JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS, 2025, Vol. 167, No. 1, pp. 83–90

ORDER, DISORDER, AND PHASE TRANSITION IN CONDENSED MEDIA

TWO-DIMENSIONAL MAGNETOPLASMONS IN THE STRIP OF FINITE WIDTH

© 2025 R. Z. Vitlina[a], L. I. Magarill[a,b,*], A. V. Chaplik[a]

[a]Rzhanov Institute of Physics, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
[b]Novosibirsk State University, Novosibirsk, Russia
*e-mail: levm@isp.nsc.ru

Received June 10, 2024

Revised August 01, 2024

Accepted September 25, 2024

Abstract. Effect of sample boundaries on the spectrum of magnetoplasmons in the 2D electron gas was investigated. using the example of a strip. As should be expected in the limit of the plasmon wave length far exceeding the strip width the dispersion law of magnetoplasmons follows the one for 1D plasma waves however the leading term in the dispersion relation depends on the magnetic field. The dispersion laws of intraband plasmons in cases when one and two subbands are populated, depolarisation shift of the interband plasmon and spatial distribution of the plasmon electric field are found. The concentration and magnetic field dependencies of the plasmon frequency have been obtained numerically.

DOI: 10.31857/S00444510250108e6

1. INTRODUCTION

The edge magnetoplasmon (EMP) in a two-dimensional (2D) electron system was first theoretically studied in the works of Volkov and Mikhailov [1, 2]. The authors conducted both classical and quantum analyses for a half-plane and found the EMP dispersion relation as $\omega(k)$ where k is the 1D wave vector of the plasmon wave along the edge of the sample. Naturally, a question arises about the role of boundaries in a real experiment, particularly concerning plasma waves in a finite-width strip, where the influence of the opposite edge must also be taken into account. This formulation of the problem was outlined in the introduction of the paper by Balev and Vasilopoulos [3]. The authors proposed a strip model with "soft" walls, described by a parabolic potential for electrons near the strip boundaries. However, in their analysis of plasma oscillations, they effectively considered only one edge, naturally obtaining the already known result for the plasmon frequency. Meanwhile, the presence of the second boundary leads to qualitatively new features of the phenomenon: strictly speaking, one should not consider an edge plasmon but rather the eigenmodes of a planar plasma waveguide. It is important to note that in such a "waveguide", the electron motion is confined in one direction, while the electric field of the plasma wave extends formally to infinity. Within the framework of the classical hydrodynamic description of 2D plasma, this problem was solved in [4, 5]. The plasmon spectrum for a 2D electron strip under conditions of strong screening by a metallic electrode was found in [6], using a classical approach within the local capacitance approximation.

In the present study, we develop a quantum theory of magnetoplasmon waves in a 2D electron gas strip of finite width $L = 2w$. The boundary conditions for the wave functions correspond to hard walls, meaning the transverse electron motion (along the x-axis) corresponds to a "truncated" harmonic oscillator at $x = \pm w$ with the cyclotron frequency ω_c and a suspension point $X = -pl^2$, where p is the conserved y-component of the electron momentum in the Landau gauge, and l is the magnetic length ($\hbar = 1$). For the Landau level with index n, the wave function has the form:

$$\Psi_{n,X}(x,y) = N_{n,X}\varphi_{n,X}(x)\frac{\exp(ipy)}{\sqrt{L_y}}. \qquad (1)$$

Here $N_{n,X}$ is the normalization coefficient, and L_y is the length of the strip. For the wave function $\psi_{n,X}(x)$, we have (see, for example, [7]):

$$\psi_{n,X}(x) = e^{-(x-X)^2/2l^2} \times$$
$$\times\Big[\Phi\Big(-q_n(X)/2, 1/2, (x-X)^2/l^2\Big) -$$
$$-B(x-X)\Phi\Big((1-q_n(X))/2, 3/2, (x-X)^2/l^2\Big)\Big]. \quad (2)$$

The first index of the confluent hypergeometric function in Equation (2) determines the energy of the Landau subbands:

$$q_n(X) = E_n(X)/\omega_c - 1/2$$

via the dispersion equation following from the boundary conditions $\psi_{n,X}(x = \pm w) = 0$. From the same conditions, the constant B is determined.

The dispersion of the Landau subbands $E_n(X)$ is well known, and its graphs have been repeatedly presented in the literature in connection with studies of the quantum Hall effect (edge channels, edge states). The functions $\psi_{n,X}(x)$ and $E_n(X)$ are required to formulate the equation for plasma waves.

