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ORDER, DISORDER, AND PHASE TRANSITION IN CONDENSED MEDIA

STATIONARY AND NON STATIONARY CURRENT IN FINITE KITAEV CHAINS

Yu. M. Bilinskii[a], P. I. Arseev[a,*], N. S. Maslova[b]

[a]Lebedev Physical Institute, Russian Academy of Sciences, Moscow, Russia
[b]Lomonosov Moscow State University, Moscow, Russia
*e-mail: ars@lpi.ru

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Abstract. We investigate the role of gap states in processes of charge transmission along finite superconducting Kitaev chain. We use the formalism of non-stationary Green's functions, which contain full information about the non-equilibrium and non-stationary properties of the system. We discuss tunneling current and non-stationary transport properties of a finite Kitaev chain in the subgap regime. Under the assumption that the finite Kitaev chain is connected at each edge to its own external lead (normal reservoir) we obtain time-dependent behavior of the tunneling current after the sudden change of bias voltage in one of the leads. Obtained results show how quickly the "Majorana mode" at one edge of the chain responds after external perturbation acts on the "Majorana mode" at the other edge. Presented calculations are completely analytical and straightforward, in contrast with many other methods.

Keywords: *Kitaev superconducting chain, tunneling in superconductors*

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1. INTRODUCTION

In recent decades, significant attention has been given to systems exhibiting "topologically nontrivial" properties. However, for practical applications, it is essential to assess the specific physical characteristics of such systems in addition to their mathematical interpretation of the ground state properties. One of the simplest models that demonstrates properties allowing for topological interpretation is the atomic chain with ppp-wave superconductivity of spinless particles, proposed by A. Kitaev [1]. The primary interest in this model in subsequent years was driven by the nontrivial topological interpretation of its ground-state properties. It was shown that, due to "topological reasons", quantum states localized at the chain edges appear within the superconducting gap. These states, often referred to as "Majorana modes", are commonly associated with the existence of quasiparticles [2] that bear resemblance to Majorana fermions [3].

Possible experimental realizations of this model are typically based on the proximity effect in semiconductor nanowires with strong spin-orbit interaction, placed on a superconducting substrate [4, 5, 6]. The latest experimental advances and discussions on the challenges encountered can be found in the review [7].

It is widely believed that further progress in this field may involve models with an effective Josephson action, accounting for Coulomb blockade-type effects [8, 9, 10, 11, 12, 13]. There is hope that long-range Coulomb interactions could facilitate signal transmission in finite Kitaev chains using "Majorana states". However, recharging effects inevitably involve charge transfer processes, so we must ensure that we accurately describe tunneling transport and charge transfer effects first in the simplest tunneling setup. Theoretical results can then be compared with tunneling experiments under various conditions [14, 15].

Some theoretical studies suggest that "Majorana states" could be utilized as an error-protected method for storing and transmitting information in quantum technology [16, 17]. However, if a state is protected from arbitrary changes due to external noise, the same protection may make intentional changes to the system's state equally difficult, potentially rendering the system impractical for real-world applications. One possible way to study how well a system responds to a signal is to investigate its nonstationary transport properties.

In [18], nonstationary effects related to tunnel barrier transparency modulation were considered in a quasiclassical approach. That work analyzed a three-terminal system, where one of the contacts was effectively used to fix the chemical potential of the superconductor. In this study, we consider a two-terminal geometry, where the superconductor is connected only to two external contacts. To explore the role of localized states in nonstationary transport properties, we employ the formalism of nonstationary Green's functions for electrons.

Below, we will demonstrate that this approach enables us to derive explicit analytical expressions for both the tunnel current and nonstationary charge transport, in contrast to more complex methods based on density matrix equations, as discussed, for example, in [19]. Furthermore, this approach allows us to compare quasiclassical calculations with microscopic methods and establish a connection between the parameters used in these different approaches.

The exact electronic Green's functions for the infinite Kitaev chain in equilibrium can be obtained analytically [20]. These functions can be used to derive the nonstationary Green's functions for a finite chain, allowing us to understand how the system evolves over time when subjected to an external perturbation. The key idea in our approach is to treat the finite Kitaev chain as a cut segment of an infinite chain or as a chain with strong defects (for a single-cut chain, see, e.g., [21]). This trick enables us to use the Green's functions of the infinite chain to study all single-particle states in the system. Our calculations do not require any special interpretation of singularities in the single-particle Green's function as specific "states". It is important to note that the poles of the single-particle Green's function, which appear inside the superconducting gap in this model, can hardly be interpreted as single-particle excitations. True Majorana particles, as discussed in the pioneering works [3], are well-defined particles (quasiparticles) with the usual algebra of creation and annihilation operators. In any physical problem, such real particles contribute to the single-particle Green's function with a residue equal to one. It is well known that bound states localized around defects, such as paramagnetic impurities [22] or resonance impurities [23] with energies lying inside the superconducting gap, frequently appear in conventional superconductors. These states are genuine single-particle states. In the present case, we observe that the appearance of poles in the electronic Green's function within the gap, with residues smaller than one, is more likely an artifact of the model, which has a degenerate (in the highly symmetric case) ground state, rather than the emergence of new quasiparticles.

2. PROPERTIES OF AN ISOLATED KITAEV CHAIN

In this section, we briefly reproduce some results related to the spectral properties of a finite Kitaev chain, using the Green's function formalism, which we will employ in later sections.

We start with the free ideal Kitaev chain, which is completely isolated from any external systems.

The model Hamiltonian of such a system can be written as

$$\widehat{\widehat{H}} = -\mu\sum_{n=1}^{N}\psi_n\psi_n - t\sum_{n=1}^{N-1}\left(\psi_n\psi_{n+1} + \psi_{n+1}\psi_n\right) + \sum_{n=1}^{N-1}\left(\Delta\psi_n\psi_{n+1} + \Delta^{*}\psi_{n+1}\psi_n\right). \quad (1)$$

Here, ψ_n and ψ_n are the creation and annihilation operators for a particle at site n, μ is the chemical potential, t is the hopping parameter between neighboring sites, Δ is the superconducting order parameter, which in this study we consider as a fixed parameter, N is the total number of sites in the lattice.

To obtain exact solutions for the Green's functions of the Hamiltonian (1), it is convenient to use the Green's functions of an infinite Kitaev chain. Indeed, the behavior of a finite chain can be modeled by considering an infinite chain with infinitely strong point defects $U\to+\infty$ added at sites 0 and N+1 (see Fig. 1). As a result, the particles located between these two sites will be completely isolated from the outer parts of the chain, and the Green's functions will be identical to those of a finite Kitaev chain of length N, as long as the node indices lie between 0 and N+1.

Thus, the behavior of the system is described by the following Hamiltonian, corresponding to the system shown in the figure:

$$\widehat{H} = \widehat{H}_0 + \widehat{V}, \tag{2}$$

where

$$\widehat{H}_0 = -\mu\sum_n \psi_n\psi_n - t\sum_n\left(\psi_n\psi_{n+1} + \psi_{n+1}\psi_n\right) + \sum_n\left(\Delta\psi_n\psi_{n+1} + \Delta^*\psi_{n+1}\psi_n\right),$$

$$\widehat{V} = U\left(\psi_0\psi_0 + \psi_{N+1}\psi_{N+1}\right).$$

This Hamiltonian (2) is identical to Hamiltonian (1) when $U\to\infty$. To determine the physical properties of the chain, we use the formalism of normal and anomalous Green's functions, denoted as $G_{nm}(t,t')$, $F_{nm}(t,t')$, respectively. In this work, we use the following definitions of Green's functions:

$$\Gamma^R_{nm}(t,t') = \begin{pmatrix} G^R_{nm}(t,t') & F^R_{nm}(t,t') \\ F^{R+}_{nm}(t,t') & G^{R+}_{nm}(t,t') \end{pmatrix} = -i\left\langle \begin{pmatrix} \left\{\psi_n(t),\psi^+_m(t')\right\} & \left\{\psi_n(t),\psi_m(t')\right\} \\ \left\{\psi^+_n(t),\psi^+_m(t')\right\} & \left\{\psi^+_n(t),\psi_m(t')\right\} \end{pmatrix} \right\rangle \theta(t-t'), \tag{3}$$

$$\Gamma^A_{nm}(t,t') = \begin{pmatrix} G^A_{nm}(t,t') & F^A_{nm}(t,t') \\ F^{A+}_{nm}(t,t') & G^{A+}_{nm}(t,t') \end{pmatrix} = i\left\langle \begin{pmatrix} \left\{\psi_n(t),\psi^+_m(t')\right\} & \left\{\psi_n(t),\psi_m(t')\right\} \\ \left\{\psi^+_n(t),\psi^+_m(t')\right\} & \left\{\psi^+_n(t),\psi_m(t')\right\} \end{pmatrix} \right\rangle \theta(t'-t), \tag{4}$$

$$\Gamma^<_{nm}(t,t') = \begin{pmatrix} G^<_{nm}(t,t') & F^<_{nm}(t,t') \\ F^{<+}_{nm}(t,t') & G^{<+}_{nm}(t,t') \end{pmatrix} = -i\left\langle \begin{pmatrix} \psi^+_m(t')\psi_n(t) & \psi_m(t')\psi_n(t) \\ \psi^+_m(t')\psi^+_n(t) & \psi_m(t')\psi^+_n(t) \end{pmatrix} \right\rangle, \tag{5}$$

where $\left\{\hat{a},\hat{b}\right\} = \hat{a}\hat{b} + \hat{b}\hat{a}$, $<\hat{a}> = \mathrm{Tr}(\hat{\rho}\hat{a})$. The indices R and A denote the retarded and advanced Green's functions, respectively.

