INVESTIGATION OF DYNAMIC PROCESSES IN AN ELASTIC LAYER ON THE SURFACE OF A COMPRESSIBLE FLUID

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Abstract. The paper investigates an interesting phenomenon found during earthquakes occurring in one area of the southern part of Azerbaijan.Taking into account the rare features of this part of the Earth's crust, the occurring event was modeled in the form of a mathematical problem of the dynamic theory of elasticity, which revealed the cause of the investigated phenomenon. Bibl.3. Fig.3.

Keywords: *earthquake, elastic layer, fluid, Lame equation*

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1. INTRODUCTION

In earthquakes, as usual, there are 2 shocks following each other. The first of them corresponds to a type of longitudinal waves appearing as oscillatory motions with increasing amplitude in a plane parallel to the earth's surface. It is followed by a second, single-moment shock, apparently corresponding to Rayleigh surface waves, in which the motion is directed upward, i.e., perpendicular to the ground surface.

We have found out that at earthquakes in Lankaran city of Azerbaijan Republic the first type of waves in the form of oscillatory motions is almost not observed even at earthquakes with high magnitudes, one of which - five-point earthquake - has happened recently in this area

Trying to find the reason, we turned to the fact of the peculiarity of the earth crust structure in this area. And it differs by the fact that there are a lot of wells here, and the level of underground groundwater is quite close to the surface. This level varies from 3 to 5 meters from the ground surface.

Taking into account these features and the fact that Lankaran is located between the Caspian Sea and the Talysh Mountains, and, usually, the epicenters of tremors are located on the sea bottom, the problem of unsteady dynamics of an elastic semi-infinite layer, the bottom part of which borders with a compressible ideal fluid, was posed (Fig. 1). The fluid motion is assumed to be potential, i.e., vortexless. Some or all part of the end face of the layer is subjected to the shock.

To solve this problem, we used some results of [1], devoted to the study of the dynamics of rectangular prisms from the position of the exact three-dimensional theory of elastodynamics. In particular, when some simplified boundary conditions are satisfied, the three-dimensional problem under study becomes two-dimensional, i.e., a solution for a layer is obtained. Here we will use exactly these ready solutions for a semi-infinite elastic layer subjected to the action of a longitudinal impact on the end region of the same layer. But in the present problem, the existence of different types of media bordering each other, of course, makes the solution process much more complicated; we obtain a problem with five unknowns.

We propose a new method for determining the originals from transform functions, which in the present paper have a very complicated form; they are represented through determinants of the fifth rank. In some sense, this method is a generalization of a similar method which, for the same purpose, was first proposed in [1], and for axisymmetric cases in [2].

Exact solutions are obtained, which are valid in the initial short time of the process, but give a rather wide possibility of seeing the whole process for subsequent times as well. The results with high accuracy confirm the correctness of determining the reasons for the absence of the first type of shocks on the crustal surface during earthquakes.

2. STATEMENT AND METHOD OF SOLUTION

Taking into account the location of Lankaran city of the Republic of Azerbaijan, the problem under consideration is modeled as follows.

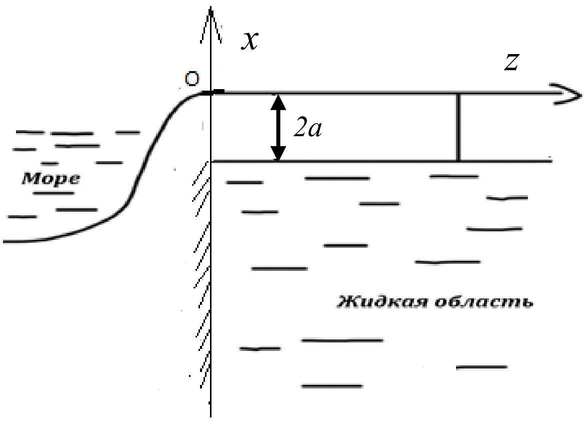

Fig.1

An elastic semi-infinite layer, thickness *2a*, is located on the surface of an ideal compressible liquid region of infinite depth. (see Fig. 1). At the boundary of the liquid region z=0, the existence of an impermeable wall is assumed. It is assumed that the impact is applied on the end region of the layer, and the fluid motion is considered to be potential. Under these conditions, the problem can be formulated as the following initial boundary value problem for a given structure consisting of two different media.