2. BASIC EQUATIONS

The problem considered here belongs to the class of plasma oscillations in multicomponent low-dimensional systems. The solution scheme, i.e., finding the eigenfrequencies of plasmons in such systems through the matrix dielectric function in the self-consistent field approximation, is described in [8] for 2D systems, such as quantum well structures with more than one populated transverse quantization level, double quantum wells, or multilayer superlattices.

In the case of magnetoplasmons in a 2D electron gas strip, the plasma components correspond to groups of electrons in different Landau levels (subbands $E_n(X)$), effectively forming 1D systems. Therefore, the Green's function of the Poisson equation takes the form of $G_k(x-x') = -K_0(|k(x-x')|)/2\pi$, where K_0 is the Macdonald function.

Another significant difference from [8] is the dependence of the transverse wave functions $\psi_{n,X}(x)$ (Equation (2)) on the longitudinal electron momentum p through the suspension point of the oscillator. Accounting for these distinctions, the equation for the matrix elements of the plasma wave potential $\varphi(x)e^{iky}$ takes the form (taking into account the selection rules for the momentum along the strip, which allow only transitions $(n,X) \to (m, X+kl^2)$:

$$\varphi_{n,X;m,X+kl^2} =$$
$$= \frac{2e^2}{\varepsilon L_y} \sum_{m',n',X'} \frac{f\left(E_{m'}(X'+kl^2)\right) - f\left(E_{n'}(X')\right)}{E_{m'}(X'+kl^2) - E_{n'}(X') + \omega + i\delta} \times$$
$$\times J_{m,n;m',n'}(X,X')\varphi_{n',X';m',X'+kl^2}, \quad (3)$$

Where ε is the average dielectric constant of the two media separated by the 2D electron gas, f is the Fermi occupation factor, and Form factors $J_{mn;m'n'}$ are defined as:

$$J_{m,n;m',n'}(X,X') =$$
$$= \int_{-w}^{w}\int_{-w}^{w} dx dx' \tilde{\psi}_{n,X}(x)\tilde{\psi}_{m,X+kl^2}(x) \times$$
$$\times K_0(|k(x-x')|)\tilde{\psi}_{n',X'}(x')\tilde{\psi}_{m',X'+kl^2}(x'). \quad (4)$$

In Equation (4), $\tilde{\psi}_{n,X}(x) = N_{n,X}\psi_{n,X}(x)$ represents the normalized wave function of the transverse motion. Thus, we obtain a system of linear homogeneous integral equations for the functions $\varphi_{n,X;m,X+kl^2}$, which we will denote by $\Phi_{nm}(X)$. For an unbounded discrete electron spectrum, the number of equations and, consequently, the number of different plasmon modes is infinite, even if only one level is populated, for example, $E_0(X)$. The off-diagonal terms in Equation (3) $m \neq n$ correspond to virtual transitions with an energy change of at least ω_c , i.e., they are responsible for inter-subband plasmons, whose spectrum has a gap $\Delta > \omega_c$ at zero wave vector $k = 0$. If one is interested only in the low-frequency part of the plasmon spectrum $\omega \ll \omega_c$, it is necessary to restrict consideration to intra-subband plasmons $m = n$ and additionally require the long-wavelength approximation $kl \ll 1$. In the following, we will consider both intra-subband plasmons, and inter-subband plasmons from the lower part of the spectrum, i.e., those associated with the levels $E_0(X)$ and $E_1(X)$.

3. INTRA-BAND PLASMON OF THE ZERO SUBBAND

In this case, instead of Equation (3), we have:

$$\Phi_{00}(X) =$$
$$= \frac{e^2}{\pi\varepsilon l^2} \int dX' \frac{f(E_0(X'+kl^2)) - f(E_0(X'))}{E_0(X'+kl^2) - E_0(X') + \omega + i\delta} \times$$
$$\times J_{00,00}(X,X')\Phi_{00}(X'). \quad (5)$$

Assuming $k \ll p \sim p_F$ (where p_F is the Fermi momentum), we expand the differences in Equation (5) up to the linear term in k. In the form factors J, we set k=0.

For $T = 0$, the numerator becomes $\delta(E_0(X') - E_F)$ (where E_F is the Fermi energy), and the integral reduces to the sum of two terms, corresponding to the values of the integrand at the points $X' = \pm X_0$, where $\pm X_0$ are the roots of the equation:

$E_0(X) = E_F$. Here ($E_0(X)$ is an even function of X.