Using Dyson's equation for Hamiltonian (2), we can express the retarded Green's functions $\Gamma^R_{nm}(t,t')$ of the finite chain in terms of the Green's functions $\Gamma^{0R}_{nm}(t,t')$ of the infinite Kitaev chain. In the Appendix, it is shown that the functions $\Gamma^R_{nm}(\omega)$ have poles at points $\omega = \pm\omega_0$, where

$$\omega_0 = \frac{|\Delta|(4t^2-\mu^2)}{it\sqrt{4(t^2-|\Delta|^2)-\mu^2}}\left(\chi_+^{N+1} - \chi_-^{N+1}\right). \tag{6}$$

Here

$$\chi_\pm = \frac{-\mu \pm i\sqrt{4t^2-\left(\mu^2+4\,|\,\Delta\,|^2\right)}}{2(t+|\Delta|)}. \tag{7}$$

As $|\chi_\pm| < 1$ (see 52), expression (6) is written for the case $|\chi_\pm|^N \ll 1$. For sufficiently large N, the parameter ω_0 is small compared to other system parameters and decays exponentially as the N chain length increases.

For the cases $|\Delta| \ll t$ and $|\Delta| < t$, $|\Delta| \to t$, the "exponential smallness" of expression (6) in N can be explicitly demonstrated:

$$\omega_0 = \begin{cases} 4|\Delta|e^{-N(|\Delta|/t)}, & |\Delta| << t, \\ 2te^{-N\ln\left(\sqrt{2t/(t-|\Delta|)}\right)}, & (t-|\Delta|) \ll t. \end{cases} \tag{8}$$

The exponential decay of ω_0 with increasing chain length is explained by the exponentially weak overlap of the two bound states at opposite edges of the chain. Using (8), we can estimate the localization length of the bound states as:

$$l_{loc} \simeq \begin{cases} a(t/|\Delta|), & |\Delta| << t, \\ a/\ln\left(\sqrt{2t/(t-|\Delta|)}\right), & (t-|\Delta|) \ll t, \end{cases} \tag{9}$$

where a is the lattice constant.

Such an exponential dependence has been observed in tunneling experiments using Coulomb blockade methods, as described in [15].

In the limit $N\to\infty$, the states near each edge begin to behave as if the chain were semi-infinite. In this case, the two poles with residues equal to 1/2 together correspond to a single Fermi excitation, which is split between the two edges of the chain. Thus, the residue in terms of Bogoliubov excitations is equal to 1, as it should be. However, when observing only one end of the chain, we "see" only half of this excitation. This Fermi excitation is very specific because it is

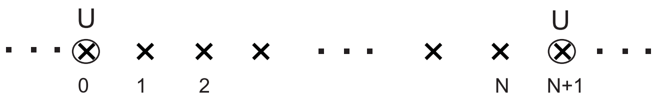

Fig. 1: Infinite Kitaev chain with two defects

the excitation that connects two degenerate ground states with different parity (i.e., a different number of electrons), but with the same energy.

This statement can be easily illustrated with a simple example of a two-site chain. The Hamiltonian (1) for two sites can be diagonalized using the Bogoliubov transformation. In terms of Bogoliubov operators, the Hamiltonian takes the form

$$\widehat{H}_2 = E_0 + \varepsilon_1 c_1^+ c_1 + \varepsilon_2 c_2^+ c_2, \qquad (10)$$

where

$$c_{1,2} = u\frac{(\psi_1 + \psi_2)}{\sqrt{2}} \pm v\frac{(\psi_1^+ - \psi_2^+)}{\sqrt{2}},$$

$$v^2, u^2 = \frac{1}{2}\left(1 \pm \frac{\mu}{t}\right),$$

$$\varepsilon_{1,2} = t\left(1 \mp \frac{\Delta}{\sqrt{t^2 - \mu^2}}\right).$$

For $\Delta = \sqrt{t^2 - \mu^2}$ (which in the case $\mu = 0$ gives $\Delta = t$), we obtain $\varepsilon_1 = 0$ (the solution 53 for the case $N = 2$, $\omega = 0$). Then, $\mid \Phi_0 >= \frac{1}{\sqrt{2}}(\psi_1^+ - \psi_2^+) \mid 0 >$ corresponds to the ground state and satisfies $c_{1,2} \mid \Phi_0 >= 0$. At the same time, the state $\mid \Phi_1 >= c_1 \mid \Phi_0 >= (v + u\psi_1^+\psi_2^+) \mid 0 >$ also has zero energy, which means that the ground state is degenerate. For the matrix elements between these ground states, we have

$$< \Phi_0 \mid \psi_1 \mid \Phi_1 >= u / \sqrt{2}, < \Phi_1 \mid \psi_1 \mid \Phi_0 >= v / \sqrt{2}.$$

This means that in the single-particle function G_{11} at $\omega = 0$, a pole appears with a residue equal to 1/2.

3. TUNNELING CURRENT

We first consider the stationary tunneling properties of the Kitaev chain. To do this, we assume that the chain is connected at sites 1 and N to two external reservoirs with a large number of degrees of freedom, labeled by indices l and r, respectively.

The total Hamiltonian can then be written as

$$\begin{aligned}\widehat{\widehat{H}} = \quad & \widehat{H} + \sum_p \tau_p^l \left(h_p^{l+}\psi_1 + \psi_1^+ h_p^l\right) + \\ & + \sum_p \tau_p^r \left(h_p^{r+}\psi_N + \psi_N^+ h_p^r\right) + \\ & + \sum_p E_p^l h_p^{l+} h_p^l + \sum_p E_p^r h_p^{r+} h_p^r.\end{aligned} \qquad (11)$$

The current flowing into the chain through site 1 is given by the standard expression ([24]):

$$I_l(t) = i\sum_p \tau_p^l < h_p^{l+}\psi_1 - \psi_1^+ h_p^l > . \qquad (12)$$

Using the nonstationary diagrammatic technique, this expression can be rewritten as

$$I_l(t) = -\sum_p \tau_p^l \left(\tilde{G}^<_{lp,1}(t,t) - \tilde{G}^<_{1,lp}(t,t)\right), \qquad (13)$$

where

$$\begin{aligned}\tilde{G}^<_{lp,1}(t,t) = \quad & \int dt_1 g^<_{lp}(t,t_1)\tau_p^l \tilde{G}^A_{1,1}(t_1,t) + \\ & + \int dt_1 g^R_{lp}(t,t_1)\tau_p^l \tilde{G}^<_{1,1}(t_1,t), \\ \tilde{G}^<_{1,lp}(t,t) = \quad & \int dt_1 \tilde{G}^<_{1,1}(t,t_1)\tau_p^l g^A_{lp}(t_1,t) + \\ & + \int dt_1 \tilde{G}^R_{1,1}(t,t_1)\tau_p^l g^<_{lp}(t_1,t).\end{aligned}$$

The parameter p corresponds to the density of states inside both reservoirs, $g_{\alpha p}(\omega)$ is the Green's function of reservoir α when it is disconnected from the chain, where α takes values l and r, $G^R_{n,m}(t, t1)$ are the exact are the retarded and advanced Green's functions of the chain, accounting for tunneling transitions into the reservoirs.

