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THE QUANTUM MODELS OF AN ELECTRON WITH THE ZERO SELF-ENERGY

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Abstract. We propose two models of an electron with the zero self-energy based on the updated regular charged Reissner-Nordström and Kerr-Newman metrics with quantum nuclei.

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1. INTRODUCTION

Since the advent of General Relativity (GR), attempts have been made to construct models of elementary particles in curved spacetime. Notable contributors to such models include G.B. Jeffery (1921), P. A.M. Dirac (1962), W. Israel (1970), C.A. López (1984), O. Gron (1984), A. Burinskii (1974–2023), and others. Unfortunately, none of the proposed models have found practical application in classical and quantum field theory calculations.

Another longstanding problem, which has engaged many researchers and is the focus of this paper, is the issue of the infinite self-energy of a charged particle in classical and quantum electrodynamics. Efforts to eliminate the linear divergence of self-energy in classical electrodynamics were made by H. Poincaré, M. Born, L. Infeld, P. A.M. Dirac, J. Wheeler, R. Feynman, and others. In quantum field theory, the renormalization procedure for fermion masses was developed to address the logarithmic divergence of self-energy.

Such efforts continue today. For example, in [1, 2], quantum electrodynamics demonstrates that the self-energy of a point charge converges when the nonlinearity of the theory is considered in any finite order of the Euler–Heisenberg Lagrangian expansion in powers of the electric field.

In this paper, using the electron as an example, we propose two quantum models of charged elementary particles with zero self-energy. By employing the quantum geometry of the Reissner–Nordström (RN) metric and neglecting extremely small gravitational coefficients, all practical calculations in classical and quantum electrodynamics can be conducted within the paradigm of elementary particles as point masses with electric charges.

Our approach is based on the phenomenological description of quantum black holes for modified Schwarzschild (Sq) and Reissner–Nordström (RNq) geometries [3, 4]. In this framework, black holes contain quantum cores described by coherent states of gravitons. The coherent-state-averaged solutions of the massless Klein–Gordon equation for longitudinal gravitons are equated, with certain coefficients, to classical potentials. Short wavelengths are eliminated by a graviton energy cut-off, introducing a maximum graviton energy:

$$k_{UV} = \frac{\hbar c}{R_S}. \quad (1)$$

For convenience, as in [3, 4], we introduce the parameter R_S. The primary quantity in this theory is the maximum graviton energy k_{UV}. The presence of a quantum core gives rise to quantum “hairs.” Quantum black holes thus possess quantum hairs.

In a future quantum theory of gravity, the graviton energy cut-off k_{UV} will be replaced by strict integration, and the absence of short wavelengths in graviton coherent states will naturally result from the application of a more advanced quantum theory.

In our previous work [5], we extended the approach of [3, 4] to modified M and Kerr–Newman (KNq) geometries, describing regular uncharged and charged quantum rotating collapsars. As with the RNq geometry, this term includes either black holes with quantum cores and event horizons or rotating quantum cores without event horizons.

In [5], for charged rotating collapsars with mass M, charge Q, and angular momentum J, we obtained full regularization of the KNq quantum metrics at the following parameter value:

$$R_S = R_S^{reg} = \frac{\pi}{8}\frac{Q^2}{Mc^2} \tag{2}$$

This regularization yielded finite values for key GR quantities, such as the mass function $m_{KNq}(r)$, $R_q^{\mu\nu}(r,\theta)$, the Kretschmann scalar $K_q(r,\theta)$, and others.

For $R_S = R_S^{reg}$, the total energy of the quantum charged rotating collapsar equals $E = Mc^2$, meaning its self-energy is zero. Due to the presence of a quantum core, the electromagnetic forces responsible for the collapsar's self-energy are counterbalanced by gravitational forces.

Similar results are obtained for the RNq quantum metric [4].

In Section 2, we propose two quantum electron models with zero self-energy based on RNq and KNq quantum geometries. Section 3 compares these models, favoring the RNq-based electron model. The conclusion summarizes the key findings of this paper.

The Appendix provides the procedure for calculating the energy of a charged rotating black hole with a quantum core (see [5]).

2. QUANTUM MODELS OF THE ELECTRON

Based on regular quantum models of charged rotating and non-rotating black holes [4, 5], we propose two quantum models of the electron with modified KNq and RNq metrics.

