JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS, 2025, Vol. 167, No. 1, pp. 42–45

NUCLEI, PARTICLES, FIELDS, GRAVITATION, AND ASTROPHYSICS

CORRECTIONS FROM NON-LOCAL GRAVITY FOR BLACK HOLE SHADOW IMAGES

© 2025 S. O. Alexeyev[a, c], A. V. Nemtinova[b], O. I. Zenin[c,]*, A. A. Baiderin[c]

[a]Sternberg Astronomical Institute, Lomonosov Moscow State University, Moscow, Russia

[b]Ural Federal University named after the First President of Russia B.N. Eltsin, Yekaterinburg, Russia

[c]Quantum Theory and High Energy Physics Department, Physics Faculty, Lomonosov Moscow State University, Moscow, Russia

*e-mail: dkiiiabu4@gmail.com

Received August 22, 2023

Revised September 13, 2024

Accepted September 14, 2024

Abstract. The work presents the necessary conditions for the "bounce" of the scale factor existence (except Big Bang at the initial moment of the Universe). We study rather wide range of parameter values. This fact seems to be significant both for the further construction of the theory of quantum gravity and for the consideration of subsequent cosmological evolution based on this model.

Keywords: *bounce, Fab Four, Horndeski model, scale factor, quantum gravity, scalar field*

DOI: 10.31857/S00444510250103e2

1. INTRODUCTION

Currently, General Relativity (GR) accurately explains nearly the entire body of astronomical data. However, from the very first cosmological solutions [1], Einstein's equations must necessarily include the energy-momentum tensor on the right-hand side. One approach is that the entire array of modern astrophysical data is well described by GR equations, and theories of gravity extending GR in various ways [2, 3, 4, 5, 6] are developed specifically to explain the physical nature of the right-hand side and its source.

One promising direction for extending GR has been scalar-tensor theories of gravity, where, as the name suggests, physical fields are included alongside geometric terms and curvature invariants. To address the issue of higher-order differential equations, theories have been constructed where higher degrees mutually cancel out, with the most general example of this approach being the Horndeski model [7, 8]. Despite significant constraints on the Horndeski model from gravitational-wave astronomy data [9, 10], interest in it (and theories derived from it that pass the GW170817 test) remains strong. This model has also been used to create nonsingular cosmology models, where the initial singularity is replaced by a "bounce" of the scale factor [11, 12]. This approach appears promising, and within the Horndeski framework, models known as the "Fab Four" were proposed, where the corrections themselves, without additional tuning parameters like the cosmological constant (Λ), ensure the accelerated expansion of the Universe [13, 14]. Nonsingular cosmological solutions within the Fab Four model, as an example of a scalar-tensor theory with a simpler structure than the general Horndeski theory, have also been discussed earlier [15].

The idea of adding quantum-field corrections to gravity models [16] allows, for example, the limitation of nonlocality size in gravity theories at the quantum limit [17]. This approach was also applied to the Fab Four model [18], and the additional inclusion of quantum-field corrections ensures that the speed of gravitational wave propagation now matches the experimental results of gravitational-wave astronomy. All of this highlights the potential of scalar-tensor models.

Therefore, we consider a nonminimal effective model of scalar-tensor gravity with third- and fourth-order field terms, formed by summing one-loop interactions [19] in the form:

$$S = \int \sqrt{-g}\left[\left(\frac{2}{\kappa^2} + \alpha\phi^2\right)R + \kappa^2\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \right.$$
$$\left. -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{3!}\lambda\phi^3 - \frac{1}{4!}g\phi^4\right]d^4x, \qquad (1)$$