Crucially, the tunneling Hamiltonian (11) and the tunneling current (12) are expressed in terms of real electron operators, and they directly provide the actual electric current in the system. It should be noted that attempts to use effective Hamiltonians in terms of Majorana quasiparticle operators often lead, in our opinion, to questionable results, as handling Majorana operators requires great caution and precision. Due to the Clifford algebra commutation relations, there is no Wick's theorem directly applicable to Majorana operators, and pair correlators do not have the meaning of Green's functions, which form the basis of conventional diagrammatic techniques. In the calculations presented in this paper, we do not encounter any difficulties that we would have faced if we had worked with Majorana operators. For the problem of a finite Kitaev chain of arbitrary length, inserted between two leads and described by Hamiltonian (11), we have exactly computed the electronic current

(12). It is not surprising that some discrepancies may arise between our results and those of [26, 27, 25] and other authors, as the latter were obtained using a number of approximations in the Majorana operator representation.

In what follows, we assume, as usual, that due to the large number of particles and degrees of freedom in each reservoir, the particle distribution function does not significantly change throughout the experiment, and thus each reservoir remains practically in equilibrium. However, the system as a whole is not in equilibrium, although in this section, we consider it stationary, meaning the current does not change over time. Thus, Equation (13) can be rewritten using frequency-dependent Green's functions as follows:

$$I_l = -\sum_p \tau_p^l \int \frac{d\omega}{2\pi}\left(\tilde{G}^{<}_{lp,1}(\omega) - \tilde{G}^{<}_{1,lp}(\omega)\right). \quad (14)$$

where

$$\begin{aligned} \tilde{G}^{<}_{lp,1}(\omega) &= g^{<}_{lp}(\omega)\tau_p^l \tilde{G}^{A}_{1,1}(\omega) + g^{R}_{lp}(\omega)\tau_p^l \tilde{G}^{<}_{1,1}(\omega), \\ \tilde{G}^{<}_{1,lp}(t,t) &= \tilde{G}^{<}_{1,1}(\omega)\tau_p^l g^{A}_{lp}(\omega) + \tilde{G}^{R}_{1,1}(\omega)\tau_p^l g^{<}_{lp}(\omega). \end{aligned} \quad (15)$$

We can simplify this expression by introducing the irreducible part

$$\Sigma_\alpha^{R(A,<)}(\omega) = \sum_p \left(\tau_p^\alpha\right)^2 g_{\alpha p}^{R(A,<)}(\omega). \quad (16)$$

Then, we can use the identity

$$\Sigma_\alpha^{<}(\omega) = n_\alpha(\omega)\left(\Sigma_\alpha^A(\omega) - \Sigma_\alpha^R(\omega)\right),$$

where $n_\alpha(\omega)$ are the Fermi-Dirac distribution functions for the l and r reservoirs.

Thus, Equation (14) can be rewritten as:

$$\begin{aligned} \hat{I}_l = & -\int \frac{d\omega}{2\pi}\left(\Sigma_l^A(\omega) - \Sigma_l^R(\omega)\right)\times \\ & \times\left(n_l(\omega)\left(\tilde{\Gamma}^A_{1,1}(\omega) - \tilde{\Gamma}^R_{1,1}(\omega)\right) - \tilde{\Gamma}^{<}_{1,1}(\omega)\right). \end{aligned} \quad (17)$$

Here, the current I_l is determined by the upper-left element of the matrix $\hat{I}_l$ ($\hat{I}_l^{11}$).

An expression of this type in terms of nonequilibrium Green's functions was first derived in [24] and later applied in [28]. At first glance, this expression appears asymmetric with respect to the left and right contacts. However, in the stationary case, a properly calculated current (17) can always be rewritten in an explicitly symmetric form.

In our case, Equation (17) can be further simplified using the relations

$$\begin{aligned} \tilde{\Gamma}^{<}_{1,1}(\omega) &= \tilde{\Gamma}^{R}_{1,1}(\omega)\Sigma_l^{<}(\omega)\tilde{\Gamma}^{A}_{1,1}(\omega) + \\ &+ \tilde{\Gamma}^{R}_{1,N}(\omega)\Sigma_r^{<}(\omega)\tilde{\Gamma}^{A}_{N,1}(\omega), \end{aligned} \quad (18)$$

$$\begin{aligned} \tilde{\Gamma}^{\delta}_{1,1}(\omega) &= \tilde{\Gamma}^{R}_{1,1}(\omega)\Sigma_l^{\delta}(\omega)\tilde{\Gamma}^{A}_{1,1}(\omega) + \\ &+ \tilde{\Gamma}^{R}_{1,N}(\omega)\Sigma_r^{\delta}(\omega)\tilde{\Gamma}^{A}_{N,1}(\omega). \end{aligned} \quad (19)$$

where

$$\tilde{\Gamma}^{\delta}_{1,1}(\omega) = \tilde{\Gamma}^{A}_{1,1}(\omega) - \tilde{\Gamma}^{R}_{1,1}(\omega),$$

$$\Sigma_\alpha^{\delta}(\omega) = \Sigma_\alpha^{A}(\omega) - \Sigma_\alpha^{R}(\omega).$$

Using the wide-band approximation for the reservoirs, we assume that for the considered values of ω, the condition $\Sigma^A_{l(r)}(\omega) \approx i\gamma_{l(r)}$, $\Sigma^R_{l(r)}(\omega) \approx -i\gamma_{l(r)}$, holds, where $\gamma_{l(r)} = \pi\nu^{l(r)}(\tau_p^{l(r)})^2$ and $\nu^{l(r)}$ are the densities of states in the rese rvoirs $l(r)$.

Direct substitution gives:

$$\hat{I}_l = 4\gamma_l\gamma_r \int \frac{d\omega}{2\pi}\tilde{\Gamma}^R_{1,N}(\omega)\tilde{\Gamma}^A_{N,1}(\omega)\left(n_l(\omega) - n_r(\omega)\right). \quad (20)$$

A formula of this type was derived in [24]. It should be noted that the obtained equation for the current through the system is symmetric with respect to its two edges. Naturally, this implies that, in the stationary case, the current flowing into the system equals the current flowing out of it. The conservation of total current cannot be violated in any system and does not require additional conditions, such as equal tunneling rates or symmetrically applied voltages at different edges. Thus, the appearance of asymmetric expressions for stationary tunneling current, as obtained in some works on Kitaev chain-type systems (e.g., [29]), signals the need to verify the applied approximations. This statement remains valid even for interacting systems, but deriving an explicitly symmetric expression in such cases is more challenging. Examples of such calculations for systems with electron-phonon interactions can be found, for example, in [30, 31]. We emphasize that Equation (20) is exact and explicitly symmetric for the left and right contacts.

Since we aim to study the low-energy bound state corresponding to the "Majorana mode", we consider the case where the applied voltage is smaller than the superconducting gap. In this case, we exclude the influence of quasiparticle states from the continuous spectrum. To express $\tilde{\Gamma}^R_{1,N}(\omega)$ through the Green's functions of the isolated chain $\Gamma^R_{n,m}(\omega)$, we use Dyson's equation:

$$\tilde{\Gamma}^R_{n,m}(\omega) = \Gamma^R_{n,m}(\omega) + \Gamma^R_{n,1}(\omega)\Sigma^R_l(\omega)\tilde{\Gamma}^R_{1,m}(\omega) + \Gamma^R_{n,N}(\omega)\Sigma^R_r(\omega)\tilde{\Gamma}^R_{N,m}(\omega). \tag{21}$$

Simple algebraic transformations yield:

$$\tilde{\Gamma}^R_{1,N}(\omega) = \Big[\hat{I} + \gamma_l\gamma_r\Big(\hat{I} + i\gamma_l\Gamma^R_{1,1}(\omega)\Big)\Gamma^R_{1,N}(\omega)\times \times\Big(\hat{I} + i\gamma_r\Gamma^R_{N,N}(\omega)\Big)\Gamma^R_{N,1}(\omega)\Big]^{-1}\times \times\Big(\hat{I} + i\gamma_l\Gamma^R_{1,1}(\omega)\Big)\Gamma^R_{1,N}(\omega)\Big(\hat{I} + i\gamma_r\Gamma^R_{N,N}(\omega)\Big).$$

where $\hat{I}$ is the identity matrix. The explicit form of the Green's functions $\Gamma^R_{n,m}(\omega)$ for $|\omega| \ll |\Delta|, t$ is derived in the Appendix. A simpler form can be obtained for $\Delta^2/(t\gamma) \gg 1$. Retaining the leading terms in (55) for this parameter, we get:

$$\tilde{\Gamma}^R_{1,N}(\omega) = \frac{C\omega_0}{\omega^2 - \omega_0^2 + 2i(\gamma_l + \gamma_r)C\omega - 4\gamma_l\gamma_r C^2}\begin{pmatrix} 1 & -\frac{\Delta}{|\Delta|} \\ \frac{\Delta^*}{|\Delta|} & -1 \end{pmatrix}, \tag{22}$$

$$C \equiv -\frac{|\Delta|(4t^2 - \mu^2)}{2t(4(t^2 - |\Delta|^2) - \mu^2)}(\chi_+ - \chi_-)^2 = \frac{|\Delta|(4t^2 - \mu^2)}{2t(t + |\Delta|)^2}. \tag{23}$$