By substituting variable X in left-hand side of Equation (5) with $\pm X_0$, we arrive at two linear homogeneous equations for the quantities $\Phi_\pm \equiv \Phi_{00}(\pm X_0)$:

$$\begin{aligned} \Phi_+ &= \beta k\left(\frac{J_{+-}}{\omega - kV_0}\Phi_- - \frac{J_{++}}{\omega + kV_0}\Phi_+\right), \\ \Phi_- &= \beta k\left(\frac{J_{--}}{\omega - kV_0}\Phi_- - \frac{J_{-+}}{\omega + kV_0}\Phi_+\right), \end{aligned} \tag{6}$$

Where $\beta = e^2 / \pi\varepsilon, V_0$ is Fermi velocity in the zero subband, while

$$J_{\pm\pm} = J_{00;00}(\pm X_0, \pm X_0),$$

$$J_{\pm\mp} = J_{00;00}(\pm X_0, \mp X_0).$$

It is evident that $J_{-+} = J_{+-}$. In the Appendix, it is shown that $J_{--} = J_{++}$. Thus, there are two independent form factors. The roots of the determinant of the system (6) determine the plasmon frequency $\omega_0(k)$:

$$\omega_0^2(k) = k^2\left(V_0^2 + \beta^2(J_{++}^2 - J_{+-}^2) + 2\beta V_0 J_{++}\right). \tag{7}$$

In the integrals defining $J_{+\pm}$, the functions $\psi_0^2(x)$ are localized near the points $\pm X_0$ within a region of order l. Therefore, for J_{++}, the argument K_0 is small under $k \to 0$, and we can use the asymptotic form of the Macdonald function:

$$K_0(|k(x - x')|) = -\ln(|k(x - x')|e^\gamma / 2),$$

where γ is the Euler constant. Then, for J_{++}, we obtain:

$$J_{++} = \ln\left(\frac{2e^{-\gamma}}{|k|l}\right) + \bar{J}_{++}, \tag{8}$$

where

$$\bar{J}_{++} = \int dx dx' \tilde{\psi}_{0,X_0}^2(x) \ln\left(\frac{l}{|x - x'|}\right) \tilde{\psi}_{0,X_0}^2(x'). \tag{9}$$

The leading term in J_{++} is $|\ln(|k|l)|$. For the form factor J_{+-}, the argument y of the K_0 function can be set to $2|k|X_0$, which may not be small, even for $kl \ll 1$. In this case $J_{+-} = K_0(2|k|X_0)$ and gives a significant contribution, provided the stronger condition $kX_0 \ll 1$ is satisfied. Under this condition, the plasmon frequency becomes

$$\omega_0^2(k) = k^2\left\{2\beta[\beta(\bar{J}_{++} - \bar{J}_{+-}) + V_0]\ln\left(\frac{2e^{-\gamma}}{|k|l}\right) + \right.$$
$$\left. + V_0^2 + \beta^2\left(\bar{J}_{++}^2 - \bar{J}_{+-}^2\right) + 2\beta V_0 \bar{J}_{++}\right\}. \tag{10}$$

Thus, we obtain the expected result for a one-dimensional (1D) plasmon, as found in [9, 10]:

$$\omega \sim k\sqrt{|\ln(|k|l)|}.$$

However, it is important to note that in the case considered here, the dependence of the magnetoplasmon frequency on the electron concentration and magnetic field cannot be expressed analytically. Another important difference is the change in the coefficient before the logarithmic term: to the Fermi velocity V_0 (for a 1D plasmon without a magnetic field), the first term in the square brackets of Equation (10) is added. This additional term can significantly exceed V_0 (for example, at $N_L = 10^6$ cm^{-1}, $H = 1.6$ T, the enhancement is more than an order of magnitude). The results of the numerical calculation are presented below.

The formulas derived in this section are valid up to the very beginning of the plasmon spectrum (k=0), when the plasmon wavelength is much larger than all characteristic lengths of the problem, including the width of the strip L. In this limit, the system effectively becomes one-dimensional. However, the transition to the half-plane limit, studied in [1, 2], is impossible, as it corresponds to an infinitely large width L. The dispersion laws differ: in the half-plane it is proportional to $\ln k$, while in stripe it is $\sqrt{\ln k}$ as expected for one-dimensional systems [9, 10].

