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PHASE TRANSITION AT THE BIG BANG POINT IN LATTICE GRAVITY THEORY

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Abstract. Lattice regularization of gravity theory provides new opportunities for studying Big Bang physics. It is proved that in the 4D lattice gravity model studied here, there exists a high-temperature phase characterized by the vanishing of the mean energy-momentum tensor of matter and the collapse of space into a point. The existence of a low-temperature phase in the long-wavelength limit is also shown, whose geometric properties and dynamics correspond to known concepts: the Universe's expansion initially follows an exponential law and then smoothly transitions to a power-law regime.

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1. INTRODUCTION

In the author's works [1–5], a model of discrete gravity on a simplicial complex was studied. This model included degrees of freedom of pure gravity (tetrads, connection), as well as fermionic Dirac fields minimally coupled to gravity. In particular, paper [5] described a discrete symmetry of the action $\mathbb{Z}_2$, called PT-symmetry. This PT-transformation changes the sign of tetrads to the opposite and mutually replaces Dirac fields with their Hermitian conjugates, i.e., mutually transposes particles and antiparticles.

In the present work, the next step is made: we show that at ultra-high temperatures, PT-symmetry is not broken, but it is broken at sufficiently low temperatures. In the high-temperature phase, the mean values of the energy-momentum tensor of matter, as well as tetrads, are equal to zero. In contrast, in the low-temperature phase, these values are non-zero. The tetrad field is the order parameter.

As is known, in the Minimal Standard Model (MSM), the number of fermionic degrees of freedom multiple times exceeds the number of bosonic degrees of freedom (even taking into account two gravitons). It follows that the total energy of vacuum zero oscillations is negative. We assume that some properties of MSM near the Big Bang point can be described within the framework of discrete gravity studied here. Here we are interested in the following question: can the continuation of solutions of continuous Einstein equations in the vicinity of the Big Bang point contain some information about the properties of the high-temperature phase in lattice theory? To answer this question, we solve Einstein's equations within the Friedmann paradigm, but with a positive bare cosmological constant. The positive cosmological constant compensates for the huge negative vacuum energy. And since we consider continuous theory as a long-wavelength limit of discrete gravity theory, all quantities in the equations (vacuum energy density ε, bare cosmological constant Λ_0 etc.) are finite. Therefore, both Einstein's equations themselves and their solutions are mathematically correct. Moreover, the obtained solutions demonstrate the necessary general properties for cosmology: near the Big Bang point, there is an exponential regime of Universe expansion (inflation phase), but then the expansion transitions to a power-law regime. It is also interesting that in the found solution, the magnitude of vacuum energy $|\varepsilon|$ decreases when approaching the Big Bang point. In other words, the energy-momentum tensor of matter in continuous theory tends to zero when approaching the Big Bang point. In our opinion, this tendency indicates that the considered continuous Einstein equation indeed models discrete gravity theory in the long-wavelength limit. However, it should be kept in mind that as the absolute value of the energy-momentum tensor decreases, the role of quantum fluctuations increases, and classical Einstein equations become inapplicable (see Section 4).

The paper is organized as follows.

To facilitate reading the work, Section 2 provides the definition of the lattice gravity theory variant that is studied here.

Section 3 establishes the main result of the work: in the studied model of lattice gravity, the discrete PT-symmetry is not broken at ultra-high temperatures. However, at low temperatures, there is a spontaneous breaking of this symmetry.

Section 4 examines the solution of Einstein's equations within the Friedmann paradigm. Such consideration makes sense since the lattice theory transitions into conventional gravity theory in the long-wavelength limit. It is shown that the solution of classical equations is possible only at some distance from the Big Bang point. The reason is that quantum fluctuations become significant when approaching the singularity.

The present work is of a model nature.

2. INTRODUCTION TO LATTICE GRAVITY THEORY

2.1. Definition of Lattice Theory

We need to define the model of lattice gravity that is studied here. More detailed information on this subject is contained in works [1–5].

In this section, we use Euclidean signature. Let γ^a be Hermitian 4×4 Dirac matrices, such that

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\delta^{ab}, \quad \sigma^{ab} \equiv \frac{1}{4}[\gamma^a, \gamma^b], \quad a = 1,2,3,4,$$
$$\gamma^5 \equiv \gamma^1\gamma^2\gamma^3\gamma^4 = (\gamma^5)^\dagger. \tag{1}$$

Consider an orientable 4-dimensional simplicial complex $\mathfrak{K}$. Suppose that each of its 4-simplices belongs to such subcomplex $\mathfrak{K}' \in \mathfrak{K}$, which has a geometric realization in $\mathbb{R}^4$, topologically equivalent to a disk without cavities. Vertices are denoted by a_v, indices v and w enumerate vertices and 4-simplices s^4_w respectively. It is necessary to use local enumeration of vertices belonging to a given 4-simplex: all 5 vertices of 4-simplex s^4_w are enumerated as $a_{v(w)i}$, $i = 1,2,3,4,5$. Further, notations with additional lower index (w) indicate that the corresponding values belong to 4-simplex s^4_W. Note that a value belonging to 4-simplex s^4_w, may also belong to adjacent 4-simplex s^4_{w}, if these 4-simplices have common simplices of lower dimension. The Levi-Civita symbol with pairwise distinct indices

$$\varepsilon_{v(w)1 v(w)2 v(w)3 v(w)4 v(w)5} = \pm 1$$

depending on whether the order of vertices $s^4_w = a_{v(w)1}a_{v(w)2}a_{v(w)3}a_{v(w)4}a_{v(w)5}$ determines even or odd orientation of 4-simplex s^4_w. The element of compact group Spin(4) and the element of Clifford algebra,

$$\Omega_{v_1v_2} = \Omega^{-1}_{v_2v_1} = \exp(\omega_{v_1v_2}) =$$
$$= \exp\left(\frac{1}{2}\sigma^{ab}\omega^{ab}_{v_1v_2}\right) \in \text{Spin}(4), \quad \sigma^{ab} \equiv \frac{1}{4}[\gamma^a\gamma^b],$$
$$\hat{e}_{v_1v_2} \equiv e^a_{v_1v_2}\gamma^a \equiv -\Omega_{v_1v_2}\hat{e}_{v_2v_1}\Omega^{-1}_{v_1v_2}, \tag{2}$$
$$\left|e_{v_1v_2}\right| < 1, \quad \left|e_{v_1v_2}\right| \equiv \sqrt{\sum_a (e^a_{v_1v_2})^2},$$

are defined on each oriented 1-simplex $a_{v_1}a_{v_2}$. The boundedness of the tetrad according to (2) is necessary for the convergence of the functional integral of the partition function. This constraint imposed on the tetrad distinguishes the present work from the author's previous works on discrete gravity theory. By assumption, the set of variables $\{\Omega, \hat{e}\}$ is a set of independent dynamical variables. Fermionic degrees of freedom (Dirac spinors) are defined on the complex vertices:

$$\Psi^\dagger_v, \quad \Psi_v. \tag{3}$$

The set of variables $\{\Psi^\dagger, \Psi\}$ are mutually independent, where spinors $\Psi^\dagger_v$ and Ψ_v are in mutual involution (or anti-involution) with respect to Hermitian conjugation operation.