Substituting this result into (20), we obtain:

$$I_l = \int \frac{d\omega}{2\pi}\frac{8\gamma_l\gamma_r C^2\omega_0^2}{\left|\omega^2 - \omega_0^2 + 2i(\gamma_l + \gamma_r)C\omega - 4\gamma_l\gamma_r C^2\right|^2}(n_l(\omega) - n_r(\omega)). \tag{24}$$

These and further calculations are performed for the following parameter hierarchy: $t > \Delta > \gamma_{l,r}$. For the case $\gamma_{l,r} > \Delta$, we cannot exclude the influence of the continuous part of the spectrum on conductivity, and information about low-energy resonances is lost, so this case is not considered here.

We see that the magnitude of the current (25) is directly proportional to ω_0^2, meaning that the current decreases exponentially with increasing chain length. Moreover, if $\omega_0 = 0$, which is typically associated with Majorana particles, then no current flows through the system at all. Note that Equation (25) is symmetric with respect to the contact parameters l and r, as expected. A similar expression for the normal component of the current was obtained in the quasiclassical approach in [18], where it was also noted that the zero-bias peak in tunneling conductance is unlikely to be observed for a realistic ratio between ω_0 and $\gamma_{l,r}$.

The tunneling conductance peak associated with Majorana states was also studied in [32]. That study considered a single NS contact, where it was assumed that the chemical potential of the superconductor was somehow fixed. The problem was solved using the effective transmission coefficient method for quasiparticles, which, in the presence of a bound state, always leads to formulas of type (25). However, the peak amplitude for the two different systems – a single NS contact and a superconductor between two normal contacts – cannot be directly compared due to the problem of fixing the superconducting chemical potential. It is worth noting that results similar to those in [32] for the current in an NS contact, considering Majorana states, can also be obtained using the methods from [33].

If in Equation (25) the applied voltage is greater than the width of localized states, but less than the superconducting gap, meaning $n_l(\omega) - n_r(\omega) = 1$ for $|\omega| \lesssim \gamma_l, \gamma_r$ then we obtain a simple final expression for the tunneling current associated with Majorana modes:

$$I_l = \frac{2\gamma_l\gamma_r C\omega_0^2}{\gamma_l + \gamma_r}\frac{1}{4\gamma_l\gamma_r C^2 + \omega_0^2}. \tag{25}$$

Thus, the magnitude of the current is always determined by the smallest transfer rate present in the system (the weakest link); in our case, these rates are defined by the parameters $\omega_0^2/(\gamma_l+\gamma_r), \gamma_l, \gamma_r$. If $\omega_0^2 \geq C^2\gamma_l\gamma_r$, then the general equation (25) leads to a current proportional to $\gamma_l\gamma_r/(\gamma_l+\gamma_r)$, which is the usual expression for tunneling through an intermediate state. For the considered system, the physically reasonable relation is $\omega_0 \ll \gamma_l, \gamma_r$. Using this, we obtain:

$$I_l = \frac{\omega_0^2}{2C(\gamma_l+\gamma_r)}. \tag{26}$$

If we use Equations (8) and (23), this formula gives:

$$I = \begin{cases} \dfrac{2\Delta t}{(\gamma_l+\gamma_r)} e^{-2N(\Delta/t)}, \Delta \ll t, \\ \dfrac{2t^2}{(\gamma_l+\gamma_r)} e^{-N\ln(2t/(t-\Delta))}, (t-\Delta) \ll t. \end{cases} \tag{27}$$

For arbitrary parameters $\mu < \Delta < t$, the current is always small in long chains. In the case of $\omega_0 = 0$, which is considered the most favorable scenario for observing unusual topological properties, we will not be able to observe a zero-bias peak in the tunneling conductance at all. This observation holds true for the model considered in this paper, where the chain has two contacts at its edges. In any experiment measuring stationary current, at least two external leads are required, connected to the "left" and "right" edges of the system. Of course, there are more complex multi-contact configurations, but their analysis is beyond the scope of this paper. Real hybrid semiconductor-superconductor structures, which simulate the Kitaev chain, require the consideration of a model Hamiltonian that describes a semiconductor nanowire with strong spin-orbit interaction, which is coupled due to the proximity effect to an underlying superconducting layer. In this case, the superconductor can be considered as a reservoir with a fixed chemical potential, and the "second contact" as the interface between the semiconductor and the superconductor. Alternatively, instead of edge connections, we could also consider a Kitaev chain lying on a substrate, where all chain atoms are weakly coupled to corresponding substrate atoms. In this case, the "second contact" with the reservoir becomes spatially distributed. This problem can be solved, but it is different from the one considered in this paper. Nevertheless, if the overlap of the localized state with the reservoir states is small, then the zero-bias current peak should also be small. Its magnitude in the case of a spatially distributed "second contact" will not decay exponentially with chain length, but will still be much smaller than what would be expected from naive formulas. This may be a possible reason why the zero-bias peak is often poorly observed in conventional tunneling experiments [14].

We want to emphasize that naively applied general formulas for the tunneling current between two contacts often lead to misleading results when used for low-dimensional systems, such as the Kitaev chain [21], due to the possible appearance of localized states in the contact region.

The lowest-order response (second order in the tunneling coupling) of quantum mechanical perturbation theory describes the current only at the initial moment after the tunneling connection is "switched on". However, the stationary tunneling current can only be calculated using the full system of kinetic equations, or equivalently, the full system of equations for the nonstationary Keldysh-Green's functions. Only in simple systems with a continuous spectrum, where rapid electron relaxation to equilibrium is implicitly assumed, is the formula based on the equilibrium local density of states of the leads guaranteed to be valid.

To clarify this idea, let us consider a tunneling contact with a localized state at the edge of one of the leads. This localized state creates a sharp peak in the local density of states and contributes to the simplest formula for tunneling current. Suppose this state is empty at the initial moment (i.e., lies above the Fermi level). Then, immediately after applying a positive bias voltage to the other lead, the current begins to flow into this empty localized state. However, after some relaxation time, determined by the tunneling rate, this state becomes occupied, and from that point onward, no more electrons can tunnel into it. The stationary tunneling current then vanishes, even though the simplest formula still predicts a "zero-bias peak" in the tunneling conductance. For this localized state to contribute to the stationary current, some inelastic processes must be included, which are responsible for removing (or adding) electrons from this localized state. For a finite system, it is also possible that this localized state at one edge has some overlap with the second contact. (This corresponds to our case and the case of a distributed "grounded contact" in a real system.)

In the usual formula for tunneling current, which relies on the local density of states of the contacts, it is implicitly assumed that at any moment, the chemical potentials of all contact states are fixed. To maintain a constant chemical potential, the system must be connected to some reservoir via a contact that allows for particle exchange. Thus, when we say that we fix the chemical potential of localized states, we are implicitly including some inelastic relaxation processes or a direct connection to a reservoir for these states.

4. NONSTATIONARY CURRENT

Now, let us attempt to answer the question of what the typical time scales are for current or charge transfer from one edge of the chain to the other. We will pose the problem differently than in [18], where the effect of periodic modulation of tunnel barrier transparency on zero-bias tunneling conductance was studied. An interesting result in that study was the discovery and analysis of resonance between the external driving frequency and the splitting of Majorana states ω_0. In our case, we are interested in the characteristic speeds of transient processes. To do this, let us assume that the system is initially in equilibrium at $t<0$, and then at $t=0$, a voltage is applied to one of the leads. This additional voltage induces a nonstationary current, which at $t\to\infty$ reaches the stationary value (25).