4. INTRA-SUBBAND PLASMONS IN A TWO-SUBBAND SYSTEM

Let us now consider the case where the states $E_0(X)$ and $E_1(X)$ are populated, but we neglect the off-diagonal contribution $\Phi_{0,1}$. The Fermi level lies between $E_1(0)$ and $E_2(0)$, intersecting the curves

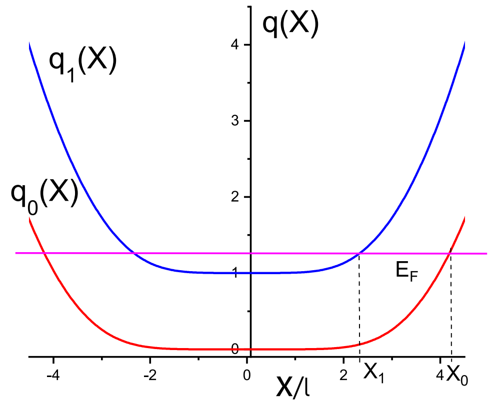

Fig. 1. Electron spectrum of the strip. The figure shows the two lowest Landau subbands. The horizontal line indicates the position of the Fermi level, $w/l = 4$.

$E_0(X)$ and $E_1(X)$ at the points $\pm X_0$ and $\pm X_1$, respectively (Fig. 1).

The four equations for $\Phi_{00}(\pm X_0)$ and $\Phi_{11}(\pm X_1)$ lead to a biquadratic equation for the plasmon frequencies, the roots of which are equal (here, the results are presented for infinitesimally small plasmon momenta $kX_0 \ll 1$, in order to clarify the behavior of $\omega(k)$ at the very beginning of the spectrum):

$$\omega_{ac}^2 = \frac{k^2}{2}\Big(V_0^2 + V_1^2 + \beta^2\Big[\bar{J}_{0+;0+}^2 - \bar{J}_{0+;0-}^2 + \bar{J}_{1+;1+}^2 - \bar{J}_{1+;1-}^2 + 2\bar{J}_{0+;1+}^2 - 2\bar{J}_{0+;1-}^2\Big] + 2\beta(V_0\bar{J}_{0+;0+} + V_1\bar{J}_{1+;1+})\Big), \quad (11)$$

$$\omega_{opt}^2 = \omega_{ac}^2 + 2k^2\ln\left(\frac{2e^{-\gamma}}{|k|l}\right)\Big[\beta(V_0 + V_1) + \beta^2\Big(\bar{J}_{0+;0+} - \bar{J}_{0+;0-} + \bar{J}_{1+;1+} - \bar{J}_{1+;1-} + 2\bar{J}_{0+;1+} - 2\bar{J}_{0+;1-}\Big)\Big]. \quad (12)$$

Here, $V_{0,1}$ are the Fermi velocities in the zero and first subbands, respectively, while six independent form factors such as $\bar{J}_{0+,0+}, \bar{J}_{0+,1+}$, etc. are defined similarly to how it was done in the previous section.

It is important to emphasize that in Equation (11), all logarithmic contributions exactly cancel. The corresponding root of the dispersion equation gives the linear dependence $\omega_{ac}(k)$ as $k \to 0$, which justifies calling this branch acoustic. The second root (optical branch, Equation (12)) exhibits the known singularity at zero at $k \to 0$:

$$\omega_{opt}^2(k) \sim k^2\big|\ln(|k|l)\big|.$$

5. INTER-SUBBAND PLASMON IN A TWO-LEVEL SYSTEM

The rank of the characteristic determinant, considering N subbands, is N^2, since the dielectric function is a 4x4 matrix. Out of the N^2 roots, N correspond to intra-subband plasmons, while in the remaining $N(N-1)$ roots each pair gives rise to one inter-subband branch, making the total number of inter-subband branches equal to $N(N-1)/2$. We focus on the lowest inter-subband branch, associated with the E_0 and E_1 levels. The solution of the problem in the general case (for arbitrary plasmon momenta k) involves extremely complex numerical calculations, as neither the dispersion relations of electrons nor the form factors can be expressed analytically. Therefore, we limit ourselves to finding the threshold frequency $\omega_{01}(k=0)$, which determines the gap in the inter-subband plasmon spectrum. The difference between this value and the minimum energy gap between the E_0 and E_1 subbands is known as the depolarization shift.

