ATOMS, MOLECULES, OPTICS

TWO-TEMPERATURE DISTRIBUTION OF ATOMS UNDER SUB-DOPPLER COOLING CONDITIONS

© 2024 A. A. Kirpichnikova, R. Ya. Ilyenkov, and O. N. Prudnikov*

Institute of Laser Physics, Siberian Branch of the Russian Academy of Sciences 630090, Novosibirsk, Russia Received April 3, 2024; revised May 8, 2024 Accepted for publication May 8, 2024

**e-mail: oleg.nsu@gmail.com*

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Abstract. The problem of sub-Doppler laser cooling of atoms under "optical molasses" conditions in fields formed by counter-propagating waves with different polarization configurations is considered, with full accounting for quantum recoil effects. It is shown that the distribution of cold atoms is not in equilibrium but can nevertheless be approximated by two Gaussian functions and, accordingly, characterized by temperatures of "cold" and "hot" fractions. A detailed analysis of the atomic fractions and their temperatures depending on the parameters of light fields is carried out. Based on the obtained results, the concept of weighted average temperature can be introduced, which corresponds to the average kinetic energy of atoms.

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1. INTRODUCTION

Laser cooling of atoms is a basic tool of modern quantum physics and contributes to the development of many directions with fundamental and practical applications. Among the main ones are the devising of modern precision frequency standards [1–4], embrace a new direction of atomic sensors based on matter wave interference [5–7], quantum computing [8,9], and quantum communications [10]. Subsequent application of evaporative cooling methods allows achieving ultra-low temperatures at which quantum properties of Bose and Fermi condensates emerge, which is of separate interest for research [11, 12].

From a classical perspective, the impact of light on atoms is described in terms of forces being by nature a radiative light pressure on moving atoms, as well as forced dipole forces, arising from the re-emission of field photons by atoms between different spatial field modes [13–15]. The "quantum" nature of atom-photon interaction within the quasi-classical approach is described by the fluctuation of forces acting on the atom, which allows describing atomic kinetics both within the Fokker-Planck equation [16, 17] for the atomic distribution function in phase space, and in its equivalent approach based on stochastic equations of motion for individual atoms – Langevin equations [18, 19].

An alternative to quasi-classical approaches is our developed fully quantum approach, which allows solving the problem of laser cooling of atoms within boundaries of quantum kinetic equation for the atomic density matrix [20–23]. The presented approach allows obtaining a stationary numerical solution of the quantum kinetic equation for the atomic density matrix, containing complete information about both internal and translational degrees of freedom of the atom in a laser field. The analysis of the problem within the quantum approach reveals features associated with the finite atoms' recoil parameter while interacting with field photons, $\varepsilon_R = E_k / \hbar\gamma$ ($E_k = \hbar\omega_R$ — kinetic energy received by a stationary atom when interacting with a field photon, ω_R — recoil frequency, γ — natural linewidth of the atomic transition), in contrast to the quasi-classical approach, where this parameter is considered extremely small, $\varepsilon_R \ll 1$.

Taking into account the influence of quantum recoil effects, discreteness of momentum and energy transferred to an atom during interaction with field photons is most relevant task both for laser cooling using narrow optical transitions [24] and for cooling atoms characterized by an insufficiently small parameterм ε_R [25]. In particular, the presented quantum approach made it possible to compare the efficiency of sub-Doppler laser cooling of atoms in fields with spatially inhomogeneous polarization, formed by counter-propagating waves with opposite circular polarizations (σ_+–σ_--field configuration), or orthogonal linear polarizations (lin (*lin*⊥*lin*-configuration) [26]. Addirionally it has been shown that the momentum distribution of cold atoms is significantly non-equilibrium and, strictly speaking, cannot be described in terms of temperature. Therefore, within theoretical approaches for describing laser cooling, we used the average kinetic energy of atoms, which can be expressed in temperature units. Experimentally, the temperature of cold atoms is calculated by approximating the momentum distribution with a Gaussian function, and the result may depend on approximation methods. For example, in work [27], besides the narrow component of momentum distribution characterized by sub-Doppler temperature, the presence of a broader component was shown, which looks like a "substrate." However, its width turns out to be comparable to the Doppler limit temperature, which generally corresponds to a two-temperature distribution of cold atoms.