where $\kappa^2 = 32\pi G$, G is the Newtonian constant, ϕ is the new scalar field, R is the scalar curvature, α and β are dimensionless constants, λ is the cubic scalar coupling with mass dimension, g is the dimensionless fourth-order scalar coupling, and $G_{\mu\nu}$ is the Einstein tensor $\left(G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)$. Despite its "extended" nature, this model remains significantly simpler than the standard version of the Horndeski or DHOST theory, increasing interest in its potential to explain dark energy and early Universe processes. To further analyze the applicability of this model to early Universe evolution, it is necessary to study its predictions for bounce and genesis realization [20]. This paper is dedicated to the first step in this direction – investigating the conditions for bounce existence. It is important to note that the absence of an initial singularity in the cosmological model significantly increases its appeal. For example, consider the search for parameter spaces where a "bounce" occurs [21] in second-order curvature correction gravity – the Gauss-Bonnet model [22, 23], one of the candidates for the semiclassical limit of string gravity [24]. Moreover, the bounce already appears with the simple addition of a scalar field, as in the Brans-Dicke model [25]. Thus, the presence of a nonsingular asymptotic solution in the considered theory serves as an additional argument for its relevance. As the first step in examining the strengths and weaknesses of the theory (1), we investigate this issue. Since additional constraints on the theory's parameters were previously proposed to pass astronomical tests (discussed at the end of Section 3) [19], it is of interest to compare these constraints with those imposed by the bounce requirement.

This paper is structured as follows. Section 2 derives the field equations for the theory proposed in [19]; Section 3 explores the parameter space constraints imposed by the bounce requirement; and Section 4 discusses the results and conclusions.

2. FIELD EQUATIONS

The Klein-Gordon equations are obtained by varying the action (1) with respect to the scalar field. Following [26], we have:

$$-\frac{1}{2!}\lambda\phi^2 - \frac{1}{3!}g\phi^3 + \Box\phi +$$
$$+2\alpha\phi R - 2\kappa^2\beta G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0. \qquad (2)$$

Varying with respect to the metric tensor and introducing the effective gravitational constant $G_{eff}(\phi)$, which depends only on the scalar field, gives:

$$\frac{2}{\kappa^2} + \alpha\phi^2 = \frac{1}{16\pi G_{eff}(\phi)}. \qquad (3)$$

As a result, Einstein's equation takes the form:

$$\mathcal{G}_{\mu\nu} = \frac{1}{16\pi G_{eff}}G_{\mu\nu} - \left(\nabla_{\mu\nu} - g_{\mu\nu}\Box\right)\alpha\phi^2 - \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi -$$
$$-\frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + \frac{1}{3!}\lambda\phi^3 + \frac{1}{4!}g\phi^4\right) -$$
$$-\kappa^2\beta\left(-\nabla_\lambda\nabla_\mu\phi\nabla^\lambda\nabla_\nu\phi + \nabla_\mu\nabla_\nu\phi\Box\phi - \right.$$
$$-R_{\alpha\mu\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi - \frac{1}{2}\left[\nabla_\mu\phi G_{\nu\lambda}\nabla^\lambda\phi + \right.$$
$$\left. +\nabla_\nu\phi G_{\mu\lambda}\nabla^\lambda\phi\right] - \frac{1}{2}\left[\nabla_\mu\phi R_{\nu\lambda}\nabla^\lambda\phi + \right.$$
$$\left. +\nabla_\nu\phi R_{\mu\lambda}\nabla^\lambda\phi\right] + g_{\mu\nu}\left[R^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - \right.$$
$$\left.\left. -\frac{1}{2}(\Box\phi)^2 + \frac{1}{2}(\nabla_{\alpha\beta}\phi)^2\right]\right) = \frac{1}{2}T_{\mu\nu}, \qquad (4)$$

where $T_{\mu\nu}$ is the effective energy-momentum tensor:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}, \qquad (5)$$

Here L_m is the matter Lagrangian.

3. COSMOLOGICAL SOLUTION WITH A "BOUNCE"

Following [22, 23], we consider an isotropic (Friedmann-like) cosmological solution of the form:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \qquad (6)$$

where both the scale factor a, and the scalar field ϕ depend only on the time coordinate t.