If we retain only the equations for $m = 0,1$ and $n = 0,1$ in the system (3) and take the limit $k \to 0$, the right-hand side will only include the off-diagonal element φ, since the diagonal elements vanish due to the difference in occupation numbers approaching zero at $\psi\ 0, X(x)$, $\psi\ 1, X(x)$. In the same limit, the function $K_0(|k(x-x')|)$ simplifies to:

$$\ln(2e^{-\gamma}/|k(x-x')|) = \ln(2e^{-\gamma}/|k|l) + \ln(l/|x-x'|).$$

The first term does not contribute to the form factor $J_{01,01}$ due to the orthogonality of the wave functions $\psi\ 0,X(x)$, $\psi\ 1,X(x)$. As a result, we arrive at the equation

$$\Phi_{01}(X) = \frac{2\beta}{l^2}\int_{-X_0}^{X_0} dX' \times \frac{\Delta(X')}{\omega^2 - \Delta(X')^2} Q(X,X')\Phi_{01}(X'), \quad (13)$$

where $\Delta(X) = E_1(X) - E_0(X)$ and ω^2 is the desired eigenvalue (its minimum vakue is required, i.e. ω_{min}^2), and the kernel factor $Q(X,X')$ is equal to

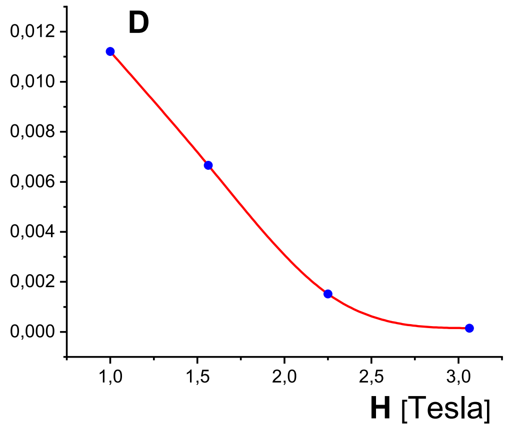

Fig. 2. Dependence of the depolarization shift of the inter-subband plasmon between levels 0 and 1 on the magnetic field; $D = \Omega/\Delta(X=0) - 1$, $N_L = 0.47 \cdot 10^6$ cm^{-1}, $L = 0.1$ μm.

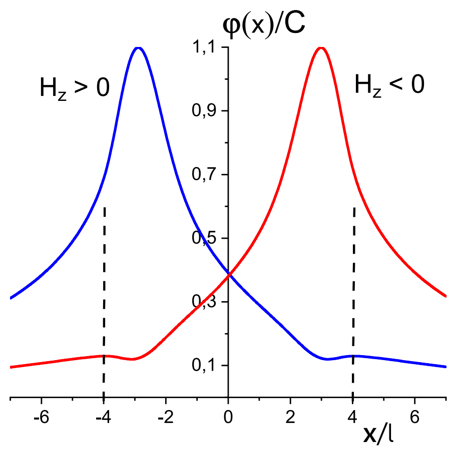

Fig. 3. Distribution of the plasmon wave potential across the transverse coordinate for two opposite propagation directions or magnetic field orientations; $N_L = 10^6$ cm^{-1}, $L = 0.2$ μm, $H = 1$ T.

$$Q(X,X') = \int_{-w}^{w}\int_{-w}^{w} dx dx' \tilde{\psi}_{0,X}(x)\tilde{\psi}_{1,X}(x) \times \\ \times \ln(l/|x-x'|)\tilde{\psi}_{0,X'}(x')\tilde{\psi}_{1,X'}(x'). \quad (14)$$

The value of ω_{min}^2 was found numerically. We replaced the integral with the corresponding Riemann sum by dividing the integration interval into a large number of points, reducing the problem to finding the eigenvalues of a system of linear homogeneous equations, the number of which equals the number of partition points. The depolarization shift Ω is defined as the difference between the minimum plasmon frequency ω_{min} and the minimum energy gap between the levels $\Delta(0)$. Its dependence on the magnetic field is shown in Fig. 2.

As is known, the depolarization shift also determines the frequency of IR absorption during an inter-subband (inter-level in an infinite plane) transition, which differs from the energy gap due to the dynamic screening of the electric field of the exciting wave.