Consider a model with action

$$\mathfrak{A} = \mathfrak{A}_g + \mathfrak{A}_\Psi + \mathfrak{A}_{\Lambda_0}. \qquad (4)$$

Here $\mathfrak{A}_g$ and $\mathfrak{A}_\Psi$ are the actions of pure gravitation and the Dirac field, respectively:

$$\mathfrak{A}_g = -\frac{1}{5!\cdot 2l_P'^2}\sum_W\sum_\sigma \varepsilon_{\sigma(\mathcal{V}_{(w)1})\sigma(\mathcal{V}_{(w)2})\sigma(\mathcal{V}_{(w)3})\sigma(\mathcal{V}_{(w)4})(\mathcal{V}_{(w)5})}\times$$
$$\times \mathrm{Tr}\gamma^5\left\{\Omega_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)1})}\Omega_{\sigma(\mathcal{V}_{(w)1})\sigma(\mathcal{V}_{(w)2})}\Omega_{\sigma(\mathcal{V}_{(w)2})\sigma(\mathcal{V}_{(w)5})}\hat{e}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)3})}\hat{e}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)4})}\right\}, \qquad (5)$$

$$\mathfrak{A}_\Psi = \frac{1}{5\cdot 24^2}\sum_W\sum_\sigma e_{\sigma(\mathcal{V}_{(w)1})\sigma(\mathcal{V}_{(w)2})\sigma(\mathcal{V}_{(w)3})\sigma(\mathcal{V}_{(w)4})\sigma(\mathcal{V}_{(w)5})}\times$$
$$\times \mathrm{Tr}\gamma^5\left\{\hat{\Theta}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)1})}\hat{e}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)2})}\hat{e}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)3})}\hat{e}_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)4})}\right\}, \qquad (6)$$

$$\hat{\Theta}_{\mathcal{V}_1\mathcal{V}_2} \equiv \Theta^a_{\mathcal{V}_1\mathcal{V}_2}\gamma^a = \hat{\Theta}^\dagger_{\mathcal{V}_1\mathcal{V}_2}, \quad \Theta^a_{\mathcal{V}_1\mathcal{V}_2} = \frac{i}{2}\left(\Psi^\dagger_{\mathcal{V}_1}\gamma^a\Omega_{\mathcal{V}_1\mathcal{V}_2}\Psi_{\mathcal{V}_2} - \Psi^\dagger_{\mathcal{V}_2}\Omega_{\mathcal{V}_2\mathcal{V}_1}\gamma^a\Psi_{\mathcal{V}_1}\right). \qquad (7)$$

Each σ is one of the 5! permutations of vertices $\mathcal{V}_{(w)i} \to \sigma(\mathcal{V}_{(w)i})$. One can verify that (cf. with (2))

$$\hat{\Theta}_{\mathcal{V}_1\mathcal{V}_2} \equiv -\Omega_{\mathcal{V}_1\mathcal{V}_2}\hat{\Theta}_{\mathcal{V}_2\mathcal{V}_1}\Omega^{-1}_{\mathcal{V}_1\mathcal{V}_2}. \qquad (8)$$

The contribution to the lattice action from the cosmological constant has the form

$$\mathfrak{A}_{\Lambda_0} = -\frac{1}{5!\cdot 12}\frac{\Lambda_0}{l_{P'}^2}\varepsilon_{abcd}\sum_W\sum_\sigma \varepsilon_{\sigma(\mathcal{V}_{(w)1})\sigma(\mathcal{V}_{(w)2})\sigma(\mathcal{V}_{(w)3})\sigma(\mathcal{V}_{(w)4})\sigma(\mathcal{V}_{(w)5})}\times$$
$$\times e^a_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)1})}e^b_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)2})}e^c_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)3})}e^d_{\sigma(\mathcal{V}_{(w)5})\sigma(\mathcal{V}_{(w)4})}. \qquad (9)$$

The partition function is represented by the integral

$$Z = \prod_{1-\text{simplices}}\int_{|e_{\mathcal{V}_1\mathcal{V}_2}|<1}\prod_a de^a_{\mathcal{V}_1\mathcal{V}_2}\int d\mu\{\Omega_{\mathcal{V}_1\mathcal{V}_2}\}\times$$
$$\times\prod_{\mathcal{V}}\int d\Psi^\dagger_{\mathcal{V}}\,d\Psi_{\mathcal{V}}\exp(\mathfrak{A}). \qquad (10)$$

Everywhere $d\mu\{\Omega_{\mathcal{V}_1\mathcal{V}_2}\}$ is an invariant measure on the Spin(4) group.

Action (4), as well as integral (10), are invariant under gauge transformations

$$\tilde{\Omega}_{\mathcal{V}_1\mathcal{V}_2} = S_{\mathcal{V}_1\mathcal{V}_2}\Omega_{\mathcal{V}_1\mathcal{V}_2}S^{-1}_{\mathcal{V}_2}, \quad \tilde{\hat{e}}_{\mathcal{V}_1\mathcal{V}_2} = S_{\mathcal{V}_1}\hat{e}_{\mathcal{V}_1\mathcal{V}_2}S^{-1}_{\mathcal{V}_1},$$
$$\tilde{\Psi}_{\mathcal{V}} = S_{\mathcal{V}}\Psi_{\mathcal{V}}, \quad \tilde{\Psi}^\dagger_{\mathcal{V}} = \Psi^\dagger_{\mathcal{V}}S^{-1}_{\mathcal{V}}, \qquad (11)$$
$$S_{\mathcal{V}} \in \mathrm{Spin}(4).$$

Verification of this fact is simplified when using the relation (cf. with the relation for $\hat{e}_{\mathcal{V}_1\mathcal{V}_2}$ in (11))

$$\tilde{\hat{\Theta}}_{\mathcal{V}_1\mathcal{V}_2} = S_{\mathcal{V}_1}\hat{\Theta}_{\mathcal{V}_1\mathcal{V}_2}S^{-1}_{\mathcal{V}_1}, \qquad (12)$$

which directly follows from (11).

The considered lattice model is invariant under global discrete $\mathbb{Z}_2$-symmetry, which is analogous to the combined PT-symmetry. Let us denote the operator of this transformation as $\hat{u}_{PT}$. Then the transformed dynamical variables are expressed through the initial variables as follows:

$$\hat{u}^{-1}_{PT}\Psi_{\mathcal{V}}\hat{u}_{PT} = U_{PT}\left(\Psi^\dagger_{\mathcal{V}}\right)^t,$$
$$\hat{u}^{-1}_{PT}\Psi^\dagger_{\mathcal{V}}\hat{u}_{PT} = -\left(\Psi_{\mathcal{V}}\right)^t U^{-1}_{PT},$$
$$U_{PT} = i\gamma^1\gamma^3, \qquad (13)$$
$$\hat{u}^{-1}_{PT}e^a_{\mathcal{V}_1\mathcal{V}_2}\hat{u}_{PT} = -e^a_{\mathcal{V}_1\mathcal{V}_2},$$
$$\hat{u}^{-1}_{PT}\omega^a_{\mathcal{V}_1\mathcal{V}_2}\hat{u}_{PT} = \omega^{ab}_{\mathcal{V}_1\mathcal{V}_2}.$$

Here, the upper index t denotes transposition of Dirac matrices and spinors. We have

$$U_{PT}^{-1}\gamma^a U_{PT} = (\gamma^a)^t,\quad U_{PT}^{-1}\sigma^{ab} U_{PT} = -(\sigma^{ab})^t. \tag{14}$$

From (13) and (14) follows

$$U_{PT}^{-1}\Omega_{\mathcal{V}_1\mathcal{V}_2}U_{PT} = \left(\Omega_{\mathcal{V}_2\mathcal{V}_1}\right)^t, \tag{15}$$

$$\hat{u}_{PT}^{-1}\Theta^a_{\mathcal{V}_1\mathcal{V}_2}\,\hat{u}_{PT} = -\Theta^a_{\mathcal{V}_1\mathcal{V}_2}. \tag{16}$$

2.2. Long-wavelength limit

Let us proceed to the long-wavelength limit, i.e., to the limit of slowly varying fields along the lattice. In this limit, action (4) transforms into the well-known continuum action of gravity in the Palatini form and the Dirac field minimally coupled to gravity, plus a cosmological constant term. This limit transition makes sense together with the transition to Minkowski signature. As a result, the compact gauge group Spin(4) transforms into the non-compact group Spin(3,1). Further in this section, all lattice variables in the case of Euclidean signature are marked with a prime. For field variables in the case of Minkowski signature, the old notations are used.