In this work, within the boundaries of our developed quantum approach [22], we conduct a detailed analysis of the non-equilibrium distribution of atoms in the problem of sub-Doppler laser cooling under conditions of optical molasses, taking into full account quantum recoil effects. This problem can also be applied as an approximate description laser cooling of atoms in a magneto-optical trap (MOT), since atoms are cooled in the center of the MOT where the magnetic field is zero. It was found that the temperatures of the "cold" and "hot" fractions of atoms and their proportions depend not only on the parameters of the used field but also on the chosen configuration of light fields and the recoil parameter ε_R. The presented results allow us to judge the cooling regimes in which a significantly two-temperature distribution of atoms emerges, and allow us to describe conditions for maximizing the proportion of atoms in the "cold" fraction, which is of separate interest for creating a source of cold atoms.

2. PROBLEM STATEMENT

An ensemble of low-density atoms with negligible interatomic interaction is cooled in a monochromatic field resonant with a closed optical transition $F_g \to F_e$, where F_g and F_e — are the total angular momenta of the ground (g) and excited (e) states. Consider configurations of monochromatic field formed by counter-propagating waves of equal intensity:

$$\mathbf{E}(z,t) = E_0(\mathbf{e}_1 e^{ikz} + \mathbf{e}_2 e^{-ikz})e^{-i\omega t} + \text{c.c.}, \quad (1)$$

where E_0 — is the complex amplitude of light waves; ω — field frequency; $k = \omega / c$ — wave vector. Polarizations $\mathbf{e}_1$ and $\mathbf{e}_2$ of counter-propagating waves in the Cartesian basis $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ are expressed through components of vectors $\mathbf{e}_{0,\pm 1}$ in the cyclic basis:

$$\mathbf{e}_n = \sum_{\sigma=0,\pm 1} e_n^{\sigma} \mathbf{e}_{\sigma}, n = 1,2. \quad (2)$$

Here $\mathbf{e}_\sigma$ — are unit vectors of the circular basis: $\mathbf{e}_{\pm 1} = \mp(\mathbf{e}_x \pm \mathbf{e}_y)/\sqrt{2}, \mathbf{e}_0 = \mathbf{e}_z$. In this work, we will consider the most common configurations of light fields formed by counter-propagating waves with orthogonal polarizations, in which sub-Doppler laser cooling mechanisms can emerge [28]:

1) *lin*⊥*lin*-configuration of the light field with $\mathbf{e}_1 = \mathbf{e}_x$, formed by a pair of counter-propagating waves with orthogonal linear polarizations,

2) $\sigma_+ - \sigma_-$-configuration of the light field with $\mathbf{e}_1 = \mathbf{e}_+$ and $\mathbf{e}_2 = \mathbf{e}_-$, formed by a pair of counter-propagating waves with circular polarizations.

A feature of these configurations is that the spatial dependence of the polarization vector (1) is determined by only one parameter of the light field. Thus, in the lin *lin*⊥*lin* configuration, only the ellipticity of the light field depends on the coordinate, periodically changing the polarization from circular to linear and back along the axis z. In the case $\sigma_+ - \sigma_-$-configuration, the polarization of the light field at each point is linear, but the tilt angle of the axis changes periodically along the$z$ z-axis (see, for example, papers [28, 29]).

To describe the evolution of a low-density atomic ensemble, we use the quantum kinetic equation for the atomic density matrix $\hat{\rho}$:

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \hat{\Gamma}\{\hat{\rho}\}, \tag{3}$$

where $\hat{H}$ is the Hamiltonian, and $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation of the atom while interacting with vacuum modes of the electromagnetic field, i.e., as a result of spontaneous decay. The atomic Hamiltonian $\hat{H}$ is divided into the sum of contributions:

$$\hat{H} = \frac{\hat{\rho}^2}{2M} + \hat{H}_0 + \hat{V}, \tag{4}$$

where the first term is the kinetic energy operator; M — atomic mass; $\hat{H}_0 = -\hbar\delta\hat{P}_e$ — Hamiltonian of a free atom in the rotating wave approximation (RWA); $\delta = \omega - \omega_0$ — detuning of the optical frequency ω from the atomic transition frequency ω_0;