6. SPATIAL DISTRIBUTION OF THE PLASMON WAVE FIELD

In this section, we derive the expression for the coordinate dependence of the plasmon potential $\varphi(x)$, corresponding to the zeroth subband, i.e., the lowest-frequency branch of the plasmon spectrum. Within the self-consistent field theory, $\varphi(x)$ satisfies the Poisson equation (quasistatic approximation, neglecting retardation), with the right-hand side containing the electron density perturbation induced by the plasmon wave. In the present case, we consider only the contribution from the zeroth subband:

$$\Delta_{x,z}\varphi_0(x,z,k) - k^2\varphi_0(x,z,k) = \\ = -\frac{4\pi e^2}{\varepsilon L_y}\delta(z)\sum_X \frac{f\left(E_0(X+kl^2)\right) - f\left(E_0(X)\right)}{E_0(X+kl^2) - E_0(X) + \omega + i\delta} \times \\ \times \Phi_{00}(X)\tilde{\psi}_{0,X}^2(x). \quad (15)$$

Equation (15) corresponds to a plasmon in the form of a plane wave Ce^{iky}, and the matrix element $\Phi_{00}(X)$ on the right-hand side is evaluated in the plane of the strip $z = 0$. The solution to Equation (15) is written using the Green's function $G(x - x')$, already defined in Section 2 for the plane $z = 0$. The resulting integral for $\varphi_0(x)$ in the long-wavelength limit and for $T = 0$ is evaluated similarly to the calculation of the plasmon frequency $\omega_0(k)$.

Now it is necessary to find the solutions of the system of two equations (6) for the matrix elements $\Phi_{00}(X)$ at the points $\pm X_0$. The result has the form (C is the wave amplitude determined by the excitation conditions):

$$\varphi_0(x) = Ck\beta\left(\frac{I_-(x)}{\omega_0(k) - kV_0} - \frac{RI_+(x)}{\omega_0(k) + kV_0}\right),$$

$$I_\pm(x) = \int_{-w}^{w} dx' K_0\left(|k(x-x')|\right)\tilde{\psi}_{0,\pm X_0}^2(x'), \quad (16)$$

$$R = \frac{\omega_0(k) + kV_0}{\omega_0(k) - kV_0}\frac{k\beta J_{++} - \omega_0(k) + kV_0}{k\beta J_{+-}}.$$

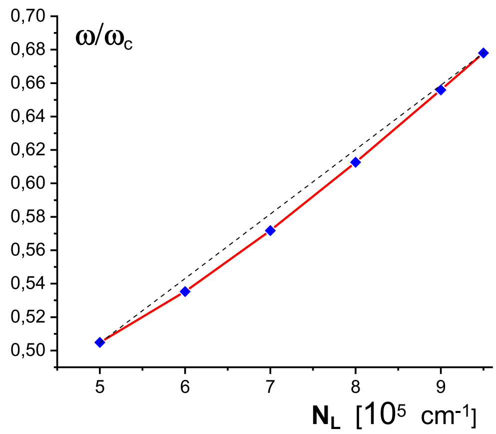

Fig. 4. Dependence of the plasmon frequency on the linear electron concentration. Magnetic field H = 1 T, strip width $L = 0.2$ μm.

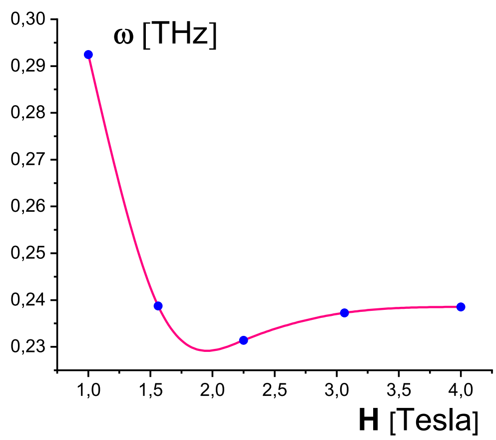

Fig. 5. Magnetic field dependence of the plasmon frequency; $N_L = 10^6$ cm^{-1}; $L = 0.2$ μm, $k = 0.4 \cdot 10^6$ cm^{-1}.

Fig. 3 shows the plasmon field $\varphi_0(x)$ for opposite propagation directions. As can be seen, for a given propagation direction, the maximum of $\varphi(x)$ is located near one edge of the strip. This result was previously obtained in [2] within the framework of the hydrodynamic approach.