$$\rho\frac{\partial \mathbf{U}}{\partial t^2} = (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{U} + \mu\Delta\mathbf{U},, \quad \mathbf{U} = \mathbf{U}(u, w) \tag{1}$$

$$u = w = 0, \qquad \frac{\partial u}{\partial t} = \frac{\partial w}{\partial t} = 0, \qquad \text{at} t = 0, \tag{2}$$

$$\sigma_{zz} = \sigma_0 f(t), u = 0 \quad \text{at} \quad z = 0 \tag{3}$$

$$\sigma_{xx} = \sigma_{xz} = 0 \qquad \text{at } .x = 0$$

In the following we will assume that $f(t) = H(t)$, where$H(t)$ is the Heaviside function

The following conditions take place at the boundary between the liquid region and the layer:

$$\sigma_{xx} = -\rho_{ж}\frac{\partial\varphi}{\partial t} \qquad \text{at ,} \qquad x = -2a$$

$$\sigma_{xz} = 0 \qquad \text{at} \qquad x = -2a\,, \tag{4}$$

$$\dot{u} = \frac{\partial\varphi}{\partial x}$$

$$\frac{\partial\varphi}{\partial z} = \quad 0 \qquad \text{при} \qquad z = 0 \tag{5}$$

and the equation describing the motion of the liquid region is as follows:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{a_{ж}^2}\frac{\partial^2\varphi}{\partial t^2}\,.$$

Here$\mathbf{U} = \mathbf{U}(u, w)$ is the displacement vector of the elastic layer, λ andμ are Lamé coefficients,$a_{ж}$ is the speed of propagation of sound waves in a liquid medium whose motion is described by a potential function - φ, ρ ,$\rho_{ж}$ are the densities of the layer and the liquid, respectively,t is time.

To solve this system, we will apply a similar method that was developed and used in [1]. Thanks to this method, the system of Lame equations is reduced to the simplest system of inhomogeneous Helmholtz equations, in the right-hand side of which there are boundary impact functions. This method involves the application of twofold integral transformations, along with the method of substitution of the sought functions, which leads to the above-mentioned excellent result. But this fact does not yet relieve us of the difficulty involved in going from the transformations to the originals. And to overcome these difficulties, the most universal method for finding analogous originals of twofold integral transformations is proposed there.

So, using the ready equations of this work for two-dimensional motion, after simple calculations with respect to the equation of motion of the fluid part of this structure, we can obtain the following algebraic system of linear equations to determine the five unknown constants appearing in the composition of the new potential functions:

$$\begin{Bmatrix} C_{01} \\ C_{02} \\ A_{01} \\ A_{02} \\ g_0 \end{Bmatrix}\{D\} = \begin{Bmatrix} 0 \\ \Omega q^2 \\ 0 \\ \Omega q^2 \\ 0 \end{Bmatrix}, \qquad \text{where}\Omega = -\frac{f(p)\sigma_0}{(\lambda+2\mu)v_1^2 q}\,. \tag{6}$$

Here $\{D\} = \{a_{ik}\}e^{2av_1}e^{2av_2}$, $\{D\}$ is a rank 5 matrix

$a_{11} = 2qv_1$,

$a_{21} = \left(1 + \frac{2\mu}{\lambda}\right)v_1^2 - q^2$,

$a_{31} = 2qv_1 e^{-2av_1}$,

$$a_{41} = \left[\left(1+\frac{2\mu}{\lambda}\right)\nu_1^2 - q^2\right]e^{-2a\nu_1} ,$$

$$a_{51} = p\nu_1 e^{-2a\nu_1} ,$$

$$a_{12} = -2q\nu_1 e^{-2a\nu_1} ,$$

$$a_{22} = \left[\left(1+\frac{2\mu}{\lambda}\right)\nu_1^2 - q^2\right]e^{-2a\nu_1} ,$$