For the specified action transformation, the following deformations of integration contours in integral (10) are necessary:

$$\omega'^{4\alpha}_{\mathcal{V}_1\mathcal{V}_2} = i\omega^{0\alpha}_{\mathcal{V}_1\mathcal{V}_2},\quad \omega'^{\alpha\beta}_{\mathcal{V}_1\mathcal{V}_2} = -\omega^{\alpha\beta}_{\mathcal{V}_1\mathcal{V}_2},\quad e'^{4}_{\mathcal{V}_1\mathcal{V}_2} = e^{0}_{\mathcal{V}_1\mathcal{V}_2},\quad e'^{\alpha}_{\mathcal{V}_1\mathcal{V}_2} = ie^{\alpha}_{\mathcal{V}_1\mathcal{V}_2}. \tag{17}$$

Variables $\omega^{ab}_{w_{ij}}$, $e^a_{w_{ij}}$ in Minkowski signature are real and their indices take values

$$a,b,\ldots = 0,1,2,3,\quad \alpha,\beta,\ldots = 1,2,3. \tag{18}$$

In the orthonormal basis (ONB), the metric tensor is $\eta^{ab} = \mathrm{diag}(1,-1,-1,-1)$. Dirac matrices transform as

$$\gamma'^4 = \gamma^0,\quad \gamma'^\alpha = i\gamma^\alpha,\quad \gamma'^5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3,\quad \frac{1}{2}(\gamma^a\gamma^b + \gamma^b\gamma^a) = \eta^{ab}, \tag{19}$$

$$\mathrm{Tr}\gamma^5\gamma_a\gamma_b\gamma_c\gamma_d = 4i\varepsilon_{abcd},\quad \varepsilon_{0123} = 1.$$

Thus, for spin matrices $\sigma^{ab} = (1/4)[\gamma^a,\gamma^b]$ we have

$$\sigma'^{4\alpha} = i\sigma^{0\alpha},\quad \sigma'^{\alpha\beta} = -\sigma^{\alpha\beta}. \tag{20}$$

Using (17)–(20), we find

$$\omega'_{\mathcal{V}_1\mathcal{V}_2} = \frac{1}{2}\omega^{ab}_{\mathcal{V}_1\mathcal{V}_2}\,\sigma_{ab} \equiv \omega_{\mathcal{V}_1\mathcal{V}_2},\quad \hat{e}'_{\mathcal{V}_1\mathcal{V}_2} = \gamma_a e^a_{\mathcal{V}_1\mathcal{V}_2} \equiv \hat{e}_{\mathcal{V}_1\mathcal{V}_2}, \tag{21}$$

$$\Omega'_{\mathcal{V}_1\mathcal{V}_2} = \exp\left(\frac{1}{2}\omega'^{ab}_{\mathcal{V}_1\mathcal{V}_2}\,\sigma'^{ab}\right) = \exp\left(\frac{1}{2}\omega^{ab}_{\mathcal{V}_1\mathcal{V}_2}\,\sigma_{ab}\right) \equiv \Omega_{\mathcal{V}_1\mathcal{V}_2} \in \mathrm{Spin}(3,1). \tag{22}$$

When transitioning to Minkowski signature, Dirac spinors transform as follows:

$$\Psi'_{\mathcal{V}} = \Psi_{\mathcal{V}},\quad \Psi'^{\dagger}_{\mathcal{V}} = \Psi^{\dagger}_{\mathcal{V}}\gamma^0 = \overline{\Psi}_{\mathcal{V}}. \tag{23}$$

The transition to the long-wavelength limit is possible for such field configurations that change sufficiently slowly during transitions from simplex to simplex, i.e., with small or significant movements across the lattice. This rule applies to any lattices. In our theory, the need to introduce local coordinates arises precisely at the stage of transition to the long-wavelength limit. Local coordinates are markers of lattice vertices. Consider some 4D-subcomplex $\mathfrak{K}' \in \mathfrak{K}$ with trivial topology of a four-dimensional disk and geometric realization in $\mathbb{R}^4$. Thus, each vertex of the subcomplex acquires coordinates x^μ, which are the coordinates of the vertex image in $\mathbb{R}^4$:

$$x^\mu_{\mathcal{V}} \equiv x^\mu(a_{\mathcal{V}}),\qquad \mu = (0,1,2,3) = (0,i). \tag{24}$$

At this stage, the coordinates are dimensionless. Consider some simplex $s^4_w \in \mathfrak{K}'$. Let's denote all five vertices of this 4-simplex as $\mathcal{V}_i$, $i = 1,2,3,4$ and $\mathcal{V}_m \neq \mathcal{V}_m$. The properties of geometric realization are such that four infinitesimal vectors

$$dx^\mu_{\mathcal{V}_m\mathcal{V}_i} \equiv x^\mu_{\mathcal{V}_i} - x^\mu_{\mathcal{V}_m} = -dx^\mu_{\mathcal{V}_i\mathcal{V}_m} \in \mathbb{R}^4,\quad i = 1,2,3,4, \tag{25}$$

are linearly independent.

In works [1-5], it is proven that in $\mathbb{R}^4$ there exist 1-forms $\omega_\mu(x)$ and $\hat{e}_\mu(x)$ that the following equalities hold

$$\omega_\mu\left(\frac{1}{2}(x_{\mathcal{V}_m} + x_{\mathcal{V}_i})\right)dx^\mu_{\mathcal{V}_m\mathcal{V}_i} = \omega_{\mathcal{V}_m\mathcal{V}_i}, \tag{26}$$

$$\hat{e}_\mu\left(\frac{1}{2}(x_{\mathcal{V}_m} + x_{\mathcal{V}_i})\right)dx^\mu_{\mathcal{V}_m\mathcal{V}_i} = \hat{e}_{\mathcal{V}_m\mathcal{V}_i}. \tag{27}$$

Further, it is assumed that the 1-forms smoothly depend on points in $\mathbb{R}^4$, which are images of complex vertices. In the long-wavelength limit, elements $\Omega_{\mathcal{V}_m\mathcal{V}_i}$ are close to unity and up to $O((dx)^2)$ inclusive, the following formula holds

$$\Omega_{\mathcal{V}_m\mathcal{V}_i}\Omega_{\mathcal{V}_i\mathcal{V}_j}\Omega_{\mathcal{V}_j\mathcal{V}_m} =$$

$$= \exp\left[\frac{1}{2}\mathfrak{R}_{\mu\nu}(x_{\mathcal{V}_m})dx^{\mu}_{(\mathcal{V}_m\mathcal{V}_i)}dx^{\nu}_{(\mathcal{V}_m\mathcal{V}_j)}\right], \quad (28)$$

$$\mathfrak{R}_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu + [\omega_\mu, \omega_\nu]. \quad (29)$$

Let's write out the long-wavelength limit of action (4):

$$\mathfrak{A}'_g \to i\mathfrak{A}_g, \quad \mathfrak{A}_g = -\frac{1}{4l_P^2}\varepsilon_{abcd}\int \mathfrak{R}^{ab}\wedge e^c \wedge e^d,$$

$$\mathfrak{R}^{ab} = \mathfrak{R}^{ab}_{\mu\nu}dx^\mu \wedge dx^\nu, \quad (30)$$

$$\mathfrak{A}'_\Psi \to i\mathfrak{A}_\Psi, \quad \mathfrak{A}_\Psi = \frac{1}{6}\varepsilon_{abcd}\int \Theta^a \wedge e^b \wedge e^c \wedge e^d,$$

$$\Theta^a = \frac{i}{2}\left[\overline{\Psi}\gamma^a D_\mu\Psi - \left(\overline{D_\mu\Psi}\right)\gamma^a\Psi\right]dx^\mu,$$

$$D_\mu = \left(\partial\mu + \omega_\mu\right), \quad (31)$$

$$\mathfrak{A}'_{\Lambda_0} \to i\mathfrak{A}_{\Lambda_0}, \quad \mathfrak{A}_{\Lambda_0} = -\frac{2\Lambda_0}{l_P^2}\int e^0 \wedge e^1 \wedge e^2 \wedge e^3. \quad (32)$$

All other terms in such transition will contain additional factors in positive powers of $l_P/\lambda \to 0$, therefore they are omitted. Here λ is the characteristic wavelength of the physical subsystem. This situation is typical when transitioning to the long-wavelength limit in any lattice theory.