2.1 Modified Kerr–Newman geometry

For the electron model, we will use the Cürses-Cürsey metric [6][1]:

$$ds_{KNq}^2 = \left(1 - \frac{2rm_{KNq}^e(r)}{\rho^2}\right)dt^2 + \frac{4a_e r m_{KNq}^e(r)\sin^2\theta}{\rho^2}dtd\varphi - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \frac{\Sigma\sin^2\theta}{\rho^2}d\varphi^2, \tag{3}$$

where $m_{KNq}^e(r)$ is the mass function,

$$\rho^2 = r^2 + a_e^2\cos^2\theta, \tag{4}$$

$$\Delta = r^2 - 2rm_{KNq}^e(r) + a_e^2, \tag{5}$$

$$\Sigma = \left(r^2 + a_e^2\right)^2 - a_e^2\Delta\sin^2\theta, \tag{6}$$

$$a_e = \frac{|J_e|}{m_e} = \frac{\hbar}{2m_e}. \tag{7}$$

In equation (7), m_e is the electron mass, and $|J_e| = \hbar/2$ is the electron spin.

In general, for a black hole with mass M, charge Q, and angular momentum J, the mass functions $m(r)$ for both classical and quantum Kerr (K) and Kerr–Newman (KN) metrics do not depend on the spin parameter $a = J/M$ and are therefore equal to the mass functions for the classical and quantum Schwarzschild and Reissner–Nordström metrics.

For the electron, the quantum mass function is

$$m_{KNq}^e(r) = m_{RNq}^e(r) = = Gm_e\frac{2}{\pi}Si\left(\frac{k_{UV}^e}{\hbar c}r\right) - \frac{Ge^2}{2r}\left[1 - \cos\left(\frac{k_{UV}^e}{\hbar c}r\right)\right] = = Gm_e\frac{2}{\pi}Si\left(\frac{r}{R_S^e}\right) - \frac{Ge^2}{2r}\left[1 - \cos\left(\frac{r}{R_S^e}\right)\right]. \tag{8}$$

where $Si(x) = \int_0^x \frac{\sin x'}{x'}dx'$ is the sine integral function. According to equation (2),

$$R_S^e = \frac{\pi}{8}\frac{e^2}{m_e c^2} = 1.11\cdot 10^{-13}\text{ cm}. \tag{9}$$

According to equation (1), the maximum (cut-off) energy of gravitons is

$$k_{UV}^e = \frac{\hbar c}{R_S^e} = 178\text{ MeV}.$$

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[1] Below we will use units with the velocity of light $c = 1$. When calculating the numerical values of the theory parameters, we will use the value $c = 3\cdot 10^{10}$ cm/s.

The asymptotics of the quantum mass function (8) are

$$m^e_{KNq}\Big|_{r\to\infty} = G m_e, \quad (10)$$

$$m^e_{KNq}\Big|_{r\to 0} = \frac{1}{18}\frac{G m_e}{\pi}\left(\frac{r}{R^e_S}\right)^3 \to 0. \quad (11)$$

According to equation (10), the quantum KN metric becomes asymptotically flat as $r \to \infty$.

For the classical KN metric, the mass function $m^{cl}_{KN} = 0$ at $r_e = e^2 / 2m_e$, i.e. at $r = r_e$, the classical metric is flat in this limit [7]. For the quantum Kq and KNq metrics, the spacetime curvature persists throughout the entire interval $r \in (0,\infty)$ [5].

2.2 Modified Reissner–Nordström Geometry

The quantum RNq metric [4] can be obtained from equation (3) by setting $a_e = 0$:

$$ds^2_{RNq} = \left(1 - \frac{2m^e_{RNq}(r)}{r}\right)dt^2 -$$

$$-\frac{1}{1-\frac{2m^e_{RNq}(r)}{r}}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (12)$$

where $m^e_{RNq}(r)$ is given in equation (8).

The quantum RNq metric is asymptotically flat as $r \to \infty$ (see equation (10)). The $g_{00} = -1 / g_{11}$ component at $r \to 0$ is

$$g_{00} = 1 - \frac{G m_e}{9\pi c^2 R^e_S}\left(\frac{r}{R^e_S}\right)^2 =$$

$$= 1 - 2.15\cdot 10^{-44}\left(\frac{r}{R^e_S}\right)^2, \quad (13)$$

meaning that the metric (12) becomes flat at $r = 0$.