The applied voltage shifts the energy levels in the reservoirs by V_α, where the index α denotes the reservoir. Thus, the reservoir Hamiltonian can now be written as

$$\widehat{H}_\alpha(t) = \sum_p \tau_p^\alpha\left(h_p^{\alpha+}\psi_1+\psi_1^+h_p^\alpha\right)+ +\sum_p\left(E_p^\alpha+V_\alpha\theta(t)\right)h_p^{\alpha+}h_p^\alpha. \quad (28)$$

The current flowing from the left reservoir into the system is given by (for the "right" contact r, all formulas can be written similarly):

$$I_l(t) = -\int dt'\left(\Sigma_l^<(t,t')\widetilde{G}^A_{1,1}(t',t)+ +\Sigma_l^R(t,t_1)\widetilde{G}^<_{1,1}(t',t)- -\widetilde{G}^<_{1,1}(t,t')\Sigma_l^A(t',t)- -\widetilde{G}^R_{1,1}(t,t')\Sigma_l^<(t',t)\right). \quad (29)$$

Here, the irreducible part takes the form:

$$\Sigma_\alpha^R(t,t') = -i\sum_p(\tau_p^\alpha)^2\theta(t-t')\times \times\exp\left(-iE_p^\alpha(t-t')-iV_\alpha\int_{t'}^t dt_1\theta(t_1)\right), \quad (30)$$

$$\Sigma_\alpha^<(t,t') = i\sum_p(\tau_p^\alpha)^2 n_p^\alpha\times \times\exp\left(-iE_p^\alpha(t-t')-iV_\alpha\int_{t'}^t dt_1\theta(t_1)\right). \quad (31)$$

In the frequency representation, these expressions correspond to the following formulas:

$$\Sigma_\alpha^R(\omega,\omega') = -i(\tau^\alpha)^2\int d\varepsilon\nu^\alpha(\varepsilon)\left[-\frac{1}{\omega-\varepsilon-V_\alpha+2i\delta}\left(-\frac{1}{\omega'-\omega-2i\delta}+\frac{1}{\omega'-\omega+V_\alpha}\right)+ +\frac{1}{\omega'-\varepsilon+2i\delta}\left(-\frac{1}{\omega-\omega'-V_\alpha}+\frac{1}{\omega-\omega'-2i\delta}\right)\right], \quad (32)$$

$$\Sigma_\alpha^<(\omega,\omega') = i(\tau^\alpha)^2\int d\varepsilon\nu^\alpha(\varepsilon)n^\alpha(\varepsilon)\left(\frac{1}{\omega-\varepsilon-V_\alpha+i\delta}-\frac{1}{\omega-\varepsilon-i\delta}\right)\left(\frac{1}{\omega'-\varepsilon-V_\alpha-i\delta}-\frac{1}{\omega'-\varepsilon+i\delta}\right). \quad (33)$$

where $\nu^\alpha(\varepsilon)$ is the density of states in the reservoir α, $\delta\to+0$. For simplicity, we assume that τ^α does not depend on p. In the wide-band approximation, where we assume that $\nu(\varepsilon)$ remains constant for $\varepsilon\sim\omega,\omega',V_{l,r}$, these expressions simplify to:

$$\Sigma_l^R(\omega,\omega') = -i\gamma_l 2\pi\delta(\omega'-\omega),$$

$$\Sigma_l^<(\omega,\omega') = \frac{i\gamma_l}{\pi}\int d\varepsilon n^l(\varepsilon)\times \times\left(\frac{1}{\omega-\varepsilon-V_l+i\delta}-\frac{1}{\omega-\varepsilon-i\delta}\right)\times \times\left(\frac{1}{\omega'-\varepsilon-V_l-i\delta}-\frac{1}{\omega'-\varepsilon+i\delta}\right).$$

As a result, in the frequency representation, Equation (29) simplifies to:

$$\hat{I}_l(\omega) = -\int\frac{d\Omega}{2\pi}\left(\Sigma_l^<(\Omega,\Omega-\omega)\widetilde{\Gamma}^\delta_{1,1}(\Omega-\omega)- -2i\gamma_l\widetilde{\Gamma}^<_{1,1}(\Omega,\Omega-\omega)\right). \quad (34)$$

where

$$\tilde{\Gamma}^{<}_{1,1}(\Omega,\Omega-\omega) = \tilde{\Gamma}^{R}_{1,1}(\Omega)\Sigma^{<}_{l}(\Omega,\Omega-\omega)\tilde{\Gamma}^{A}_{1,1}(\Omega-\omega) + \\ + \tilde{\Gamma}^{R}_{1,N}(\Omega)\Sigma^{<}_{r}(\Omega,\Omega-\omega)\tilde{\Gamma}^{A}_{N,1}(\Omega-\omega),$$

$$\tilde{\Gamma}^{\delta}_{1,1}(\Omega-\omega) = \tilde{\Gamma}^{A}_{1,1}(\Omega-\omega) - \tilde{\Gamma}^{R}_{1,1}(\Omega).$$

Using Dyson's equations for retarded and advanced Green's functions, we can show that:

$$\tilde{\Gamma}^{\delta}_{1,1}(\Omega-\omega) = \omega\sum_{n=1}^{N}\Gamma^{R}_{1,n}(\Omega)\Gamma^{A}_{n,1}(\Omega-\omega) + \\ +\tilde{\Gamma}^{R}_{1,1}(\Omega)2i\gamma_l\tilde{\Gamma}^{A}_{1,1}(\Omega-\omega) + \\ +\tilde{\Gamma}^{R}_{1,N}(\Omega)2i\gamma_r\tilde{\Gamma}^{A}_{N,1}(\Omega-\omega).$$

Substituting these last expressions into (34), we obtain:

$$\hat{I}_l(\omega) = -\int\frac{d\Omega}{2\pi}\Bigg(\omega\sum_{n=1}^{N}\tilde{\Gamma}^{R}_{1,n}(\Omega)\tilde{\Gamma}^{A}_{n,1}(\Omega-\omega) \\ \times\Sigma^{<}_{l}(\Omega,\Omega-\omega) \\ +2i\Big(\gamma_r\Sigma^{<}_{l}(\Omega,\Omega-\omega) - \gamma_l\Sigma^{<}_{r}(\Omega,\Omega-\omega)\times\Big) \\ \times\tilde{\Gamma}^{R}_{1,N}(\Omega)\tilde{\Gamma}^{A}_{N,1}(\Omega-\omega)\Bigg). \quad (35)$$

Here

$$\Sigma^{<}_{r}(\omega,\omega') = \frac{i\gamma_r}{\pi}\int d\varepsilon n^{l}(\varepsilon)\times \\ \times\left(\frac{1}{\omega-\varepsilon-V_r+i\delta} - \frac{1}{\omega-\varepsilon-i\delta}\right)\times \\ \times\left(\frac{1}{\omega'-\varepsilon-V_r-i\delta} - \frac{1}{\omega'-\varepsilon+i\delta}\right). \quad (36)$$

We see that the first term in (35) exists only if $V_l \neq 0$ and does not directly depend on the properties of the right reservoir *r*.

This means that this term corresponds to the filling of states at the left edge of the chain due to a change in its chemical potential.

Consequently, the second term represents the current that flows from one reservoir to another through the entire chain.