The action (30)–(32) is the Hilbert-Einstein action minimally coupled with the Dirac field and written in the Palatini form. It is invariant under diffeomorphisms. This fact is not accidental, as in (24) the method of introducing coordinates itself is such that the independence of action from the arbitrariness of coordinate introduction is already visible at this stage. We say "almost arbitrary" because diffeomorphisms are not arbitrary coordinate changes, but locally one-to-one and differentiable the required number of times.

For clarity, let us point out that on the lattice, all variables and constants are dimensionless and of order unity. In particular, the constant $l'_P \sim 1$ in (5) and (9), as well as the differentials $dx^\mu_{\mathcal{V}_m\mathcal{V}_i}$ in (25) are dimensionless. When transitioning to dimensional quantities, we assume

$$dx^\mu_{\mathcal{V}_m\mathcal{V}_i} = dx^\mu / l_p \sim 1, \quad (33)$$

where the differential dx^μ is measured in centimeters and $l_P \sim 10^{-32}$ cm (see (53)). From (33), it is evident that the irregular lattice step has a size of order l_P. And with this, all terms of the action are dimensionless, but variables and constants acquire dimensionality. For example, in term (32) the cosmological constant $\Lambda_0 \sim l_P^{-2}$.

In Minkowski signature, the PT-symmetry of the action is defined by formulas (13), (14), (16) with the only difference that in these formulas one should make the substitution $\Psi^\dagger \to \overline{\Psi}$.

3. ULTRA-HIGH TEMPERATURES IN LATTICE GRAVITY THEORY

Let us consider the state of lattice theory at ultra-high temperatures. As is known, the partitional function Tr $\exp(-\beta\mathcal{H})$ in quantum theory differs from the transition amplitude over finite time Δt, presented in the form of a functional integral, by transition to imaginary time $\Delta t = -i\beta$, $\beta = 1/T$, and taking the trace. We must study certain properties of the partitional function (10) in the case of ultra-high temperatures, i.e., $\beta \to 0$.

In application to the lattice theory studied here, this means the following.

Let the 4D lattice have two 3D sublattices Σ_1 and Σ_2, which form its boundary. For simplicity, we assume that between Σ_1 and Σ_2 there are $N < \infty$ 4D lattice layers, each one edge thick. Sublattices Σ_1 and Σ_2 are considered identical, i.e., there is a one-to-one correspondence between all elements of these sublattices. The latter property makes it possible to calculate the partitional function.

Let us denote by $\Phi_{1\xi}$ and $\Phi_{2\xi}$ holomorphic functions of fermionic variables with real coefficients, defined on Σ_1 and Σ_2 respectively. To calculate the trace over fermionic variables, it is necessary to use a complete set of holomorphic wave functions

$$\Phi_{1\xi} \equiv \Phi_{1\xi}\{\Psi^\dagger_\mathcal{V}\}$$

and their Hermitian conjugate functions

$$\Phi^{\dagger}_{2\xi} \equiv \left(\Phi_{2\xi}\{\Psi^{\dagger}_{\nu}\}\right)^{\dagger} = \Phi_{2\xi}\{\Psi_{\nu}\}.$$

The index ξ enumerates independent orthonormalized functions from their complete set. The functional

$$\Phi^{\dagger}_{2\xi}\exp(\beta\mathfrak{A})\Phi_{1\xi} \tag{34}$$

must be placed under the integral (10) and the sum over ξ must be calculated. In integral (10), on the identified 1-simplices belonging to Σ_1 and Σ_2, the variables $\{\Omega\}$ and $\{e^a\}$ must be identified.

We are interested in the case $\beta \ll 1$. Let us prove that the discrete PT-symmetry (13)–(16) is not broken at ultrahigh temperatures.

The following

Statement holds. In some finite neighborhood of the point $\beta = 0$, the free energy of the statistical sum (10), except for the term of the form $-(C_1\mathfrak{M} + 4\mathfrak{N}\nu_{\Psi})\ln\beta$ (see (42)), is a holomorphic function of the variable β. All symmetries of action (4), including discrete PT-symmetry, are preserved.

Let us present some arguments in favor of the Statement. Let us consider the high-temperature expansion of the partition function in the 2D Ising model, which is a sum over all closed paths with self-intersections on the lattice. Let the lattice contain $\mathfrak{L} \to \infty$ edges and have a fixed path (possibly several non-intersecting paths) with a total of $\mathfrak{l}$ edges. Note that is an even number and $\mathfrak{l} \geq 4$. When calculating the partition function, it is convenient to isolate the factor $(\mathrm{ch}\beta)^{\mathfrak{L}}$, which does not contribute to the singularity in the free energy. Then each path edge should be assigned the weight $\mathrm{th}\beta$. Let $g_{\mathfrak{l}}$ be the number of such closed paths. We have a strict inequality

$$g_{\mathfrak{l}} < \frac{\mathfrak{L}!}{\mathfrak{l}!(\mathfrak{L}-\mathfrak{l})!}, \tag{35}$$

since the number on the right side of inequality (35) also includes the number of all unclosed paths of weight $(\mathrm{ch}\beta)^{\mathfrak{l}}$. Therefore, the Ising partition function Z

$$\frac{Z}{(\mathrm{ch}\beta)^{\mathfrak{L}}} = 1 + \sum_{\mathfrak{l}} g_{\mathfrak{l}}(\mathrm{th}\beta)^{\mathfrak{l}} < \\ < \sum_{\mathfrak{l}=0}^{\mathfrak{L}} \frac{\mathfrak{L}!}{\mathfrak{l}!(\mathfrak{L}-\mathfrak{l})!}(\mathrm{th}\beta)^{\mathfrak{l}} = \\ = (1+\mathrm{th}\beta)^{\mathfrak{L}} \equiv Z_M. \tag{36}$$

From this, it can be seen that the specific free energy per edge,

$$f_M = -\ln Z_M/\mathfrak{L} = -\ln(1+\mathrm{th}\beta),$$

is a holomorphic function in a finite neighborhood of the point $\beta = 0$ and in the case $\mathfrak{L} \to \infty$. This observation is a consequence of the fact that the quantity Z_M is essentially local for small β, i.e., effectively Z_M is a product of local holomorphic functions. In other words, there is no long-range order at small β. But if unclosed contours are removed from the sum Z_M no long-range order will emerge. However, the size of the holomorphicity neighborhood may change. This means that the limit $f = -\ln Z/\mathfrak{L}$ at $\mathfrak{L} \to \infty$ exists. Indeed, the quantity on the left side of (36) can be represented as

$$Z/(\mathrm{ch}\beta)^{\mathfrak{L}} = [1 + \rho(\mathrm{th}\beta)]^{\mathfrak{L}}, \tag{37}$$

where the holomorphic function $\rho(\mathrm{th}\beta) \to 0$ at $\beta \to 0$. Therefore,

$$f = -\ln \mathrm{ch}\beta - \ln[1 + \rho(\mathrm{th}\beta)] \tag{38}$$

is also a holomorphic function in some finite neighborhood of the point $\beta = 0$.

