Apophis. For asteroid Duende ($P \approx 8$ h) during its close approach to Earth in 2013, the value of ΔP did not exceed a few hours. Deviations in the orientation of Duende's rotation axis could amount to tens of degrees. The above values of ΔP and $\Delta\gamma$ are probably characteristic of asteroids with relatively slow rotation (P > 5 h) during close approaches to Earth (at a distance of several Earth radii), which is consistent with the results of other model numerical experiments (Scheeres et al., 2000; 2004; Boldrin et al., 2020; Melnikov, 2022; Lobanova, Melnikov, 2023; Lobanova, Melnikov, 2024).

For asteroids with rapid rotation ($P < 1$ h), the perturbations will be very small (see also the discussion in (Lee et al., 2021)). For example, in the case of asteroid 2012 TC4 ($P \approx 12$ min.) during its close approach to Earth in 2017, our numerical experiments showed: $|\Delta P| < 10^{-5}$ min., $|\Delta\gamma| <$ 0.01°. Similar estimates were obtained for asteroid 2023 BU ($D \sim 10$ m, $P < 2$ min.) when analyzing its rotational dynamics during a very close (about 9900 km) approach to Earth in 2023. Note that asteroids with rapid rotation are evidently monolithic bodies. Larger bodies, as well as "rubble pile" type asteroids, break apart when reaching the critical point corresponding to $P \approx 2.2$ h (see more details in (Pravec, Harris, 2000; Hu et al., 2021)).

Usually, the shape parameters of small asteroids and, consequently, their moments of inertia are unknown or determined with significant error. We examined the influence of uncertainty in knowledge of moments of inertia on estimates of ΔP and $\Delta\gamma$. It was established that in the case of Apophis, uncertainties in the values of moments of inertia during numerical modeling of its approach to Earth in 2029 can lead to errors in the estimation of ΔP reaching several hours, and in the estimation of $\Delta\gamma$ - tens of degrees. A similar conclusion was made by us for Duende when modeling its approach to Earth in 2013. It should be expected that the significant influence of errors in the knowledge of moments of inertia on estimates of ΔP and $\Delta\gamma$ takes place for other asteroids with relatively slow rotation ($P > 5$ h). In contrast, for asteroids with very rapid rotation, as we demonstrated in the case of 2012 TC4, uncertainty in the knowledge of moments of inertia does not significantly affect the estimates of perturbation magnitudes in rotational dynamics arising from the close approach.

The influence of perturbations in the rotational motion of asteroids on the magnitude of the Yarkovsky effect is examined. The analysis of theoretical dependencies of the parameter A_2 on P and γ, constructed for all studied asteroids, showed that perturbations in period for Apophis and Duende practically do not affect the magnitude of the Yarkovsky effect, noticeable changes in A_2 can occur only with a significant change in γ. For asteroids with rapid rotation, such as 2012 TC4, perturbations in both rotation speed and orientation of the rotation axis noticeably affect the magnitude of A_2. In the case of Apophis, due to its close approach to Earth in 2029, there may be either a decrease or increase in the magnitude of A_2 within the range from -2.4×10^{-14} AU/day^2 to -3.2×10^{-14} AU/day^2. The expected change in the magnitude of A_2 will have a noticeable effect on the evolution of Apophis's orbit after the close approach. For Duende, the perturbations in rotational motion that occurred in 2013 probably did not affect the magnitude of A_2. Since perturbations in the rotational dynamics of asteroid 2012 TC4 during its close approach to Earth in 2017 were very small, they did not affect the magnitude of parameter A_2.

Thus, based on numerical modeling of the dynamics of a number of asteroids, we examined perturbations in their rotational motion during close approaches to Earth and their impact on the magnitude of the Yarkovsky effect. Through additional numerical experiments modeling the dynamics of various NEAs with slow ($P > 5$ h) and fast ($P < 1$ h) rotation, the conclusions obtained in this work can be generalized to these two types of asteroids. Specifically, asteroids with slow rotation are characterized by significant perturbations in rotational motion during close approaches to Earth; also, when estimating the magnitude of perturbations, the inaccuracy in knowing the actual values of the asteroid's moments of inertia should be taken into account. The change in the orientation of the rotation axis due to the close approach has a significant effect on the magnitude of the Yarkovsky effect for asteroids with slow rotation. In contrast, for asteroids with fast rotation, perturbations are small and can be neglected when modeling the asteroid's dynamics after the close approach. However, for fast-rotating asteroids, one should consider the possibility of significant changes in the magnitude of the Yarkovsky effect due to perturbations of a different nature, leading both to displacement of the rotation axis and changes in the asteroid's rotation rate. Such changes can be caused, for example, by external physical impact on the asteroid (Bottke et al., 2020; Daly et al., 2023).

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