2.3 Characteristics of electron models

Let's present some characteristic values for the electron:

$$m_e = 9.1\cdot 10^{-28}\ \text{g},\ e^2 = 2.31\cdot 10^{-19}\ \text{erg}\cdot\text{sm},$$

$$\text{spin}: \frac{\hbar}{2} = 0.5\cdot 1.054\cdot 10^{-27}\ \text{erg}\cdot\text{sm},$$

$$G = 6.67\cdot 10^{-8}\ \frac{\text{cm}^3}{\text{g}\cdot\text{s}^2},\ c = 3\cdot 10^{10}\ \frac{\text{sm}}{\text{s}},$$

$$R^e_H = \frac{2G m_e}{c^2} = 1.35\cdot 10^{-55}\ \text{cm},$$

$$\frac{G e^2}{c^4} = \left(1.38\cdot 10^{-34}\right)^2\ \text{cm}^2,$$

$$a_e^2 = \left(\frac{\hbar}{2m_e c}\right)^2 = \left(1.93\cdot 10^{-11}\right)^2\ \text{cm}^2,$$

$$\beta_1 = \frac{G e^2}{c^4}\frac{4}{\left(R^e_H\right)^2} = 4.2\cdot 10^{42},$$

$$\beta_2 = \frac{4a_e^2}{\left(R^e_H\right)^2} = 8.2\cdot 10^{88},\ \text{т.е. } \beta_1 + \beta_2 \gg 1,$$

$$R_{cl} = \frac{e^2}{m_e c^2} = 2.82\cdot 10^{-13}\ \text{cm},$$

$$R^e_S = \frac{\pi}{8}\frac{e^2}{m_e c^2} = 1.11\cdot 10^{-13}\ \text{cm},$$

$$k^e_{UV} = \frac{\hbar c}{R^e_S} = 178\ \text{MeV},$$

$$\frac{R^e_S}{R^e_H} = \frac{1.11\cdot 10^{-13}}{1.35\cdot 10^{-55}} = 0.82\cdot 10^{42}.$$

We see that for the electron, $\beta_1 + \beta_2 \gg 1$, $R^e_S / R^e_H \gg 1$. This means that in the models of electron with the RNq and KNq quantum metrics, the event horizons are absent [8]. The proposed electron models represent either rotating (KNq) or non-rotating (RNq) collapsars without event horizons and with quantum cores defined by coherent states of gravitons with a maximum energy of $k^e_{UV} = 178$ Me V.

2.4 Electromagnetic potentials

For the classical Reissner–Nordström and Kerr–Newman metrics with mass M and charge Q, the mass function consists of two terms:

$$m^{cl}(r) = \left(m^{cl}(r)\right)_M + \left(m^{cl}(r)\right)_Q = G M - \frac{G Q^2}{2r}. \quad (14)$$

The "charge" part of the mass function

$$\left(m^{cl}(r)\right)_Q = -G Q^2/2r$$

ensures that the "charge" components of the Einstein tensor, divided by $8\pi G$, match the corresponding components of the electromagnetic field energy-momentum tensor derived from Maxwell's equations:

$$\frac{\left(G_\mu{}^\nu\right)_Q}{8\pi G} = \left(T_\mu{}^\nu\right)_{em}.$$

For the classical KN geometry, the electromagnetic potentials A_μ are chosen as follows [9]:

$$A_\mu = \frac{Qr}{\rho^2}\left(1,0,0,-a\sin^2\theta\right). \quad (15)$$

Electromagnetic fields at $r \to \infty$ manifest as a superposition of the Coulomb field and the magnetic dipole field $\mu = Qa$. The gyromagnetic ratio $\mu / |J| = Q/m$, which coincides with the gyromagnetic ratio for a Dirac electron. The complex internal electromagnetic structure of the classical KN metric source is discussed, for example, in [10].

For the classical Reissner–Nordström (RN) metric, when $(a = 0)$ in equation (15), only the scalar Coulomb potential remains $A_0 = Q/r$.

For the regular quantum electron metrics (considering the relation between m_e and e^2 from equation (9)), the "charge" part of the mass function can be retained as in the classical RN and KN metrics. In this case, the mass function (8) becomes:

$$m^e_{RNq}(r) = m^e_{KNq}(r) =$$
$$= G m_e\left[\frac{2}{\pi}\mathrm{Si}\left(\frac{r}{R_S^e}\right) + \frac{4}{\pi}\frac{\cos\left(r/R_S^e\right)}{r/R_S^e}\right] - \frac{Ge^2}{2r}. \quad (16)$$

Thus, the electromagnetic properties of the proposed electron models coincide with the electromagnetic properties of the sources of the classical Reissner–Nordström and Kerr–Newman metrics.