$$a_{32} = -2q\nu_1 ,$$

$$a_{42} = \left(\left(1+\frac{2\mu}{\lambda}\right)\nu_1^2 - q^2\right),$$

$$a_{52} = p\nu_1 ,$$

(*)

$$a_{13} = -(q^2+\nu_2^2)\nu_2,$$

$$a_{23} = \frac{2\mu}{\lambda} q\nu_2^2 ,$$

$$a_{33} = (q^2+\nu_2^2)\nu_2 \times e^{-2a\nu_2} ,$$

$$a_{43} = \frac{2\mu}{\lambda} q\nu_2^2 e^{-2a\nu_2} ,$$

$$a_{53} = pq\nu_2 e^{-2a\nu_2} ,$$

$$a_{14} = (q^2+\nu_2^2)\nu_2 e^{-2a\nu_2} ,$$

$$a_{24} = -\frac{2\mu}{\lambda} q\nu_2^2 e^{-2a\nu_2} ,$$

$$a_{34} = (q^2+\nu_2^2)\nu_2 ,$$

$$a_{44} = -\frac{2\mu}{\lambda} q\nu_2^2 ,$$

$$a_{54} = pq\nu_2 ,$$

$$a_{15} = 0 ,$$

$$a_{25} = 0 ,$$

$$a_{35} = 0 ,$$

$$a_{45} = -\rho_{ж}\frac{p}{\lambda}e^{-2a\nu_{ж}} ,$$

$$a_{55} = \nu_{ж} e^{-2a\nu_{ж}} .$$

The following notations are adopted here :

$\nu_k = \sqrt{\left(\frac{p^2}{c_k^2} + q^2\right)}$, $k = 1,2$ and $\nu_{ж} = \sqrt{\left(\frac{p^2}{c_{ж}^2} + q^2\right)}$, $c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$, $c_2 = \sqrt{\frac{\mu}{\rho}}$ are the propagation velocities of longitudinal and transverse waves in the layer material.

Let's modify $\{D\}$ as follows:

$$\{D\} = e^{-2a\nu_{ж}}\{D_0\}.$$

Then only the minus degrees of $e^{-2a\nu_k}$ will appear in the new matrix $\{D_0\}$ $(k = 1,2)$

The transformation of the longitudinal velocity at the free surface of the layer at $x = 0$, according to [1], and at the selected coordinate system (Fig. 1) is expressed by the following formula:

$$\dot{\tilde{w}} = -\frac{\sigma_0}{(\lambda+2\mu)\nu_1^2} + C_{01}q + C_{02}q - \nu_2^2 A_{01} - \nu_2^2 A_{02} \tag{7}$$

Thus, in the variable parameters of transformations the solution is completely defined. But it is expressed through rank 5 determinants, and, therefore, finding the originals by the usual methods is almost impossible. In such cases, it is appropriate to apply the method based in [1]. The principle on which this method is based becomes more relevant when we deal with very complex function-transformations.

First of all, according to the above-mentioned method, in order to determine the behavior of transformations at infinity, when $p \to \infty$, it is necessary to decompose them into convergent series on functions $1/\nu_1^n$. Just in this case, each term of this series will turn out to be a function-transformation for the originals by both Laplace and Fourier.

Let us first take the basic determinant $\{D_0\}$ and modify it as follows: we put 0 where the expression $e^{-2a\nu_k}$, $(k = 1,2)$ appears, since all summands involving these terms generated by the expansion of this determinant approach zero faster in total than any degree of . ν_1^{-n}

In such a case, the expression for $\{D_0\}$ is noticeably simplified - it has only eight terms left of the total sum. Of these 8 terms, let us keep the term which has the greatest degree in infinity $p \to \infty$:

$$|D_0| \approx -\Omega q^2 a_{21} a_{13} a_{34} \cdot (a_{55} a_{42} - a_{45} a_{52}).$$