Similar reasoning can be applied to the case of the partition function (10), although this case is qualitatively more complex. Both fermionic and bosonic integrals in (10) are well-defined. Let us qualitatively consider the results of these integrations.

Let $\mathfrak{N} \to \infty$ and ν_{Ψ} denote the numbers of complex vertices and Dirac fields respectively. Integration in (10) over the Dirac field gives a factor under the remaining integrals of the form

$$\beta^{4\mathfrak{N}\nu_{\Psi}}\,\mathfrak{P}\{\Omega, e^a\}, \tag{39}$$

where $\mathfrak{P}\{\Omega, e^a\}$ is a gauge-invariant homogeneous polynomial of variables Ω and e^a of degrees $4\mathfrak{N}\nu_{\Psi}$ and $12\mathfrak{N}\nu_{\Psi}$ respectively.

Next, let us consider the integrals over bosonic variables. First, let us consider integration over variables Ω.

Let's consider in the polynomial $\mathfrak{P}\{\Omega, e^a\}$ a term that is odd with respect to the variable $\Omega_{\mathcal{V}_1\mathcal{V}_2}$. The integral $\int d\mu\{\Omega_{\mathcal{V}_1\mathcal{V}_2}\}\ldots$ of the tensor product of an odd number of elements $\Omega_{\mathcal{V}_1\mathcal{V}_2}$ equals zero. Let's expand the exponent by terms containing the element $\Omega_{\mathcal{V}_1\mathcal{V}_2}$. Suppose that after a finite number of such steps (for a finite subcomplex) we will have a tensor product of an even number of elements $\Omega_{\mathcal{V}_1\mathcal{V}_2}$ on each 1-simplex $a_{\mathcal{V}_1}a_{\mathcal{V}_2}$. Therefore, as a result of integration over the gauge group, the first non-zero term will receive an additional factor $\beta^{C\mathfrak{M}}$, partition function where $\mathfrak{M} \to \infty$ is the number of 1-simplexes of the complex and $0 \le C \sim 1$. All other terms of this expansion, as a result of integration over the gauge group, give an additional gaugeinvariant term $\mathfrak{F}\{e^a;\beta\}$ to the statistical sum. This term is a functional of variables $\{e^a\}$ and a well-convergent holomorphic function of the parameter β in the neighborhood of zero, where

$$\frac{\mathfrak{F}\{e^a;\beta\}}{\beta^{C\mathfrak{M}+4\mathfrak{N}\nu_\Psi}} \to 0 \quad \text{as} \quad \beta \to 0.$$

There are only two types of invariants with respect to the action of the gauge group (11) and associated with vertex $a_\mathcal{V}$:

$$e^a_{\mathcal{V}\mathcal{V}_1}e^a_{\mathcal{V}\mathcal{V}_2}, \tag{40}$$

$$\varepsilon_{abcd}e^a_{\mathcal{V}\mathcal{V}_1}e^b_{\mathcal{V}\mathcal{V}_2}e^c_{\mathcal{V}\mathcal{V}_3}e^d_{\mathcal{V}\mathcal{V}_4}. \tag{41}$$

In (40), vertices $a_{\mathcal{V}_1}$ and $a_{\mathcal{V}_2}$ are not necessarily different, and in (41), 1-simplexes $(a_\mathcal{V}a_{\mathcal{V}_1})$, $(a_\mathcal{V}a_{\mathcal{V}_2})$, $(a_\mathcal{V}a_{\mathcal{V}_3})$, $(a_\mathcal{V}a_{\mathcal{V}_4})$ do not necessarily belong to the same 4-simplex.

Let's briefly consider the integral over variables $\{e^a\}$. Note that if the integrand contains expression (40) in the first degree and with different $a_{\mathcal{V}_1}$ and $a_{\mathcal{V}_2}$ and it does not intersect with other similar expressions, then the integral is identically zero. A similar statement is true regarding expression (41). Consequently, expressions (40) and (41) must be under the integral in such degrees and combinations that the variable $e^a_{\mathcal{V}_1\mathcal{V}_2}$ on each 1-simplex $(a_{\mathcal{V}_1}a_{\mathcal{V}_2})$ would be in an even (possibly zero) power. It is possible that for this, the expansion in (34) by the contribution to the action from $\beta\mathfrak{A}_{\Lambda_0}$ (see (9)) is necessary.

Action (4) is local, i.e., it consists of terms (local operators), each of which is defined on the nearest lattice elements. Let's call one of these operators A, another — B, and assume that operators A and B are separated by a significant distance along the lattice. Consider the correlator $\langle AB\rangle$. For this correlator to be non-zero, it is necessary to expand the exponential in these operators so that between operators A and B there is a sufficient number of adjacent local operators. But such expansion will give the parameter $\beta \ll 1$ in a sufficiently high power, at least proportional to the number of 1-simplexes between A and B, with a coefficient of order unity. Therefore, the partition function in the high-temperature phase has a local character.

Let's draw a conclusion from this consideration. At ultra-high temperature $(\beta = 1/T \to 0)$ the partition function integral (10) is a holomorphic function in the neighborhood of point $\beta = 0$ and has the form (cf. with (37))

$$Z = \text{const}\,\beta^{C_1\mathfrak{M}+4\mathfrak{N}\nu_\Psi}(1+\mathfrak{f}(\beta))^{C_2\mathfrak{M}}, \tag{42}$$
$$\mathfrak{f}(\beta) \to 0 \quad \text{as} \quad \beta \to 0,$$
$$C_1 \sim C_2 \sim 1.$$

From this follows the validity of the Statement. In turn, the Statement implies that in the high-temperature phase, no symmetries are broken, including the discrete PT-symmetry.

Based on the validity of the Statement, let's proceed with calculating the averages of some operators.

The discrete PT-transformation (13)–(16) does not change the measure and the integrand (except possibly for the averaged quantity), but exchanges the initial and final states:

$$\left(\Phi^\dagger_{2\xi}\right)' \equiv \Phi^\dagger_{2\xi}\hat{u}^\dagger_{PT} = \Phi_{2\xi}\{U_{PT}(\Psi^\dagger_\mathcal{V})^t\},$$
$$\Phi'_{1\xi} \equiv \hat{u}_{PT}\Phi_{1\xi} = \Phi_{1\xi}\{-\Psi^t_\mathcal{V}U^{-1}_{PT}\}, \tag{43}$$
$$\left(\Phi^\dagger_{2\xi}\right)'\Phi'_{1\xi} = \Phi_{1\xi}\{\Psi^t_\mathcal{V}U^{-1}_{PT}\}\Phi_{2\xi}\{U_{PT}(\Psi^\dagger_\mathcal{V})^t\}.$$

Here, the prime above the wave function symbol means that it is PT-transformed. From the last equality, it is evident that the initial and final wave functions are interchanged as a result of PT-transformation, and their scalar product is preserved. This means that the operator $\hat{u}_{PT}$ is antiunitary.

According to the general rule, the integral does not change under a change of integration variables. In our case, the integral over fermion and tetrad variables and the sum over fermion states ξ is of interest. The integral over connection variables $\{\Omega\}$ should be excluded. Otherwise, the subsequent reasoning would lose meaning. Let's consider two integrals.