2.5 Electron's self-energy

In the study [5], we established that for

$$R_S = R_S^{reg} = \pi Q^2/8M$$

the energy of a rotating charged quantum black hole equals $E = M$ (see also the Appendix). A similar equality holds for the RNq quantum metric at any value of R_S. For electron models in natural units:

$$R_S^e = \pi e^2/8m_e c^2 = 1.11\cdot 10^{-13}\,\mathrm{cm}.$$

The equality $E = m_e$ means that the electron's self-energy E_{em} is zero.

3. DISCUSSION

We have examined two quantum models of the electron based on modified Reissner–Nordström (RNq) and Kerr–Newman (KNq) metrics. Can we currently favor one model over the other? To answer this question, let us compare some characteristics of the considered models under the condition

$$R_S = R_S^e = \frac{\pi e^2}{8m_e}.$$

Table: Comparison of electron model characteristics in Reissner–Nordström (RNq) and Kerr–Newman (KNq) quantum geometries

	Electron model characteristic	RNq	KNq
1	$E_e = m_e$, $E_{em} = 0$	+	+
2	Weak energy condition	+	–
3	$\lvert J\rvert = \frac{\hbar}{2}$, Dirac gyromagnetic ratio $\frac{\mu}{\lvert J\rvert} = \frac{e}{m_e}$	–	+
4	Absence of event horizons	+	+
5	Finiteness of the GRT quantities, such as the mass function, Ricci tensor, Kretschmann scalar, etc.	+	+
6	Compatibility with the Maxwell equations	+	+
7	Stationary bound states in the fields of regular black holes	+	–

In the table, the symbols "+" and "–" indicate the presence or absence of key characteristics in the considered models.

Let us briefly discuss points 1–7 of the table.

Point 1. For both models:

$$E_e = m_e c^2, E_{em} = 0.$$

We found an important aspect: gravity in the charged quantum Kerr–Newman (rotating) and Reissner–Nordström (non-rotating) metrics with $R_S = R_S^e$ compensates for the electromagnetic component in the expressions for the total energy of the quantum black hole.

In classical electrodynamics, the self-energy of a charged particle $E_{em}^{cl} = e^2/2r$ diverges linearly as $r \to 0$. In quantum field theory, the self-energy of a charged particle is determined by an infinite series in perturbation theory with logarithmic divergence terms.

Point 2. For the RNq quantum geometry, the energy density $\rho_\varepsilon(r)$, radial pressure $p_1(r)$, and stresses $p_2(r) = p_3(r)$ take the following form [4]:

$$\rho_\varepsilon(r) = -p_1(r) = \frac{m_e}{\pi^2\left(R_S^e\right)^3}\times$$

$$\times\left[\frac{1}{\left(r/R_S^e\right)^4}\left(1-\cos\left(\frac{r}{R_S^e}\right)\right)-\right.$$

$$\left.-\frac{1}{2\left(r/R_S^e\right)^3}\sin\left(\frac{r}{R_S^e}\right)\right], \quad (17)$$

$$p_2(r)=p_3(r)=\frac{m_e}{\pi^2\left(R_S^e\right)^3}\times$$

$$\times\left[\frac{1}{\left(r/R_S^e\right)^4}\left(1-\cos\left(\frac{r}{R_S^e}\right)\right)+\frac{1}{4\left(r/R_S^e\right)^2}\cos\left(\frac{r}{R_S^e}\right)-\right.$$

$$\left.-\frac{3}{4\left(r/R_S^e\right)^3}\sin\left(\frac{r}{R_S^e}\right)\right]. \quad (18)$$

At $r\to 0$, we have $\rho_\varepsilon(r)\to K/24$, $p_i(r)\to -K/24$, where $i=1,2,3$ and $K=m_e\big/\pi^2\left(R_S^e\right)^3$. Thus, for the RNq quantum geometry near $r=0$, the weak energy condition $\rho_\varepsilon\geq 0$, $\rho_\varepsilon+p_i\geq 0$, $i=1,2,3$ is satisfied.

Specifically, equations (17) and (18) show that at $r=0$ $\rho_\varepsilon=K/24$, $\rho_\varepsilon+p_i=0$, $i=1,2,3$.