To study the behavior at the bounce point, we examine the system (2)–(4). At the bounce point, the scale factor must be positive and finite, i.e., $a = const > 0$. To ensure the scale factor reaches a minimum at the bounce point and to avoid a cosmological singularity $a = 0$ at any other point, it is necessary that $\dot{a} = 0$ and $\ddot{a} > 0$. With this, Einstein's equations at the bounce point can be rewritten as:

$$\frac{3}{4}\dot{\phi}^2 = -\frac{1}{12}\lambda\phi^3 - \frac{1}{48}g\phi^4, \quad (7)$$

$$-2\frac{\ddot{a}}{a}\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) - 2\alpha\phi\ddot{\phi} + \frac{1}{4}\dot{\phi}^2 - -5\kappa^2\beta\frac{\ddot{a}}{a}\dot{\phi}^2 + \frac{1}{12}\lambda\phi^3 + \frac{1}{48}g\phi^4 = 0, \quad (8)$$

The Klein-Gordon-Fock equation (2) takes the form:

$$\ddot{a} = \frac{a}{12\alpha\phi}\left(\ddot{\phi} - \frac{1}{2}\lambda\phi^2 - \frac{1}{6}g\phi^3\right). \quad (9)$$

If we consider the case where the energy-momentum tensor is represented by the scalar field, its absence would imply the absence of a nontrivial cosmological solution: $\phi = 0 \Rightarrow a = 0$. Since this would lead to the singularity we aim to avoid, we introduce the additional conditions:

$\dot{\phi} = 0$, $\phi = const > 0$ and $\ddot{\phi} > 0$.

From equation (8) and (9), we obtain an equation for the scalar field:

$$\phi = -4\frac{\lambda}{g}.$$

From equations (8) and (9), we derive an expression for the second derivative of the scalar field:

$$\ddot{\phi} = -\frac{\lambda}{36\alpha}\left(\frac{\frac{1}{\alpha\kappa^2} + 8\frac{\lambda^2}{g^2}}{1 + \frac{1}{12\alpha} + \frac{g^2}{96\kappa^2\alpha^2\lambda^2}}\right).$$

The final system of inequalities (after substituting into (9) with equations (7) and (8)) is:

$$\phi = -4\frac{\lambda}{g} > 0, \quad (10)$$

$$\ddot{\phi} = -\frac{\lambda}{36\alpha}\left(\frac{\frac{1}{\alpha\kappa^2} + 8\frac{\lambda^2}{g^2}}{1 + \frac{1}{12\alpha} + \frac{g^2}{96\kappa^2\alpha^2\lambda^2}}\right) > 0, \quad (11)$$

$$a > 0, \quad (12)$$

$$\ddot{a} = \frac{ag}{1728\alpha^2}\left(\frac{\frac{1}{\alpha\kappa^2} + 8\frac{\lambda^2}{g^2}}{1 + \frac{1}{12\alpha} + \frac{g^2}{96\kappa^2\alpha^2\lambda^2}}\right) + \frac{a\lambda^2}{18\alpha g} > 0. \quad (13)$$

From inequality (10), we obtain that λ and g must have opposite signs. It is also necessary for the stability of the model that $g > 0$. Otherwise, the scalar potential would be unbounded from below, rendering the model unstable. From inequality (11), it follows that $\lambda < 0$, then $\alpha > 0$.The final inequality (13) is automatically satisfied under conditions (10)–(12). We can also consider the case $\alpha < 0$. From (13), we obtain:

$$\left(\frac{\frac{1}{\alpha\kappa^2} + 8\frac{\lambda^2}{g^2}}{1 + \frac{1}{12\alpha} + \frac{g^2}{96\kappa^2\alpha^2\lambda^2}}\right) > -\frac{1728\alpha\lambda^2}{18g^2} > 0.$$

This implies that the expression inside the parentheses is positive. Thus, condition (11) also holds if $\lambda > 0$ and $g < 0$. However, this condition contradicts the necessary stability condition of the model. Therefore, these conditions are not suitable for the given problem.

4. CONCLUSION AND FINDINGS

In the non-minimal effective model of scalar-tensor gravity with third- and fourth-order field terms formed by summing one-loop interactions [19], the realization of a "bounce" solution is possible. The necessary conditions for the realization of the bounce solution are as follows: parameters $\lambda < 0$, $g > 0$ and $\alpha > 0$. A similar model was previously studied in [27], where $\alpha = 0$, the scalar field ϕ was absent, but the cosmological constant Λ was present, ensuring the same effect. The bounce solution is realized under the conditions $\Lambda = 0$ (although the case when $\lambda = g = 0$ is not possible in our model), $\rho = 0$ (similarly, in our case, the volume density is zero), $a_0 > 0$ (in our case, the scale factor $a > 0$) and $\beta < 0$ (which does not contradict our conditions). Thus, our results partially coincide with those previously obtained for a simpler version of the discussed model, except for the zero value of the cosmological constant and the parameter α (which was initially zero in the simpler version of the theory).