First case:

$$Z\{\Omega\} = \sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \Phi^{\dagger}_{2\xi} \exp(\beta\mathfrak{A})\Phi_{1\xi} \overset{PT}{\rightarrow} Z\{\Omega\}.$$

Second case:

$$\langle\Theta^{a}_{\nu_1\nu_2}\rangle_{e,\Psi} = N\sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \Phi^{\dagger}_{2\xi}\Theta^{a}_{\nu_1\nu_2} \exp(\beta\mathfrak{A})\Phi_{1\xi} \overset{PT}{\rightarrow} -\langle\Theta^{a}_{\nu_1\nu_2}\rangle_{e,\Psi}.$$

Let's examine the second case in more detail. We have a chain of equalities:

$$\begin{aligned}
\Theta^{a}_{\nu_1\nu_2}\rangle_{e,\Psi} &= N\sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \Phi^{\dagger}_{2\xi}\Theta^{a}_{\nu_1\nu_2} \exp(\beta\mathfrak{A})\Phi_{1\xi} = \\
&= N\sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \left(\Phi^{\dagger}_{2\xi}\right)'\Theta^{a}_{\nu_1\nu_2} \exp(\beta\mathfrak{A})\Phi'_{1\xi} = \\
&= N\sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \Phi^{\dagger}_{1\xi}\left\{\hat{u}^{-1}_{PT}\Theta^{a}_{\nu_1\nu_2} \exp(\beta\mathfrak{A})\hat{u}_{PT}\right\}^{\dagger}\Phi_{2\xi} = \\
&= -N\sum_{\xi}\int_{e}\int_{\Psi^{\dagger}\Psi} \Phi^{\dagger}_{1\xi}\Theta^{a}_{\nu_1\nu_2} \exp(\beta\mathfrak{A})\Phi_{2\xi} = \\
&= -\langle\Theta^{a}_{\nu_1\nu_2}\rangle_{e,\Psi}
\end{aligned} \quad (44)$$

Here $N\{\Omega\}$is a normalization constant, the integral is calculated only over the tetrad and Dirac field variables, but not over the connection variables. If the integration also included the integral over variables $\{\Omega\}$, then any gauge non-invariant quantity, like $\Theta^{a}_{\nu_1\nu_2}$, would vanish identically. But integral (44) is meaningful since the gauge is fixed in it. In (44), the first equality is the definition of the mean value, the second equality follows from the Statement, the third and fourth equalities follow from equalities (43) and (16) respectively.

From the chain of equalities (44), the main conclusion of this work follows:

$$\langle\Theta^{a}_{\nu_1\nu_2}\rangle_{e,\Psi} \equiv \langle\Theta^{a}_{\nu_1\nu_2}\rangle_{\text{Gauge Fix}} = 0. \quad (45)$$

The vacuum average of the quantity $\Theta^{a}_{\nu_1\nu_2}$ in the long-wavelength limit in Minkowski signature at zero temperature was calculated in [6]:

$$\begin{gathered}
\langle 0\left|\Theta^{a}_{\mu}\right|0\rangle = \frac{2}{\pi^2 l_P^4} e^{a}_{\mu} \neq 0, \\
\Theta^{a}_{\mu} = \frac{i}{2}\left[\overline{\Psi}\gamma^{a}D_{\mu}\Psi - \left(\overline{D_{\mu}\Psi}\right)\gamma^{a}\Psi\right], \quad (46) \\
D_{\mu} = \left(\partial_{\mu} + \omega_{\mu}\right).
\end{gathered}$$

Comparison of equations (45) and (46) shows that in lattice gravity theory coupled to the Dirac field, there is a temperature phase transition.

The phase transition from a phase with zero mean value of the quantity Θ^{a}_{μ} to a phase with non-zero value of this quantity and the physical significance of this transition was considered in paper [7]. Paper [8] may also be of interest in this context.

4. EINSTEIN EQUATIONS AND THEIR SOLUTIONS

Here we use the standard spatially flat Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)(dx^{\alpha})^2, \quad \alpha = 1, 2, 3, \quad (47)$$

$H = \dot{a}/a$ and $a(t)$are the Hubble parameter and cosmic scale factor. We assume $c = 1$. In formulas with restored dimensionality, corresponding remarks are made.

First of all, let's show that at the end of the inflation phase and beyond, quantum vacuum fluctuations are insignificant when considering the dynamics of macroscopic regions of space and the matter contained in them.

Our approach assumes that all physical quantities are defined taking into account vacuum zero-point fluctuations of quantized fields. In particular, the energy density and vacuum pressure include vacuum energy and pressure. It is known that the following simultaneous commutation relations exist for the energy-momentum tensor components in Minkowski space [9]:

$$\begin{aligned}
&\left[T^{00}(\mathrm{x}), T^{00}(\mathrm{y})\right] = \\
&= -i\hbar\left(T^{0k}(\mathrm{x}) + T^{0k}(\mathrm{y})\right)\partial_k\delta^{(3)}(\mathrm{x}-\mathrm{y}) + \\
&+ \text{Schwinger Terms},
\end{aligned} \quad (48)$$

and so on. The Schwinger terms are higher derivatives of δ-functions and they are not of interest here. The same commutation relations hold in curved space if they are written near the center of Riemann normal coordinates.

Let us denote l_P as the order of lattice step size, which we will call the Planck scale. Then $\delta^{(3)}(0) \sim l_P^{-3}$. Using (48) and the known rule we find the order of quantum fluctuations of energy density at wavelengths of order $\lambda \gg l_P$:

$$\Delta\varepsilon \sim \sqrt{\frac{\hbar|\varepsilon|}{l_P^3\lambda}}, \quad \frac{\Delta\varepsilon}{|\varepsilon|} \sim \sqrt{\frac{\hbar}{l_P^3\lambda|\varepsilon|}}. \tag{49}$$

In the present epoch, as well as in a significant part of the inflation phase, the vacuum temperature $T_{vac} \sim \hbar H$ (see subsection 4.1) is very small compared to the maximum (in absolute value) energy of particles in the Dirac sea $|\varepsilon| \sim \hbar / l_P$. Therefore, the vacuum energy density $|\varepsilon| \sim \hbar/l_P^4$, and we obtain the estimate

$$\begin{gathered}\Delta\varepsilon/|\varepsilon| \sim \sqrt{l_P / \lambda} \sim 10^{-16} \\ \lambda \sim 1\text{ cm}, \quad l_P \sim 10^{-32}\text{ cm}.\end{gathered} \tag{50}$$

Note that under the condition of long-wavelength limit validity, it makes sense to assume $\lambda/a \sim \text{const}_1$, $l_P/a \sim \text{const}_2$, and therefore $l_P/\lambda \sim \text{const}$. Consequently, the estimate (50) remains valid in a very wide range, including a significant part of the inflation phase. In this range, the use of classical Einstein equations to describe macroscopic dynamics is justified. However, when approaching the Big Bang point, the vacuum temperature becomes too high. As a result, the energy density $|\varepsilon|$ rapidly decreases in absolute value due to the population of positive energy levels by fermions. Estimates (49) and (50) become incorrect, and in the situation $\Delta\varepsilon/|\varepsilon| \sim 1$ classical equations are inapplicable. The dynamics becomes completely quantum. A more detailed study of the transition boundary from classical to quantum description is not within the scope of this work.

Let us demonstrate the tendency of mean energy to approach zero with increasing temperature using a simple example. Consider a single fermi-particle that can only be in two states with energies $\pm\varepsilon$. Let the particle be placed in a thermostat with zero chemical potential and such temperature that $\varepsilon/T \to 0$. Then $\varepsilon = \langle\varepsilon\rangle \to 0$.