For the RNq quantum geometry at $r=0$, the energy dominance condition $\rho_\varepsilon\geq|p_i|$, $i=1,2,3$ also holds. In our case: $\rho_\varepsilon=|p_i|$.

For the Kerr–Newman quantum geometry, the asymptotics of the energy density $\rho_\varepsilon(r,\mu)$ at $r\to 0$ follow from equation (7) in [5] (here and below, $\mu=\cos\theta$):

$$\rho_\varepsilon(r,\mu)=\frac{K}{12}\frac{\mu^2-1}{\mu^4}\left(\frac{r}{a_e}\right)^2, \mu\neq 0,$$
$$\rho_\varepsilon(r,\mu)=84K, \mu=0. \quad (19)$$

At $\mu\neq 0,\pm 1$ the energy density near $r=0$ is negative. In this case, none of the energy conditions are satisfied.

Point 3. In the KNq quantum model, it is possible to introduce the spin modulus $|J|=\hbar/2$, satisfying the Dirac gyromagnetic ratio. However, introducing the quantum spin operator $S=(\hbar/2)\sigma$ is complicated when the classical definition of angular momentum is used in the Kerr–Newman geometry. Above,, σ_i are two-dimensional Pauli matrices.

In the RNq quantum geometry, the angular momentum J is zero. In the RNq electron quantum model, the spin operator S and the gyromagnetic ratio e/m_e are pure quantum properties defined externally.

Point 4. In both these models, event horizons are absent.

Point 5. In both models, general relativity (GR) quantities such as the mass function, Ricci tensor, Kretschmann scalar, and others remain finite.

Point 6. The RNq and KNq quantum geometries are consistent with Maxwell's equations (see Section 2.4 of this study). However, the electromagnetic structure of the RNq model is significantly simpler than that of the KNq model. In the RNq quantum model, the source of the electromagnetic field is a point electric charge e located at the system's center $(r=0)$. At large distances, the electromagnetic field behaves as a Coulomb field.

In contrast, the source of the electromagnetic field in the KNq quantum model is a system of surface currents and electric charges distributed over a disk of radius $a_e=|J_e|/m_ec$ with the center at $r=0$ [10]. For $r\to\infty$, the electromagnetic field is a superposition of the Coulomb field and a magnetic dipole $\mu=ea$.

Point 7. In the RNq quantum geometry, the metric (12) becomes asymptotically flat as $r\to\infty$. Importantly, for both $R_S=R_S^e$ and $r\to 0$, the metric (12) is also flat (see Equation (13)). In this case, the problem of determining the eigenfunctions and eigenvalues of the Dirac equation for motion of fermions in the RNq fields can be solved by using single-valued boundary conditions from the analogous problem for the fermion motion in the Coulomb field in flat Minkowski space.

In the Kerr–Newman quantum geometry, the situation is different. At $r\to 0$ и $R_S=R_S^e$, the metric (3) remains non-flat and takes the following form:

$$ds_{KNq}^2=dt^2-\cos^2\theta dr^2-a_e^2\cos^2\theta d\theta^2-$$
$$-a_e^2\sin^2\theta d\varphi^2. \quad (20)$$

In [11, 12], it was shown that in this case, the Dirac equation has two quadratically integrable solutions, making it impossible to formulate a well-defined eigenvalue problem for fermions in the classical or quantum KN spacetime.

To establish a well-defined quantum mechanical problem, one must perform a self-adjoint extension of the Hamiltonian, which usually results in new boundary conditions near $r = 0$ (see, for example, [13, 14]).

4. CONCLUSION

We proposed two quantum electron models with zero self-energy based on the Reissner–Nordström [4] and Kerr–Newman [5] quantum geometries. A critical parameter for regularizing key GR quantities is the choice of $R_S^e = \pi e^2/8m_e c^2 \simeq 1.11\cdot 10^{-13}$ cm, where the cut-off energy of gravitons $k_{UV}^e = \hbar c/R_S^e \approx 178$ Me V.

The proposed models solve the long-standing problem of linear divergence in the self-energy of a charged particle in classical electrodynamics. In the considered models, gravity compensates for the electromagnetic component in the total energy expressions for the electron.

It can be hypothesized that with more advanced quantum gravity theories, the problem of infinite self-energy of charged fermions in quantum field theory will be resolved similarly.