If we consider only the second term, we obtain:

$$\hat{I}_l(t) = \frac{2\gamma_l\gamma_r}{\pi}\int d\varepsilon dV\widehat{M}_{1,N}(t,\varepsilon,V)\Big(\widehat{M}_{1,N}(t,\varepsilon,V)\Big)\times \\ \times\Big[n^{l}(\varepsilon)\delta(V-V_l) - n^{r}(\varepsilon)\delta(V-V_r)\Big],$$

$$\widehat{M}_{1,N}(t,\varepsilon,V) = \int\frac{d\Omega}{2\pi}e^{-i\Omega t}\tilde{\Gamma}^{R}_{1,N}(\Omega)\times \\ \times\left(\frac{1}{\Omega-\varepsilon-V+i\delta} - \frac{1}{\Omega-\varepsilon-i\delta}\right).$$

Since our goal is to study the propagation of perturbations through the chain, we assume that at time $t = 0$, the voltage changes only at the right contact, and we observe the time-dependent current at the left contact under the condition $V_l = 0$. Then, by direct calculations, we obtain that:

$$\widehat{M}_{1,N}(t,\varepsilon,V) = -i\theta(-t)\frac{e^{-i\varepsilon t}C\omega_0}{(\varepsilon+iC(\gamma_l+\gamma_r))^2-\bar{\omega}^2}\begin{pmatrix}1 & -\frac{\Delta}{|\Delta|}\\ \frac{\Delta^*}{|\Delta|} & -1\end{pmatrix} - i\theta(t)\frac{e^{-i(\varepsilon+V_r)t}C\omega_0}{(\varepsilon+V+iC(\gamma_l+\gamma_r))^2-\bar{\omega}^2}\begin{pmatrix}1 & -\frac{\Delta}{|\Delta|}\\ \frac{\Delta^*}{|\Delta|} & -1\end{pmatrix} - \\ -\frac{iC\omega_0\theta(t)}{2\bar{\omega}}e^{-C(\gamma_l+\gamma_r)t-i\bar{\omega}t}\begin{pmatrix}1 & -\frac{\Delta}{|\Delta|}\\ \frac{\Delta^*}{|\Delta|} & -1\end{pmatrix}\left(-\frac{1}{\varepsilon+V+iC(\gamma_l+\gamma_r)-\bar{\omega}} + \frac{1}{\varepsilon+iC(\gamma_l+\gamma_r)-\bar{\omega}}\right) - \\ -\frac{iC\omega_0\theta(t)}{-2\bar{\omega}}e^{-C(\gamma_l+\gamma_r)t+i\bar{\omega}t}\begin{pmatrix}1 & -\frac{\Delta}{|\Delta|}\\ \frac{\Delta^*}{|\Delta|} & -1\end{pmatrix}\left(-\frac{1}{\varepsilon+V+iC(\gamma_l+\gamma_r)+\bar{\omega}} + \frac{1}{\varepsilon+iC(\gamma_l+\gamma_r)+\bar{\omega}}\right).$$

$$I_l(t) = \frac{4\gamma_l\gamma_r C^2\omega_0^2}{\pi}\theta(t)\int d\varepsilon n^l(\varepsilon)\left|\frac{1}{(\varepsilon + iC(\gamma_l+\gamma_r))^2 - \bar{\omega}^2}\right|^2 -$$
$$-\frac{4\gamma_l\gamma_r C^2\omega_0^2}{\pi}\theta(t)\int d\varepsilon n^r(\varepsilon - V_r)\times$$
$$\times\left|-\frac{e^{-i\varepsilon t}}{(\varepsilon + iC(\gamma_l+\gamma_r))^2 - \bar{\omega}^2} - \frac{1}{2\bar{\omega}}e^{-C(\gamma_l+\gamma_r)t - i\bar{\omega}t}\left(-\frac{1}{\varepsilon + iC(\gamma_l+\gamma_r) - \bar{\omega}} + \frac{1}{\varepsilon - V_r + iC(\gamma_l+\gamma_r) - \bar{\omega}}\right) + \right.$$
$$\left. + \frac{1}{2\bar{\omega}}e^{-C(\gamma_l+\gamma_r)t + i\bar{\omega}t}\left(-\frac{1}{\varepsilon + iC(\gamma_l+\gamma_r) + \bar{\omega}} + \frac{1}{\varepsilon - V_r + iC(\gamma_l+\gamma_r) + \bar{\omega}}\right)\right|^2.$$

Here

$$\bar{\omega} = \sqrt{\omega_0^2 - C^2(\gamma_l - \gamma_r)^2}. \quad (37)$$

As expected, if $t\to\infty$, the current approaches its stationary value (25):

$$I_l(t\to\infty) = \frac{4\gamma_l\gamma_r C^2\omega_0^2}{\pi}\theta(t)\times \int d\varepsilon(n^l(\varepsilon) - n^r(\varepsilon - V_r))\times \left|\frac{1}{(\varepsilon + iC(\gamma_l+\gamma_r))^2 - \bar{\omega}^2}\right|^2.$$

If $t\to +0$, the current at the opposite edge of the chain is not observed, illustrating the continuity of the current change when passing through $t = 0$:

$$I_l(t\to +0) = \frac{4\gamma_l\gamma_r C^2\omega_0^2}{\pi}\theta(t)\times \int d\varepsilon(n^l(\varepsilon) - n^r(\varepsilon))\times \left|\frac{1}{(\varepsilon + iC(\gamma_l+\gamma_r))^2 - \bar{\omega}^2}\right|^2 = 0.$$

If now, as in the previous section, we are interested in the role of "Majorana states," we apply an additional voltage to the right contact, which is greater than the width of the localized states but less than the value of the superconducting gap. This means that the conditions

$$n^l(\varepsilon) = n^r(\varepsilon) = 0, \quad n^r(\varepsilon - V_r) = 1.$$

are satisfied for $\varepsilon \lesssim \gamma_l, \gamma_r$

The current is defined as

$$I_l(t) = -\frac{2\gamma_l\gamma_r C\omega_0^2}{\gamma_l+\gamma_r}\frac{1}{\omega_0^2 + 4C^2\gamma_l\gamma_r}\theta(t) + \frac{2\gamma_l\gamma_r C\omega_0^2}{(\gamma_l+\gamma_r)\bar{\omega}^2}e^{-2C(\gamma_l+\gamma_r)t}\theta(t) - i\gamma_l\gamma_r\frac{C^2\omega_0^2}{\bar{\omega}^2}\frac{e^{-2C(\gamma_l+\gamma_r)t + 2i\bar{\omega}t}\theta(t)}{\bar{\omega} + iC(\gamma_l+\gamma_r)} + i\gamma_l\gamma_r\frac{C^2\omega_0^2}{\bar{\omega}^2}\frac{e^{-2C(\gamma_l+\gamma_r)t - 2i\bar{\omega}t}\theta(t)}{\bar{\omega} - iC(\gamma_l+\gamma_r)}. \quad (38)$$

We consider the case $\gamma_r, \gamma_l \gg \omega_0$ under the assumption that ω_0 is always small. However, for very symmetric tunneling coupling with the leads, we could have $\omega_0^2 \gg (\gamma_r - \gamma_l)^2$. This case appears unrealistic, but it demonstrates an oscillating current signal at the left edge:

$$I_l(t) = -\frac{\omega_0^2}{2C(\gamma_l+\gamma_r)}\times\left[1 - e^{-2C(\gamma_l+\gamma_r)t}\right] - \left[\frac{\omega_0}{2}\sin(2\omega_0 t) - \frac{C(\gamma_l+\gamma_r)}{2}(1 - \cos(2\omega_0 t))\right]\times e^{-2C(\gamma_l+\gamma_r)t}. \quad (39)$$

If $\omega_0 \ll |\gamma_r - \gamma_l|$ and $t > 0$, Equation (38) simplifies to:

$$I_l(t) = -\frac{\omega_0^2}{2C(\gamma_l+\gamma_r)}\times\left[1 + \frac{4\gamma_l\gamma_r}{(\gamma_l-\gamma_r)^2}e^{-2C(\gamma_l+\gamma_r)t} - \frac{(\gamma_l+\gamma_r)}{(\gamma_l-\gamma_r)^2}\left(\gamma_l e^{-4C\gamma_r t} + \gamma_r e^{-4C\gamma_l t}\right)\right]. \quad (40)$$

Note that the negative sign indicates that the current flows from r to l. For significantly different tunneling rates, for example, $\gamma_r \gg \gamma_l$, the time evolution of the leading contribution to the current is determined by the slowest rate:

$$I_l(t) = -\frac{\omega_0^2}{2C\gamma_r}\left[1 - e^{-4C\gamma_l t}\right]. \tag{41}$$

The final formula shows that if $\gamma_l \to 0$, the current signal at the other end of the chain increases very slowly.

5. CONCLUSION

This paper demonstrates that the transport properties of a finite-length Kitaev chain can be fully investigated using the conventional Green's function technique. For any nonstationary problem, this formalism appears much more convenient than the language of Majorana fermions or other methods, allowing for the exact analytical results. Our calculations bridge the gap between phenomenological parameters for quasiparticles in quasiclassical calculations and the microscopic description of quasi-one-dimensional superconductors.

It has been shown that the stationary tunneling current through a finite chain is always determined by the lowest transfer rate among the parameters $\omega_0^2/(\gamma_l+\gamma_r), \gamma_l, \gamma_r$, provided the applied voltage is less than the superconducting gap. For arbitrary $\mu < |\Delta| < t$, the stationary current is always exponentially small for long chains. It should be noted that for a finite Kitaev chain placed between two external thermostat contacts, no significant peak can be observed at ω_0 in the tunneling conductance. Furthermore, in the case of $\omega_0 = 0$, the stationary current completely vanishes.