4.1. Solving Einstein's Equations with Bare Cosmological Constant

We assume that the energy-momentum tensor of an ideal relativistic fluid is suitable for solving the given problem:

$$T_b^a = (\varepsilon + p)U^a U_b - p\delta_b^a. \tag{51}$$

We work in an orthonormal basis in which the metric tensor

$$\eta_{ab} = \text{diag}(1,-1,-1,-1).$$

In the right-hand side of equation (51), the symbols ε and p denote energy density and pressure respectively, and these quantities include their vacuum values. Since fermion fields, unlike boson fields, contribute negatively to vacuum energy, and according to the standard model, there are multiple times more fermion fields than boson fields, we assume $\varepsilon < 0$. Moreover, lattice regularization means that $|\varepsilon|,|p| < \infty$. U^a is the average 4-velocity of a macroscopic lattice region. In our case $U^a = (1,0,0,0)$. To compensate for vacuum energy, a finite positive cosmological constant Λ_0 is introduced into Einstein's equations:

$$\mathfrak{R}_b^a - \frac{1}{2}\delta_b^a\mathfrak{R} = 8\pi G T_b^a + \Lambda_0\delta_b^a. \tag{52}$$

In lattice theory, the cosmological constant is introduced naturally (see (9)). We assume that the cosmological constant (of restored dimension)

$$\begin{gathered}\Lambda_0 = \text{const} \sim l_P^{-2}, \\ l_P \sim \sqrt{\frac{8\pi G\hbar}{c^3}} \sim 10^{-32}\text{cm}.\end{gathered} \tag{53}$$

Note that the bare cosmological constant on the lattice, like all other constants and variables, is dimensionless and of order unity. For the metric, we use the ansatz (47). To avoid overloading the formulas, we introduce the notation

$$8\pi G\varepsilon = \tilde{\varepsilon}, \quad 8\pi G p = \tilde{p}. \tag{54}$$

All components of Einstein's equations reduce to two independent ones:

$$3\frac{\dot{a}^2}{a^2} = \Lambda_0 + \tilde{\varepsilon}, \quad 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \Lambda_0 - \tilde{p}. \tag{55}$$

The equation $\nabla_a T_b^a = 0$ is a consequence of equations (55) and therefore redundant. Using the Hubble parameter $H(t)$ equations (55) can be rewritten as follows:

$$2\dot{H} + (\tilde{\varepsilon} + \tilde{p}) = 0, \quad 3H^2 - (\Lambda_0 + \tilde{\varepsilon}) = 0. \quad (56)$$

Thus, we have three unknown functions $\{\tilde{\varepsilon}(t), \tilde{p}(t), H(t)\}$ and two equations (56). The missing equation is the equation of state relating energy density and pressure. Regarding the equation of state of relativistic matter, the following facts are known: (i) for real dust matter $p = 0$; (ii) for real ultra-relativistic matter $p = \varepsilon/3$; (iii) for energy density and pressure of vacuum in de Sitter space $p = -\varepsilon$. In all three cases, energy density and pressure are linearly related. Moreover, these quantities have the same dimension. Therefore, for energy density and pressure including vacuum values, we adopt the following hypothesis of linear relationship:

$$\tilde{p} = \varkappa\Lambda_0 + (\varkappa - 1)\tilde{\varepsilon} \leftrightarrow \tilde{\varepsilon} + \tilde{p} = \varkappa(\tilde{\varepsilon} + \Lambda_0). \quad (57)$$

This equation is linear and inhomogeneous with an unknown dimensionless function $\varkappa(t)$, whose asymptotics are further determined based on known dynamics. The system of equations (56) and (57) has the following solution:

$$\dot{H} = -\frac{3}{2}\varkappa H^2 \rightarrow$$

$$H(t) = H_0\left(1 + \frac{3}{2}H_0\int_{t_0}^{t}\varkappa(t')dt'\right)^{-1}, \quad (58)$$

$$\tilde{\varepsilon}(t) = -\Lambda_0 + 3H_0^2\left(1 + \frac{3}{2}H_0\int_{t_0}^{t}\varkappa(t')dt'\right)^{-2}, \quad (59)$$

$$\tilde{p}(t) = \Lambda_0 + 3(\varkappa(t) - 1)H_0^2\left(1 + \frac{3}{2}H_0\int_{t_0}^{t}\varkappa(t')dt'\right)^{-2}. \quad (60)$$

Here H_0 is an integration constant acting as the Hubble parameter at the beginning of the inflation phase and $t_0 > 0$.

From solutions (58)–(60), it is evident that case (i) corresponds to the value $\varkappa(t) = 1$, case (ii) corresponds to the value $\varkappa(t) = 4/3$, case (iii) corresponds to the value $\varkappa(t) = 0$.

Let us indicate some properties of the solution (58)–(60). The estimates given below are rather rough and are of a model nature. Let us accept the following estimates for the duration of the inflation phase t_{inf} and for the constant H_0:

$$t_{inf} \cong 10^{-37}\,\text{c}, \quad H_0 \cong 10^{39}\,\text{c}^{-1} \quad (61)$$

Hence $H_0 t_{inf} \cong 100$.

Estimates of the Hubble parameter value vary greatly in different sources. We accepted the value $H \sim 1.7 \cdot 10^{15}\,\text{GeV}$, which is equivalent to value (61) [10]. In paper [11], an estimate of the Hubble parameter value at the end of the inflation phase is given: $\sqrt{G}H < 10^{-5}$. This value corresponds to $H \sim 10^{38}\,\text{cm}^{-1}$, which is close to value (61).

Let's take $\varkappa(t_0) \cong 1/150$ and assume that during time t_{inf} the function $\varkappa$ changes insignificantly. This assumption means (see the first of equations (58)) that in the inflation phase $|\dot{H}| \ll H^2$. The latter inequality is a necessary condition for inflation [12]. Then in the time interval $t_0 < t < t_0 + t_{inf}$ solutions (58)-(60) take the form

$$H(t) \cong H_0, \quad \tilde{\varepsilon}(t) \cong -\tilde{p} \cong -\Lambda_0 + 3H_0^2. \quad (62)$$

Thus, during inflation, the scale factor $a(t)$ increases by $\exp(H_0 t_{inf}) \approx \exp 100 \approx 10^{43}$ times.

Let's assume that when $t > t_0 + t_{inf}$, the function $\varkappa(t)$ becomes equal to $\varkappa = 4/3$. In this case, solutions (58)–(60) transform into solutions expanding according to the power law:

$$H(t) \cong \frac{1}{2t}, \quad \tilde{\varepsilon}(t) \cong -\Lambda_0 + \frac{3}{4t^2}, \quad \tilde{p} \cong \Lambda_0 + \frac{1}{4t^2}. \quad (63)$$

Solution (63) shows that the scale factor and real matter density change according to the wellknown law, as well as the correct equation of state in the case of ultra-relativistic matter:

$$a(t) \propto \sqrt{t}, \quad \rho_{real} = \frac{3}{32\pi G t^2}, \quad p_{real} = \frac{1}{3}\varepsilon_{real}. \quad (64)$$

Here the dimension is restored.

From equalities (59) and (60), it can be seen that as we approach the Big Bang point, the values $|\tilde{\varepsilon}|$ and $\tilde{p}$ decrease and become equal to

$$\tilde{\varepsilon} = -(\Lambda_0 - 3H_0^2),$$
$$\tilde{p} = \Lambda_0 - 3(1 - \varkappa(t_0))H_0^2$$

at $t = t_0$. If we assume that at $t \to 0$ the function $\varkappa \to 0$ and $3H_0^2 \to \Lambda_0$, then the energy-momentum tensor tends to zero at the Big Bang point. However, it was shown above that such continuation of the classical solution in the immediate vicinity of the Big Bang point is impossible due to the development of quantum fluctuations. Nevertheless, the tendency of the energy-momentum tensor to approach zero in the classical solution as we approach the Big Bang point shows the consistency of the presented quantum and classical approaches.