The same mirror reflection occurs when the magnetic field direction is reversed: it is easy to see that under X, one should understand $-pl^2 sign(H)$, while $l^2 = c/|eH|$. Therefore, when the sign of H is changed, the points X_0 and $-X_0$ are swapped. This “reflection” of the plasmon field relative to the midline of the strip when the magnetic field sign is reversed is, in principle, accessible to experimental observation. When $l \ll w$ and the Fermi energy is such that the points $\pm X_0$ are close to the strip edges, the maximum of $\varphi(x)$ is also near one of the edges, and in this sense, such a wave can be called an edge magnetoplasmon.

7. DEPENDENCE ON CONCENTRATION AND MAGNETIC FIELD

The electron dispersion $E_0(p)$ (see Fig. 1) differs significantly from the standard parabolic law $p^2/2m$. Accordingly, all characteristics of the magnetoplasmon in the strip (the frequency dependence on electron concentration and magnetic field) appear unusual. For the intra-subband plasmon of the zeroth subband, the system is effectively one-dimensional, so $p_F = \pi N_L/2$, where N_L is the linear electron density (spin splitting is neglected), and $X_0 = \pi N_L l^2/2$. The dependence of E_F on V_0 is given by the right half of the lower curve in Fig. 1. The dependence of the plasmon frequency ω_0 on the linear density is determined by the Fermi velocity V_0 and the form factors X_0, appearing in formula (7). The results are presented in Fig. 4.

The dashed line in this figure is drawn to highlight the superlinear character of the dependence. Recall in this context that the classical 2D plasmon has a frequency that depends sublinearly on the surface density N_s:

$$\omega = (\omega_c^2 + \omega_p^2)^{1/2},$$

where $\omega_p^2 \propto N_s$.

The magnetic dispersion of the plasmon is even more unusual: the curve in Fig. 5 has a minimum at $H \approx 2$ T. This occurs because, as seen from (10), the dependence of the plasmon frequency on the magnetic field is due to two types of contributions. The terms containing the Fermi velocity V_0 provide the descending part of the curve in Fig. 5, as at a given density, the Fermi level rapidly decreases with increasing H and approaches the flat region of the electronic dispersion $E_0(p)$ where V_0 vanishes. Then the main contribution remains the first (Coulombic) term in (10), which leads to a logarithmically slow increase in the frequency.

For the depolarization shift (see Fig. 2), a rapid decrease is characteristic with a relatively small increase in H: more than an order of magnitude decrease at $\delta H / H = 75\%$. As the field increases, the behavior of the electron wave functions approaches that realized in an infinite plane, as the influence of the strip boundaries decreases. However, in an infinite plane, $\Omega = 0$, because in a strong magnetic field, screening (at least linear screening) is absent, along with the electron density perturbations linear in the perturbing potential.

8. CONCLUSION

We have demonstrated that the consideration of sample boundaries significantly affects the magnetoplasmonic oscillations of a two-dimensional electron gas. Mathematically, the problem becomes considerably more complex due to the non-standard dispersion law of "magnetized" electrons – the dependence of energy on the conserved momentum component in the Landau gauge. In the simple case of a straight strip, it is possible to analytically obtain only the dispersion of intra-subband plasmons in the long-wavelength limit, corresponding to the lower part of the plasmon spectrum, which generally contains an infinite number of branches. The dependence of the plasmon frequency on concentration and magnetic field was determined using numerical methods.

ACKNOWLEDGMENTS

The authors are grateful to V.A. Volkov and I.V. Kukushkin for their assistance in reviewing the relevant bibliography and to A.V. Nenashev for helpful advice during the numerical calculations.

APPENDIX

Here we demonstrate the validity of the relationship $J_{--} = J_{++}$.. For this, we need the expression for $\psi_{n,X}(x)$, which already accounts for the boundary conditions. It has the form:

$$\psi_{n,X}(x) = e^{-(x-X)^2/2l^2} \times \left[\Phi\left(-q_n(X)/2,1/2,(x-X)^2/l^2\right) - \Phi\left((1-q_n(X))/2,3/2,(x-X)^2/l^2\right) \times \frac{(x-X)\Phi\left(-q_n(X)/2,1/2,(w-X)^2/l^2\right)}{(w-X)\Phi\left((1-q_n(X))/2,3/2,(w-X)^2/l^2\right)}\right]. \quad (17)$$