Notably, when using the RNq quantum electron model, all classical and quantum electrodynamics effects can be calculated within the standard paradigm of an elementary particle with point mass m_e and electric charge $e < 0$. This is due to the extremely small values of the parameters $Gm_e/c^2 \simeq 0.7\cdot 10^{-55}$ cm and $Ge^2/c^4 \simeq 1.9\cdot 10^{-68}$ cm^2 in Equation (16) for the mass function $m_{RNq}^e(r)$.

As a result of neglecting the coefficients $\frac{Gm_e}{c^2}$ and $\frac{Ge^2}{c^4}$ the RNq geometry becomes the flat Minkowski space-time. In this case, we return to the domain of classical and quantum electrodynamics for charged leptons within the Standard Model.

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APPENDIX: ENERGY OF A CHARGED ROTATING BLACK HOLE WITH A QUANTUM CORE [5]

For the KN quantum metric, the total energy, defined by the volume integral of the energy density $T_0^{\ 0} \equiv \rho_\varepsilon(r,\theta)$, is given by:

$$E = \int T_0^{\ 0}\sqrt{-g}\,dV = \frac{1}{4G}\int_0^\infty dr\int_{-1}^{1} d\propto\left(r^2 + a^2\propto^2\right)G_0^{\ 0}(r,\propto) =$$

$$= \frac{1}{4G}\int_0^\infty dr\int_{-1}^{1} d\mu\left[2\frac{r^4 + \left(\rho^2 - r^2\right)^2 + a^2\left(2r^2 - \rho^2\right)}{\rho^4}m'_{KN_q} - \frac{ra^2\left(1-\mu^2\right)}{\rho^2}m''_{KN_q}\right] =$$

$$= \frac{1}{4G}\int_0^\infty dr\left\{\left[8 - 4\frac{r}{a}\operatorname{arctg}\frac{a}{r}\right]\left[GM\frac{2}{\pi}\frac{\sin(r/R_S)}{r} + \frac{CQ^2}{2r^2}\left(1-\cos\left(\frac{r}{R_S}\right)\right) - \frac{CQ^2}{2rR_S}\sin\left(\frac{r}{R_S}\right)\right] + \right.$$

$$+\left[2r - 2\frac{r^2}{a}\operatorname{arctg}\frac{a}{r} - 2a\operatorname{arctg}\frac{a}{r}\right]\left[GM\frac{2}{\pi}\frac{\cos(r/R_S)}{r/R_S}\frac{1}{R_S^2} - GM\frac{2}{\pi}\frac{\sin(r/R_S)}{(r/R_S)^2}\frac{1}{R_S^2} - \frac{CQ^2}{r^3}\left(1-\cos\left(\frac{r}{R_S}\right)\right) + \right.$$

$$\left.\left. + \frac{CQ^2}{r^2R_S}\sin\left(\frac{r}{R_S}\right) - \frac{CQ^2}{2rR_S^2}\cos\left(\frac{r}{R_S}\right)\right]\right\} = M + \frac{1}{2}\frac{|J|}{R_S} - \frac{\pi}{16}\frac{Q^2|J|}{M}\frac{1}{R_S^2} = M + \frac{1}{2}\frac{|J|}{\hbar}k_{UV} - \frac{\pi}{16}\frac{Q^2|J|}{M}\frac{k_{UV}^2}{\hbar^2}.$$

For the K and KN metrics:

$$\sqrt{-g} = \rho^2 \sin\theta; \rho^2 = r^2 + a^2\mu^2, \mu = \cos\theta;$$

$$m'_{KNq} \equiv \frac{dm_{KNq}}{dr}, m''_{KNq} \equiv \frac{d^2 m_{KNq}}{dr^2},$$

$$m_{KNq} = GM\frac{2}{\pi}Si\left(\frac{r}{R_S}\right) - \frac{GQ^2}{2r}\left(1 - \cos\left(\frac{r}{R_S}\right)\right).$$

When the condition

$$R_S = R_S^{reg} = \frac{\pi}{8}\frac{Q^2}{Mc^2}$$

is satisfied, the total energy of the quantum charged rotating collapsar equals zero: $E = Mc^2$.

Under this condition, the key general relativity (GR) quantities, such as the mass function $m(r)$, the Ricci tensor $R_{\mu\nu}(r,\theta)$, and the Kretschmann scalar $K(r,\theta)$, become regular and finite throughout the entire spacetime.

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