Let's estimate the temperature of the T_c phase transition that breaks PT-symmetry. For this, we will use Volovik's result [13–16], who exactly calculated the local temperature of the "vacuum" in de Sitter space:

$$T_{vac} = \frac{\hbar H}{\pi}. \quad (65)$$

Although in the studied theory the Hubble parameter, unlike in the case of de Sitter space, is not constant, here we will use formula (65) to estimate the temperature.

It was shown that at the phase transition point, the classical Einstein equations (56) are not valid. However, we use them only for qualitative estimation. Since at the phase transition point the average energy-momentum tensor of fermions equals zero ($\varepsilon = 0$), then according to the second equation (56) we have $H_c \sim \sqrt{\Lambda_0}$. From this and using (65), (53) we obtain the estimation

$$T_c \sim \frac{\hbar c}{l_P} \sim 10^{18}\,\text{GeV}, \quad \text{or} \quad T_c \sim 10^{31}\,\text{K}. \quad (66)$$

The phase transition temperature can also be estimated as the energy of the Dirac sea contained in the Planck volume

$$V_P \sim l_P^3 : T_c \sim (\hbar c / l_P^4) l_P^3 \sim \hbar c / l_P.$$

This temperature is of the same order of magnitude as the Grand Unification temperature.

Let's also estimate the temperature value in Kelvin at the beginning of the inflation phase, when according to some estimates $H_0 \sim 10^{39}\ \text{c}^{-1}$. Then

$$T_0 \sim \frac{\hbar H_0}{k} \sim 10^{28}\,\text{K}. \quad (67)$$

The temperature estimate (67) corresponds to known estimates of temperature in the initial phase of inflation.

4.2. Brief review of the divergent cosmological constant problem

The cosmological constant problem lies in the fact that the energy density of quantum zeropoint fluctuations in vacuum diverges as the fourth power of the momentum space cutoff parameter, and currently there is no generally accepted solution for how to compensate for this enormous energy density.

It appears to us that above in this section, a possible solution to the problem is presented, which is correct in the case when spacetime has the property of granularity (lattice) at the smallest scales. Indeed, the introduction of a finite bare cosmological constant leads to a meaningful solution of Einstein's equations: in the initial phase, we have exponential expansion of the Universe (inflation regime), which transitions into the known power-law expansion in the ultra-relativistic matter regime.

We find it appropriate to give here a very brief and incomplete review of attempts to solve this problem within the framework of traditional quantum field theory.

In the fundamental review [17], the following statements were made regarding the divergent vacuum energy. (i) In flat Minkowski spacetime, these divergences generally occur, but in the case of supersymmetric theories, they completely cancel out. (ii) In curved spacetime, even in the case of supergravity, the cosmological constant diverges. (iii) String theory also does not save the situation.

Among more recent and specialized works, we note papers [18–23]. In these works, efforts are directed at solving the cosmological constant problem through microscopic analysis. The probabilities of the following processes were calculated. Consider a massive particle in de Sitter space. Such a particle generates similar particles over a sufficiently long time. This problem was studied for both free and interacting fields. A similar process occurs in flat spacetime for massive charged particles in a constant and uniform electric field: particle-antiparticle pairs are created, which reduce the initial electric field. In the case of de Sitter space, the idea is that pair production also leads to a decrease in the cosmological constant over time. Unfortunately, the cited works did not study the back-reaction of quantized matter fields on spacetime geometry. Perhaps further efforts in this direction will lead to solving the cosmological constant problem.

In paper [24], the average energy-momentum tensor of a quantized scalar field was calculated in the case of anisotropic and time-varying classical metric. Regularization was carried out as follows: the average energy-momentum tensor calculated for the stationary vacuum was subtracted from the obtained value.

The authors of paper [25] study such field theory models which, while not being supersymmetric, have an equal number of bosonic and fermionic degrees of freedom. In this case, the highest fourth-order divergences cancel out in the quantum average of the energy-momentum tensor. It is shown what conditions the renormalized field masses must satisfy for the remaining divergences to cancel. This approach can be called a fine-tuning method of the theory, resulting in the disappearance of infinite vacuum energy.

Paper [26] appears interesting and complementary to the present work, as it also introduces a seed cosmological constant. The reduction of vacuum energy is a dynamic effect rather than a result of fine-tuning.

Another interesting approach to solving the problem, using thermodynamic ideology, is presented in paper [27] (see also references to works by F. R. Klinkhamer and G. E. Volovik). The main idea is as follows. Suppose that the system reaches a state of thermodynamic equilibrium and consider the grand thermodynamic potential Ω, referred to the spatial volume V:

$$\Omega(\beta, \mu, V) = -P(T, \mu)V. \tag{68}$$

right-hand side of equation (68) tends to zero, as the Universe has no external pressure at all. But the effective energy-momentum tensor of matter is formed by potential (68). Therefore, the effective energy density of matter, including vacuum energy, is estimated as $\varepsilon \sim \Omega/V \to 0$.

Paper [28] criticizes the use of metric (47) due to strong fluctuations of all fields at Planck scales of frequencies and wavelengths. This conclusion is made based on studying the correlators of energy-momentum tensor components of quantized material fields. The calculation shows that even in the case of free quantized fields, the vacuum averages of energy-momentum tensor components and their fluctuations are of the same order and they diverge as the fourth power of the cutoff parameter. However, correlators between long-wavelength and short-wavelength degrees of freedom tend to zero. Therefore, long-wavelength dynamics can be considered independently of fluctuations at the Planck scale. Thus, the use of metric (47) is legitimate in the case of describing low-frequency physics. However, this assumption becomes incorrect with decreasing scale factor, increasing temperature, and decreasing average energy density (see Section 3).

5. CONCLUSIONS

In works [29–32] and [5], the equality was proposed

$$\left\langle e^{a}_{\ \nu_1\nu_2} \right\rangle = \kappa^{(0)}_{(\nu_1\nu_2)} \left\langle \Theta^{a}_{\ \nu_1\nu_2} \right\rangle, \tag{69}$$

which holds both in the lattice theory of gravity, identical to the one studied here, and in the continuous theory. Relation (69) is also confirmed by the fact that the quantities under the average sign transform equally under all symmetries of the theory, including discrete PT-symmetry. Relation (46), obtained by direct calculation in the continuum limit at zero temperature, is another argument in favor of the validity of relation (69).

Therefore, if hypothesis (69) is true, then in the high-temperature phase due to (45), the left side of equation (69) vanishes:

$$\left\langle e^{a}_{\ \nu_1\nu_2} \right\rangle_{\text{Gauge Fix}} = 0. \tag{70}$$

This result is also obtained directly, in the same way as result (45) was obtained. The last equality means that in the studied model of lattice gravity theory, space collapses to a point in the high-temperature phase.

However, in the long-wavelength limit at low temperature, the energy-momentum tensor of the Dirac field is non-zero (46), the quantized tetrad field fluctuates weakly, and its average is non-zero.

The above means that in the studied lattice theory of gravity there is a temperature phase transition (possibly more than one). In the high-temperature phase, space collapses to a point, the average of the energy-momentum tensor is zero, and discrete PT-symmetry is not broken. Conversely, in the low-temperature phase, these quantities are non-zero and PT-symmetry is broken. The role of the order parameter is played by the average $\langle e^a_\mu \rangle$, which becomes nonzero in the low-temperature phase. In the low-temperature phase, the process of exponential space expansion begins, transitioning to a power-law expansion. During the phase transition from high-temperature to low-temperature phase, domains with opposite values of averages $\langle e^a_\mu \rangle$ and $\langle \Theta^a_\mu \rangle$. may form. The domain wall between such domains was studied in work [6].

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