$$\hat{P}_e = \sum_{\mu} |F_e,\mu\rangle\langle F_e,\mu| \tag{5}$$

— projection operator for excited state levels $|F_e,\mu\rangle$, characterized by total angular momentum F_e and angular momentum projection μ on the quantization axis. The last term $\hat{V}$ describes the interaction of the atom with field (1). The interaction of the atom with the field resonant to the electric dipole transition is described by the interaction operator of the following form:

$$\hat{V} = \hat{V}_1\exp(ikz) + \hat{V}_2\exp(-ikz), \tag{6}$$
$$\hat{V}_n = \hbar\frac{\Omega}{2}(\hat{D}e_n) = \hbar\frac{\Omega}{2}\sum\hat{D}_\sigma e_n^\sigma,$$

where Ω is the Rabi frequency of the electric dipole transition, and is determined by the polarization vectors of counter-propagating waves and vector operator $\hat{D}$, whose matrix components $\hat{D}_\sigma$ in the circular basis are expressed through Clebsch-Gordan coefficients:

$$\hat{D}_\sigma = \sum_{\mu,m} C^{F_e,\mu}_{F_g,m;1,\sigma} |F_e,\mu\rangle\langle F_g,\mu|. \tag{7}$$

The last term of the kinetic equation (3), describing the relaxation of the atomic density matrix accounting recoil effects, is determined by the expression (see, for example, papers [20–23])

$$\hat{\Gamma}\{\hat{\rho}\} = \frac{\gamma}{2}(\hat{P}_e\hat{\rho} + \hat{\rho}\hat{P}_e) - \frac{3\gamma}{2}\times$$
$$\times\left\langle \sum_{\xi=1,2} (\hat{D}e_\xi(\mathbf{k}))^\dagger \exp(-i\mathbf{k}\hat{\mathbf{r}})\hat{\rho}\exp(i\mathbf{k}\hat{\mathbf{r}})(\hat{D}e_\xi(\mathbf{k})) \right\rangle_{\Omega_k}, \tag{8}$$

where $\langle\ldots\rangle_{\Omega_k}$ denotes averaging over the directions of spontaneous photon emission with momentum $\hbar k$ with two orthogonal polarizations $e_\xi(\mathbf{k})$.

Note that the solution of the quantum kinetic equation (3) for the considered type of optical transition $F_g \to F_e$ can be characterized by three parameters: the ratio of detuning to the natural line width δ/γ, the recoil parameter ε_R and the light shift determined by the depth of the optical potential [20–23]:

$$U = \frac{\hbar|\delta|}{3}\frac{|\Omega|^2}{(\delta^2 + \gamma^2/4)}, \tag{9}$$

proportional to the laser field intensity. To find a stationary solution of the quantum kinetic equation (3) and analyze the achievable limits of laser cooling, we further use our proposed approach, detailed in papers [20–23].

3. TWO-TEMPERATURE DISTRIBUTION

Note that during laser cooling, the state of the cold atom ensemble is significantly non-equilibrium [30] and, strictly speaking, cannot be described in terms of temperature. Therefore, in works [17, 26] the average kinetic energy of atoms was used as a measure of cooling

$$\langle E_{kin}\rangle = \int \frac{p^2}{2M} W(p)dp, \tag{10}$$

where $W(p)$ is the momentum distribution function. This expression allows determining the temperature T_E as a measure of average kinetic energy for the atom ensemble,

$$\langle E_{kin}\rangle = \frac{N}{2}k_B T_E, \tag{11}$$

where N is the dimension of the problem, k_B is the Boltzmann constant. For a thermodynamically