In the same way we determine the principal terms of other determinants $|D_n| \cdot e^{-2a\nu_{ж}} (n = 1,2,3,4)$, formed from the system (6), according to Cramer's rule for determining constants $C_{01}, C_{02}, A_{01}, A_{02}$:

$$|D_1| \approx -\Omega q^2 \cdot a_{13} \cdot a_{34}(a_{55} \cdot a_{42} - a_{45} \cdot a_{52}) \cdot e^{2a\nu_1} \cdot e^{2a\nu_2},$$

$$|D_2| \approx -\Omega q^2 \cdot a_{13} \cdot a_{34} \cdot a_{55} \cdot e^{2a\nu_2},$$

$$|D_3| \approx -\Omega q^2 \cdot a_{11} \cdot a_{34} \cdot (a_{55} \cdot a_{42} - a_{45} \cdot a_{52}) \cdot e^{2a\nu_1} \cdot e^{2a\nu_2},$$

$$|D_4| \approx -\Omega q^2 \cdot a_{21} \cdot a_{13} \cdot a_{32} \cdot a_{55} \cdot e^{2a\nu_1}.$$

For short time intervals during which the shock load is applied, the behavior of the ratios$\frac{D_k}{D_0}$, k=1,2.3, 4 , at infinity, of course, will be determined mainly from the ratios of these same terms of the highest degree. Then we obtain

$$C_{01} = \frac{\Omega q^2}{a_{21}}, \quad C_{02} = \frac{\Omega q^2 \cdot a_{55} \cdot e^{-2av_1}}{(a_{55} \cdot a_{42} - a_{45} \cdot a_{52})} \quad ,$$

$$A_{01} = \frac{\Omega q^2 \cdot a_{11}}{a_{21} \cdot a_{13}}, A_{02} = -\frac{\Omega q^2 \cdot a_{32} \cdot a_{55} \cdot e^{-2av_2}}{a_{34} \cdot (a_{55} \cdot a_{42} - a_{45} \cdot a_{52})} \quad . \tag{8}$$

Putting the values of these constants in the formulas (7), we obtain the expression of the desired solution in the parameters of the transformations. Using the existing analytical method [3], it is easy to determine the twofold originals of these transformations

Let us separately determine the originals of the first, second and fourth terms in the sum of the right-hand side of (7):

$$1) - \frac{\sigma_0}{(\lambda+2\mu)\cdot v_1^2} \leftrightarrow -\frac{\sigma_0 \cdot c_1}{(\lambda+2\mu)} H\left(t - \frac{z}{c_1}\right); \tag{9}$$

$$2) \; C_{01} q = \frac{\Omega q^3}{a_{21}} = -\frac{\sigma_0 \, q^2}{(\lambda+2\mu) v_1^2} \frac{1}{\left(1+\frac{2\mu}{\lambda}\right) v_1^2 - q^2} = -\frac{\sigma_0 \cdot c_1^2}{(\lambda+2\mu)} \left[\frac{H\left(t - \frac{z}{\sqrt{2} c_2}\right)}{\sqrt{2} \cdot c_2} - \frac{H\left(t - \frac{z}{c_1}\right)}{c_1} \right], \tag{10}$$

$$3) \; -v_2^2 A_{01} = -v_2^2 \cdot \frac{\Omega q^2 \cdot a_{11}}{a_{21} \cdot a_{13}} = \frac{\sigma_0 \cdot v_2^2}{(\lambda+2\mu) \cdot v_1^2} \cdot \frac{2 q^2 v_1}{\left[\left(1+\frac{2\mu}{\lambda}\right) v_1^2 - q^2\right] \cdot \left[(q^2 + v_2^2) v_2\right]} \leftrightarrow$$

$$\leftrightarrow \frac{\sigma_0 \cdot (c_1^2 - 2c_2^2)}{(\lambda+2\mu)} \frac{c_1}{c_2} \frac{H\left(t - \frac{z}{\sqrt{2} c_2}\right)}{\sqrt{2} \cdot c_2}. \tag{11}$$

Note that in the last formula an obvious approximation is used

$$\frac{v_2}{v_1} \approx \frac{c_1}{c_2} \quad \text{at} \quad p \to \infty \quad .$$

As can be seen from these formulas, their sum is identically zero for the ratio$c_1 = 2c_2$, which is valid for the value of Poisson's ratio $\nu = \frac{1}{3}$, and that for most materials this value is exactly this value.