Thus, in the discussed scalar-tensor gravity model, instead of an initial singularity, a bounce is possible even in the simplest configuration, provided the initial constraints are met. This means that the model, with a simpler structure than most scalar-tensor models based on Horndeski's theory, not only solves the initial singularity problem but also brings us closer to the development of quantum gravity while offering the potential for the realization of both bounce and genesis scenarios.

FUNDING

The work of O.I.Z. was funded by the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS," grant 22-2-2-11-1.

REFERENCES

1. *A. Friedmann*, *Über die Krümmung des Raumes (About the curvature of space)*, Z. Phys. **10**, 377 (1922).

2. *S. Alexeyev, E. Pamyatnykh, A. Ursulov et al.*, *General theory of relativity: Introduction. Modern development and applications*, URSS publishing group Moscow, 400 (2022).

3. *S. Capozziello, M. De Laurentis*, Phys.Rept. **509**, 167 (2011).

4. *E. Berti, E. Barausse, V. Cardoso, L. Gualtieri, P. Pani*, Class. Quant. Grav. **32**, 243001 (2015).

5. *L. Barack et al*, Class. Quant. Grav. **36**, 143001 (2019)

6. *S. Alexeyev, V. Prokopov*, Universe **8**, 283 (2022)

7. *G. Horndeski*, Int. J. Theor. Phys. **10**, 363 (1974).

8. *T. Kobayashi*, Rept. Prog. Phys. **82**, 086901 (2019).

9. *J. Ezquiaga, M. Zumalac'arregui*, Phys. Rev. Lett. **119**, 251304 (2017).

10. *P. Creminelli, F. Vernizzi*, Phys. Rev. Lett. **119**, 251302 (2017).

11. *A. Starobinsky*, Phys. Lett. B **91**, 99 (1980); Adv. Ser. Astrophys. Cosmol. **3**, 130 (1987).

12. *Y. Ageeva, P. Petrov, V. Rubakov*, Phys. Rev. D **104**, 063530 (2021).

13. *C. Charmousis, E. J. Copeland, A. Padilla, P. M. Saffin*, Phys. Rev. Lett. **108**, 051101 (2012).

14. *E. J. Copeland, A. Padilla, P. M. Saffin*, JCAP **12**, 026 (2012).

15. *Torres, J. C. Fabris, O. F. Piattella*, Phys.Lett.B **798**, 135003 (2019).

16. *X. Calmet, D. Croon, C. Fritz*, Eur. Phys. J. C, **75**, 605 (2015).

17. *S. Alexeyev, X. Calmet, B. Latosh*, Phys. Lett. B, **776**, 111 (2018).

18. *B. Latosh*, Eur. Phys. J. C, **78**, 991 (2018).

19. *B. Latosh*, Eur. Phys. J. C, **80**, 845 (2020).

20. *S. Mironov, V. Rubakov, V. Volkova*, Phys.Rev.D **100**, 083521 (2019).

21. *S. Alexeyev, A. Toporensky, V. Ustiansky*, Class. Quant. Grav **17**, 2243 (2000).

22. *S. Alexeyev, K. Rannu*, JETP **141**, 463 (2012).

23. *S. Alexeyev, M. Senduk*, Universe, **6**, 25 (2020).

24. *P. K. Townsend, P. van Nieuwenhuizen*, Phys.Rev.D **19**, 3592 (1979).

25. *I. Novikov, A. Shatsky, S. Alexeyev, D. Tretyakova*, UFN **184**, 379 (2014).

26. *T. Kobayashi, M. Yamaguchi, J. Yokoyama*, Prog.Theor. Phys. **126** 511 (2011).

27. *S. Sushkov, R. Galeev*, Phys.Rev.D **108** 044028 (2023).