We have also obtained the time-dependent behavior of the tunneling current following a sudden change in the bias voltage at one of the leads. It was shown that the typical timescales of tunneling current evolution are primarily determined by the tunneling rates γ_l, γ_r from the left and right edge sites of the chain to the corresponding leads. Although the results presented here are for an ideal system, we can be confident – based on the conclusions of [34, 35]—that weak disorder does not significantly affect the properties of the ideal Kitaev chain. Therefore, only strong disorder can completely alter our results.

In conclusion, it is worth noting that when considering systems of multiple Kitaev chains, an effective description based on Coulomb blockade effects is often constructed. However, such an effective description is sensitive to charge transfer rates, which may be important for modern proposals related to signal transmission, quantum information exchange, and storage using Kitaev chains.

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APPENDIX ANALYTICAL DESCRIPTION OF THE ISOLATED KITAEV CHAIN

In this section, we present the formulas for the Green's functions of the isolated Kitaev chain.

As shown in [20], the exact solution for the Green's functions of the infinite chain can be written as:

$$\Gamma_{nm}^{0R}(\omega) = -\frac{1}{4(|\Delta|^2 - t^2)(A_+ - A_-)} \times \left[\chi_+^{|n-m|}\widehat{M}_1 - \chi_-^{|n-m|}\widehat{M}_2\right]. \tag{42}$$

Here

$$\widehat{M}_1 = \begin{pmatrix} \frac{\omega - \mu - 2tA_+}{\sqrt{A_+^2 - 1}} & 2\Delta \operatorname{sign}(n-m) \\ -2\Delta^* \operatorname{sign}(n-m) & \frac{\omega + \mu + 2tA_+}{\sqrt{A_+^2 - 1}} \end{pmatrix},$$

$$\widehat{M}_2 = \begin{pmatrix} \frac{\omega - \mu - 2tA_-}{\sqrt{A_-^2 - 1}} & 2\Delta \operatorname{sign}(n-m) \\ -2\Delta^* \operatorname{sign}(n-m) & \frac{\omega + \mu + 2tA_-}{\sqrt{A_-^2 - 1}} \end{pmatrix}.$$

The complex value of the square root $\sqrt{A_\pm^2 - 1}$ is defined such that it has a branch cut along the interval $\sqrt{A_\pm^2 - 1}$ and takes positive values when $A_\pm > 1$.

$$A_\pm = \frac{t\mu \pm |\Delta|\sqrt{\mu^2 + 4(|\Delta|^2 - t^2)\left(1 - \frac{(\omega + i\delta)^2}{4|\Delta|^2}\right)}}{2(|\Delta|^2 - t^2)}, \tag{43}$$

$$\chi_{\pm} = A_{\pm} - \sqrt{A_{\pm}^2 - 1}. \quad (44)$$

We assume $\delta \to +0$. The Green's function for the Hamiltonian (2) can be written in terms of the Green's function of the infinite chain, using Dyson's equation with the perturbation $\dot{V}$,

$$\Gamma^R_{nm}(\omega) = \Gamma^{0R}_{nm}(\omega) + \Gamma^{0R}_{n0}(\omega)U\sigma_z\Gamma^R_{0m}(\omega) + \\ +\Gamma^{0R}_{n,N+1}(\omega)U\sigma_z\Gamma^R_{N+1,m}(\omega). \quad (45)$$

If we solve Equation (45) for $\Gamma^R_{nm}(\omega)$ and take the limit $U \to \infty$, we can find the exact solution for the Green's functions $\Gamma^R_{nm}(\omega)$:

$$\begin{aligned}\Gamma^R_{nm}(\omega) = \; & \Gamma^{0R}_{nm}(\omega) - \Gamma^{0R}_{n0} \times \\ & \times(\omega)\Big(\Gamma^{0R}_{0,0}(\omega) - \Gamma^{0R}_{0,N+1}(\omega)(\Gamma^{0R}_{N+1,N+1}(\omega))^{-1}\Gamma^{0R}_{N+1,0}(\omega)\Big)^{-1} \times \\ & \times\Big(\Gamma^{0R}_{0,m}(\omega) - \Gamma^{0R}_{0,N+1}(\omega)(\Gamma^{0R}_{N+1,N+1}(\omega))^{-1}\Gamma^{0R}_{N+1,m}(\omega)\Big) - \\ & -\Gamma^{0R}_{n,N+1}(\omega)\Big(\Gamma^{0R}_{N+1,N+1}(\omega) - \Gamma^{0R}_{N+1,0}(\omega)(\Gamma^{0R}_{00}(\omega))^{-1}\Gamma^{0R}_{0,N+1}(\omega)\Big)^{-1} \times \\ & \times\Big(\Gamma^{0R}_{N+1,m}(\omega) - \Gamma^{0R}_{N+1,0}(\omega)(\Gamma^{0R}_{00}(\omega))^{-1}\Gamma^{0R}_{0m}(\omega)\Big).\end{aligned} \quad (46)$$

The matrix elements of $\Gamma^R_{nm}(\omega)$ describe the Green's functions of the finite chain, provided the indices satisfy the condition $0 < n,m < N+1$. It can be directly verified that $\Gamma^R_{nm}(\omega) = 0$ if one of the arguments n or m is positive, while the other is negative, giving us direct proof that our procedure effectively removes the site $n = 0$ from the system. The same is true for the site $n = N+1$.

We can see that the function $\Gamma^R_{nm}(\omega)$ may have a set of poles at values ω determined by the equation:

$$\det\Big(\Gamma^{0R}_{00}(\omega) - \Gamma^{0R}_{N+1,0}(\omega)(\Gamma^{0R}_{00}(\omega))^{-1}\Gamma^{0R}_{0,N+1}(\omega)\Big) = 0. \quad (47)$$

Since $\Gamma^{0R}_{nm}(\omega)$ has no poles inside the superconducting gap, it can be assumed that the solutions of this equation correspond to the energies of states localized at the chain edges. Direct substitution of the Green's functions (42) allows us to find the solution for ω at arbitrary parameter values.

For the semi-infinite chain, if $N \to \infty$, the situation simplifies significantly. Equation (47) simplifies to:

$$\det\Big(\Gamma^{0R}_{0,0}(\omega)\Big) = 0, \quad (48)$$

and it has only one solution in the gap $\omega = 0$. This solution does not arise if $|\mu| > 2t$. This pole at $\omega = 0$ exists in the Green's function $\Gamma^R_{nm}(\omega)$ only if both n and m are positive or both are negative, for any set of parameters t, μ, Δ satisfying the condition

$$t^2 > (\mu/2)^2 + \Delta^2,$$

the condition that separates the topologically nontrivial and trivial phases. This means that the system described by the Hamiltonian (2) has two states with energy $\omega = 0$: one to the left and one to the right of the defect, which cuts the chain into two subsystems.

If we now consider a long finite chain of length T, we can write the equation for localized states as

$$\det\Big[\Gamma^{(X)R}_{N+1,N+1}(\omega)\Big] = 0, \quad (49)$$

where $\Gamma^{(X)R}_{N+1,N+1}(\omega)$ is the Green's function for the semi-infinite chain:

$$\Gamma^{(X)R}_{nm}(\omega) \equiv \\ \equiv \Gamma^{0R}_{nm}(\omega) - \Gamma^{0R}_{n0}(\omega)\Big(\Gamma^{0R}_{0,0}(\omega)\Big)^{-1}\Gamma^{0R}_{0,m}(\omega). \quad (50)$$

Since we are interested in bound states within the gap with energies close to zero, the calculations can be simplified using the following fact. For $\omega \to 0$, the values of χ satisfy the condition $|\chi_{\pm}| < 1$. Indeed, for $\omega = 0$, Equation (44) gives

$$\chi_{\pm} = \frac{-\mu \pm i\sqrt{4t^2 - \left(\mu^2 + 4|\Delta|^2\right)}}{2(t + |\Delta|)}. \quad (51)$$

As a result,

$$|\chi|^2 = \left|\frac{t - |\Delta|}{t + |\Delta|}\right|. \quad (52)$$

This means that $|\chi| < 1$ for $t^2 > ((\mu/2)^2 + \Delta^2)$ and $\omega \ll |\Delta|$. Thus, quantities like $|\chi|^N$ appearing in the Green's functions Γ_{0N}, are small parameters for large *N*. Henceforth, we will refer to such quantities as "exponentially small," implying exponential decay with chain length (or number of sites).