This is an unusually interesting result, confirming the high accuracy of determining the cause of the investigated phenomenon, since the main tone in the formation of longitudinal motions on the surface of the layer is set by these components. The other two components in (7) represent diffraction waves coming from the low side contacting the liquid. They are insignificant, and are not able to change the wave pattern formed from the above constructed solutions. Nevertheless, below we give ready solutions and graphs of distributions for the following values of time, longitudinal velocities at the upper boundary of the layer, corresponding to each wave separately.

First, let us consider the longitudinal diffraction waves reflected from the low side of the layer. From formulas (7), (8) and from the expressions of the components of the principal determinant($*$) one can easily determine the mathematical expression of this wave. It is as follows

$$\dot{\breve{w}}_{прод.} = C_{02}q = \frac{\Omega q^3 \cdot a_{55} \cdot e^{-2a\nu_1}}{(a_{55}\cdot a_{42} - a_{45}\cdot a_{52})} = \frac{\sigma_0 \cdot c_1}{(\lambda+2\mu)} \frac{c_1 \cdot (c_1^2 - 2c_2^2) q^2 e^{-2a\nu_1}}{\nu_1^2 \cdot \left(p^2\left(1+\frac{\rho_ж}{\rho}\frac{a_ж}{c_1}\right) + 2\cdot c_2^2 \cdot q^2\right)} \quad . \qquad (12)$$

Let $\frac{\rho_ж}{\rho} \cdot \frac{a_ж}{c_1} = \alpha$, then the inverse Laplace transform of formula (12), can be represented in the following form:

$$\frac{\dot{\breve{w}}_{прод}}{\frac{\sigma_0 \cdot c_1}{(\lambda+2\mu)}} = \frac{0.5}{(1+\alpha)} \int_2^t \left(\int_2^{\tau_1} \left(J_0(q(\tau_1 - \tau))\right) \left(J_0(q\sqrt{\tau^2-4})\right) d\tau \right) q \sin\left(0.5\sqrt{2\frac{1-\alpha}{1+\alpha}}\, q(t-\tau_1) \right) d\tau_1 .$$

Let us only note that the Efros theorems, convolution theorems and tables given in [3] were used to obtain this formula. Now, having made the inverse Fourier-cosine transformation, we can give the plots of the longitudinal velocity on the upper surface of the layer.

Calculated for $\nu = \frac{1}{3}$ and for the following values of time $t = \frac{a}{c_1}n$, and for the dimensionless longitudinal velocity $\dot{\breve{w}}_{прод} = \frac{\dot{\breve{w}}_{прод}}{\frac{\sigma_0 \cdot c_1}{(\lambda+2\mu)}}$.

The first diffraction wave will cross this surface at $n = 2$,

(a) (b) (c)