Furthermore, we need the explicit form of the equation defining the electron spectrum, i.e., the parameter $q_n(X)$. For it, we have:

$$F(q_n(X)) = 0, \quad (18)$$

$$F(q) = \frac{\Phi\left(-q/2,1/2,(w-X)^2/l^2\right)}{(w-X)\Phi\left((1-q)/2,3/2,(w-X)^2/l^2\right)} + \frac{\Phi\left(-q/2,1/2,(w+X)^2/l^2\right)}{(w+X)\Phi\left((1-q)/2,3/2,(w+X)^2/l^2\right)}. \quad (19)$$

Using the explicit expressions for the form factors $J_{\pm\pm}$, we write the difference $J_{--} - J_{++}$ as:

$$J_{--} - J_{++} = \int_{-w}^{w}\int_{-w}^{w} dx dx' K_0(|k(x-x')|) \times \left[N_{0,-X}^4 \psi_{0,-X}^2(x)\psi_{0,-X}^2(x') - N_{0,X}^4 \psi_{0,X}^2(x)\psi_{0,X}^2(x')\right], \quad (20)$$

Here, $\psi_{n,X}(x)$ is defined in (17). By changing the integration variable in the first term within the square brackets, we arrive at the expression:

$$J_{--} - J_{++} = \int_{-w}^{w}\int_{-w}^{w} dx dx' K_0(|k(x-x')|) \times \left[N_{0,-X}^4 \psi_{0,-X}^2(-x)\psi_{0,-X}^2(-x') - N_{0,X}^4 \psi_{0,X}^2(x)\psi_{0,X}^2(x')\right]. \quad (21)$$

It is evident that to prove the equality $J_{--} = J_{++}$, it is sufficient to show that the relationships $\psi_{0,-X}(-x) = \psi_{0,X}(x)$ and $N_{0,-X} = N_{0,X}$. hold. Using (17), we obtain:

$$\psi_{0,-X}(-x) - \psi_{0,X}(x) = e^{-(x-X)^2/2l^2}(x-X) \times \Phi\left((1-q_0(X))/2,3/2,(x-X)^2/l^2\right) \times \left[\frac{\Phi\left(-q_0(X)/2,1/2,(w-X)^2/l^2\right)}{(w-X)\Phi\left((1-q_0(X))/2,3/2,(w-X)^2/l^2\right)} + \frac{\Phi\left(-q_0(X)/2,1/2,(w+X)^2/l^2\right)}{(w+X)\Phi\left((1-q_0(X))/2,3/2,(w+X)^2/l^2\right)}\right]. \quad (22)$$

Thus, the expression under the square brackets in (22) is the function $F(q_0(X))$, defined in (19), and therefore:

$$\psi_{0,-X}(-x) = \psi_{0,X}(x). \quad (23)$$

For $N_{0,-X}$, we have:

$$N_{0,-X} = \left(\int_{-w}^{w} dx \psi_{0,-X}^2 (x) \right)^{-1/2}.$$

Performing the variable change $x \to -x$ in the integral over x and considering (23), the evenness of the normalization coefficient with respect to X is thus proven.

REFERENCES

1. *V. A. Volkov, S. A. Mikhailov*, JETP Lett. **42**, 556 (1985) [Eksp. Teor. Fiz. Pis. Red. **42**, 450 (1985)].
2. *V. F. Volkov, S. A. Mikhailov*, Sov. Phys. JETP, **67**, 1639 (1988) [Zh. Eksp. Teor. Fiz **94**, 217 (1988)].
3. *O. G. Balev, V. Vasilopulos*, Phys. Rev. B **59**, 2807 (1999).
4. *I. L. Aleiner, D. Xue, and L. I. Glazman*, Phys. Rev. B **51**, 13467 (1995).
5. *G. Eliasson, J. Wu, P. Hawrylak, and J. J. Quinn*, Solid State Commun. **60**, 41 (1986).
6. *D. A. Rodionov, I. V. Zagorodnev*, JETP Letters **118**, 100 (2023) [Pis'ma Zh. Eksp. Teor. Fiz **118**, вып.2, 90 (2023)].
7. *A. H. MacDonald, P. Streda*, Phys. Rev.B **29**, 1616 (1984).
8. *R. Z. Vitlina, A. V. Chaplik*, Sov. Phys. JETP **54**, 536 (1981) [Zh. Eksp. Teor. Fiz **81**, 1011 (1981)].
9. *A. V. Chaplik and M. V. Krasheninnikov*, Surface Science **98**, 533 (1980).
10. *S. Das Sarma and W. Y. Lai*, Phys. Rev. B **32**, 1401 (1985).