Expanding Equation (49) in terms of ω and $\chi_\pm^N$, which we treat as small, as explained above, we obtain

$$0 = \det\left[-\hat{I}\omega\frac{t}{|\Delta|(4t^2-\mu^2)} - \right.$$
$$-\begin{pmatrix} 1 & \frac{\Delta}{|\Delta|} \\ \frac{\Delta^*}{|\Delta|} & 1 \end{pmatrix}\frac{1}{\omega}\frac{|\Delta|(4t^2-\mu^2)}{2t(4(t^2-|\Delta|^2)-\mu^2)}\times \quad (53)$$
$$\left.\times\left(\chi_+^{N+1}-\chi_-^{N+1}\right)^2\right],$$

where $\hat{I}$ is the identity matrix. The solution $\omega = 0$ corresponds to the pole of the Green's function, which exists only on the semi-infinite chain segments. The other pair of solutions has finite but small energies $\omega = \pm\omega_0$, where

$$\omega_0 = \frac{|\Delta|(4t^2-\mu^2)}{it\sqrt{4(t^2-|\Delta|^2)-\mu^2}}\times$$
$$\times\left(\chi_+^{N+1}-\chi_-^{N+1}\right). \quad (54)$$

Here we see that this solution satisfies the approximations we made, if $\left|\chi_\pm^{N+1}\right| \ll 1$. Considering Equation (51), the condition $t^2 = ((\mu/2)^2 + \Delta^2)$ separates the two regions with oscillating and non-oscillating solutions for ω_0. If ω_0 crosses zero with varying μ, this implies a change in fermion parity, as discussed in [36].

The leading term in the expansion of the Green's function $\Gamma_{nm}^R(\omega)$ near $\omega \to \pm\omega_0$, which in quantum mechanics would describe the spatial structure of the wavefunctions of the two localized states, takes the following form:

$$\Gamma_{nm}^R(\omega) = -\frac{\omega}{(\omega+i\delta)^2-(\omega_0)^2}\frac{|\Delta|(4t^2-\mu^2)}{2t(4(t^2-|\Delta|^2)-\mu^2)}\times$$
$$\times\left((\chi_+^n-\chi_-^n)(\chi_+^m-\chi_-^m)\begin{pmatrix} 1 & \frac{\Delta}{|\Delta|} \\ \frac{\Delta^*}{|\Delta|} & 1 \end{pmatrix} + (\chi_+^{N+1-n}-\chi_-^{N+1-n})(\chi_+^{N+1-m}-\chi_-^{N+1-m})\begin{pmatrix} 1 & -\frac{\Delta}{|\Delta|} \\ -\frac{\Delta^*}{|\Delta|} & 1 \end{pmatrix}\right) -$$
$$-\frac{\omega_0}{(\omega+i\delta)^2-(\omega_0)^2}\frac{|\Delta|(4t^2-\mu^2)}{2t(4(t^2-|\Delta|^2)-\mu^2)}\times \quad (55)$$
$$\times\left((\chi_+^n-\chi_-^n)(\chi_+^{N+1-m}-\chi_-^{N+1-m})\begin{pmatrix} 1 & -\frac{\Delta}{|\Delta|} \\ \frac{\Delta^*}{|\Delta|} & -1 \end{pmatrix} + (\chi_+^{N+1-n}-\chi_-^{N+1-n})(\chi_+^m-\chi_-^m)\begin{pmatrix} 1 & \frac{\Delta}{|\Delta|} \\ -\frac{\Delta^*}{|\Delta|} & -1 \end{pmatrix}\right).$$

Diagonal elements Γ_{nn} show the spatial distribution of density in localized states. In the limit $\Delta = t$, only Γ_{11} and Γ_{NN} remain non-zero, since Equation (52) gives

$$\chi_+^n \propto \chi_-^n \propto |t - |\Delta||^{n/2}.$$

In the high-symmetry case $\mu = 0$ and $|\Delta| \to t$, the energy levels are equal:

$$\omega_0 = \frac{4|\Delta|t}{t+|\Delta|}\left(\frac{t-|\Delta|}{t+|\Delta|}\right)^{\frac{N}{2}}\times$$
$$\times\sin\left(\frac{\pi(N+1)}{2}\right) \to 0. \quad (56)$$

As noted earlier (see, for example, [37]), for an odd number of sites is equal to zero for any values of t and Δ.

REFERENCES

1. *A. Yu. Kitaev*. PhysicsUsp. **44**, 131, (2001)
2. *A. F. Andreev*, Sov. Phys. JETP **19**, 1228 (1964); 49, 655 (1966); 22, 455 (1966)
3. *V. V. Val'kov, M. S. Shustin, S. V. Aksenov et al.*, Phys. Usp. **65,** 2 (2022)
4. *V. Mourik, K. Zuo, S. M. Frolov et al.* Science **336**, 1003 (2012).
5. *M. T. Deng et al.* Science **354**, 1557 (2016).
6. *H. J. Suominen, M. Kjaergaard, A. R. Hamilton et al.* Phys. Rev. Lett. **119**, 176805 (2017)
7. *E. Prada, P. San-Jose, M. W. A. de Moor et al.* Nature Reviews Physics **2**, 575 (2020)
8. *D. Aasen, M. Hell, R. V. Mishmash et al.* Physical Review X **6**, 031016 (2016)
9. *T. Karzig, Y. Oreg, G. Refael and M. H. Freedman*, Physical Review X **6**, 031019 (2016)
10. *F. Pientka, L. Jiang, D. Pekker et al.* New Journal of Physics **15**, 115001 (2013)
11. *T. Hyart, B. van Heck, I. C. Fulga et al.* Phys. Rev. B **88**, 035121 (2013)
12. *L. Jiang, D. Pekker, J. Alicea et al.* Phys. Rev. Lett. **107**, 236401 (2011)
13. *B. Van Heck, A. R. Akhmerov, F. Hassler et al.* New Journal of Physics **14**, 035019 (2012)
14. *M. Valentini, M. Borovkov, E. Prada et al.* Nature **612**, 442 (2022)
15. *S. M. Albrecht, A. P. Higginbotham, M. Madsen et al.* Nature **531**, 206 (2016)
16. *S. D. Sarma, M. Freedman, and C. Nayak*, npj Quantum Inf. **1**, 15001 (2015)
17. *C. Nayak, S. H. Simon, A. Stern et al.* Rev. Mod. Phys. **80** 1083 (2008)
18. *J. Jin, and X. Q. Li*, New J. Phys. **24** 093009 (2022)
19. *L. Arrachea, G. S. Lozano, and A. A. Aligia*, Phys. Rev. B **80**, 014425 (2009).
20. *A. Zazunov, R. Egger, and A. Levy*, Yeyati Phys. Rev. B **94**, 014502 (2016).
21. *A. I. Rusinov*, Sov. Phys. JETP. **29**, 1101 (1969)
22. *P. I. Arseyev, and B. A. Volkov*, Sol.St.Comm. **78**, 373 (1991)
23. *C. Caroli, R. Combescot, P. Nozieres, and D. SaintJames*, J. Phys. C: Solid State Phys. **4**, 916 (1971)
24. *K. Flensberg*, Phys. Rev. B **82**, 180516 (2010)
25. *S. Smirnov*, Phys. Rev. B **105**, 205430 (2022)
26. *L. Fu*, Phys. Rev. Lett. **104**, 056402 (2010)
27. *Y. Meir, and N. S. Wingreen*, Phys. Rev. Lett. **68**, 2512 (1992)
28. *N. Leumer, M. Grifoni, B. Muralidharan, and M. Marganska*, Physical Review B. **103**, 165432 (2021)
29. *C. Caroli, R. Combescot, P. Nozieres, and D. SaintJames*, J. Phys. C: Solid State Phys. **5**, 21 (1972)
30. *P. I. Arseev, N. S. Maslova*, Physics-Usp. **53**, 1151 (2010)
31. *S. N. Thomas, S. D. Sarma, and J. D. Sau*, Phys. Rev. B **106**, 174501 (2022)
32. *Y. H. Lai, S. D. Sarma, and J. D. Sau*, Phys. Rev. B **106**, 094504 (2022)
33. *S. Hegde et al.*, New J. Phys. **17**, 053036 (2015)
34. *K. Kawabata, R. Kobayashi, N. Wu, and H. Katsura*, Phys. Rev. B **95**, 195140 (2017)