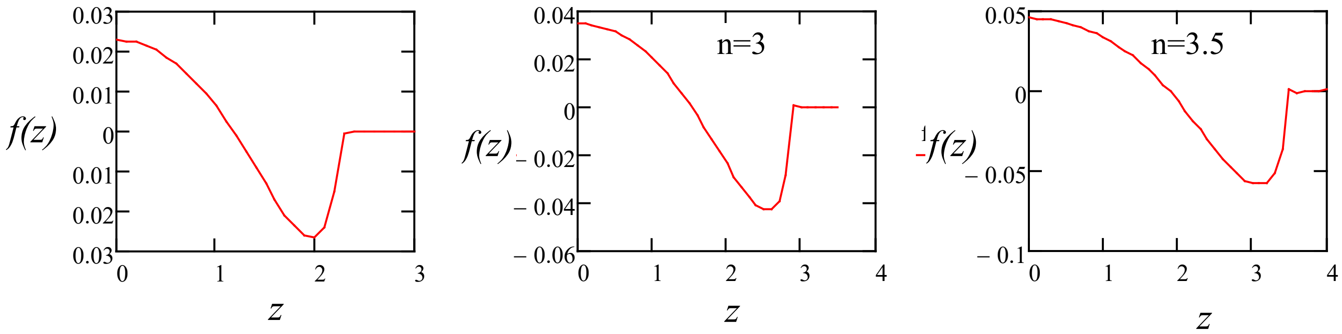

Fig. 2

Now let's calculate the corresponding dimensionless component of the diffraction transverse wave that will cross the upper layer at the moment $n = 4$. Then we

$$\dot{\breve{w}}_{попер} = -\nu_2^2 A_{02} = \nu_2^2 \frac{\Omega q^2 a_{32} a_{55} e^{-2a\nu_2}}{a_{34}(a_{55}a_{42} - a_{45}a_{52})} =$$

$$= \frac{\sigma_0 c_1}{(\lambda+2\mu)} \frac{(c_1^2 - 2c_2^2)^2 q^2 e^{-2a\nu_1}}{c_2(p^2 + 2c_2^2)\left(p^2\left(1 + \frac{\rho_ж}{\rho}\cdot\frac{a_ж}{c_1}\right) + 2c_2^2 q^2\right)} ,$$

$$\dot{\breve{w}}_{попер} = -\frac{1}{2\cdot\alpha(1+\alpha)} \int_4^t \left(\left(\cos\left(\frac{q}{2}(\tau-4)\right)\right) - \left(\cos\left(\frac{q}{2}\frac{1}{\sqrt{1+\alpha}}(\tau-4)\right)\right) \right) \cdot J_0\left(\frac{q}{2}\sqrt{t^2-\tau^2}\right) d\tau \ ,$$

(а) (б) (в)

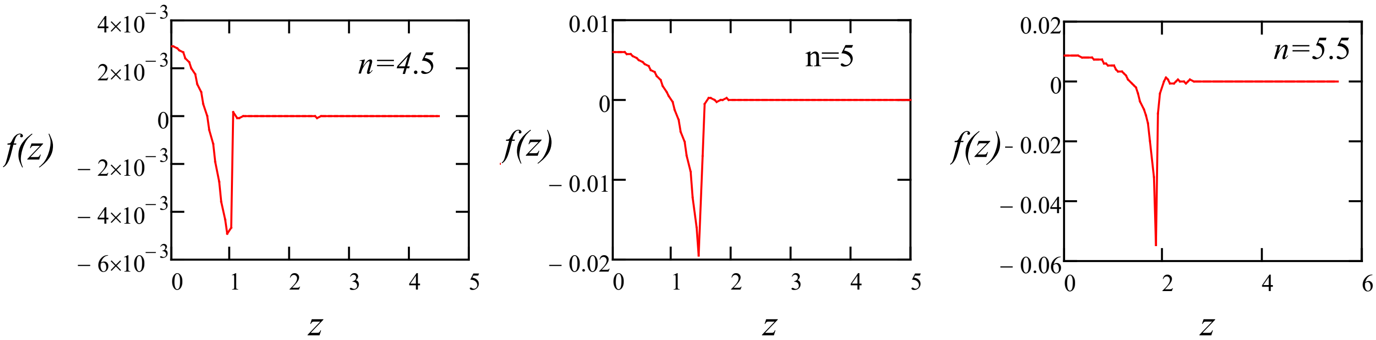

Fig.3

As can be seen from these graphs, their values are quite small; they are less than one hundredth of the value of each wave (9)-(11), which is formed at the upper boundary of the layer. Naturally, they cannot cause perceptible oscillations. One more interesting fact should be noted: in these components, the motion along the impact direction quickly changes its sign to the opposite one, as the graphs show.

3. CONCLUSION

The obtained result allows us to formulate the following interesting conclusion: the top layer of objects on the liquid surface almost does not experience longitudinal impact loading.

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