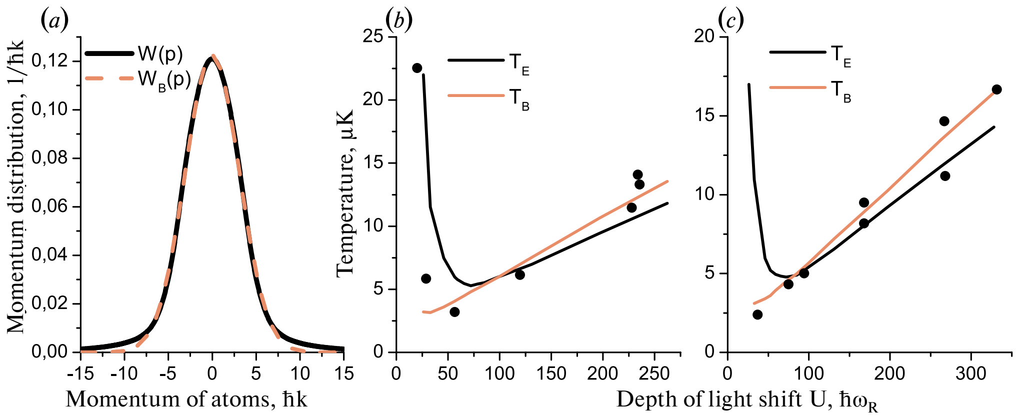

Fig. 1. (*a*) — Momentum distribution of the atom ensemble ^{85}Rb $W(p)$ — black line — and its approximation with a single Gaussian function $W_B(p)$ — dashed red line ($T = 3.5$ μK) — in the field of lin *lin*⊥*lin*-lin-configuration, resonant to the closed optical transition $5S_{1/2}(F_g = 3) \to 5P_{3/2}(F_e = 4)$ at field detunings $\delta = -8\gamma$, $U = 50\hbar\omega_R$ ($\Omega = 0.9\gamma$). (*b*, *c*) — Temperature of cold atoms ^{85}Rb as a function of light field intensity at field detuning $\delta = -4\gamma$ (*b*) and $\delta = -8\gamma$ (*c*). Here, the black line indicates the temperature as a measure of the average kinetic energy of atoms T_E (11), the red line shows the Boltzmann temperature T_B, calculated by approximating the momentum distribution with a Gaussian function. Black dots represent the results of measuring atom temperature in the lin *lin*⊥*lin*-configuration field [32]. Recoil parameter $\varepsilon_R = 6.4 \cdot 10^{-4}$

equilibrium state, the temperature T_E coincides with the classical definition of temperature. Mostly usage of term "temperature" implies momentum distribution to be described by the Maxwell-Boltzmann distribution for an ideal gas of non-interacting particles. The probability density for such distribution can be written as

$$W_B(p) = C\exp\left(-\frac{p^2}{2Mk_BT_B}\right), \tag{12}$$

where C — is the normalization constant, and T_B — is the Boltzmann (classical) temperature.

Note that the non-equilibrium state of atoms emerges even for extremely small recoil parameters $\varepsilon_R < 10^{-3}$, i.e., under conditions where quasi-classical approaches are applicable [17]. For example, Fig. 2 *a* shows the momentum distribution of cold atoms ^{85}Rb in the field of *lin*⊥*lin*-configuration, calculated by numerical solving of equation (3), and its approximation by a Gaussian function. For atoms ^{85}Rb the recoil parameter $\varepsilon_R = 6.4 \cdot 10^{-4}$ can be considered extremely small. Nevertheless, a deviation of the distribution function $W(p)$ from the normal distribution (12) is observed to lead to discrepancies in temperature determinations T and T_E (see Fig. 1 *b*, *c*). Such deviation from equilibrium distribution can explain the scatter in ensemble atom temperature in laser cooling experiments. The scatter in temperature measurements in paper [31], obtained through numerical solving of equation (3), corresponds to definitions T_B and T_E (Fig. 1 *b*, *c*).

Furthermore, for atoms with insufficiently small recoil parameters,

$$10^{-3} < \varepsilon_R < 1 \tag{13}$$

the momentum distribution of the atomic ensemble, calculated from the numerical solving of the quantum kinetic equation (3), significantly differs from the Maxwell-Boltzmann distribution (Fig. 2 *a*). This leads to the classical temperature T_B (12) to differ significantly from the characteristic temperature T_E (11) and cannot be used to describe the ensemble kinetics not only quantitatively but also qualitatively (Fig. 2 *b*, *c*). Thus, for the thermodynamic description of the cooled atomic system, the introduction of an alternative characteristic is required. One way to describe non-equilibrium systems is with a two-temperature distribution, where instead of one Gaussian function (12), the momentum distribution is approximated with two Gaussian functions:

$$W_D(p) = \frac{N_{hot}}{\sqrt{2\pi Mk_BT_{hot}}}\exp\left(-\frac{p^2}{2Mk_BT_{hot}}\right) + \\ + \frac{N_{cold}}{\sqrt{2\pi Mk_BT_{cold}}}\exp\left(-\frac{p^2}{2Mk_BT_{cold}}\right). \tag{14}$$

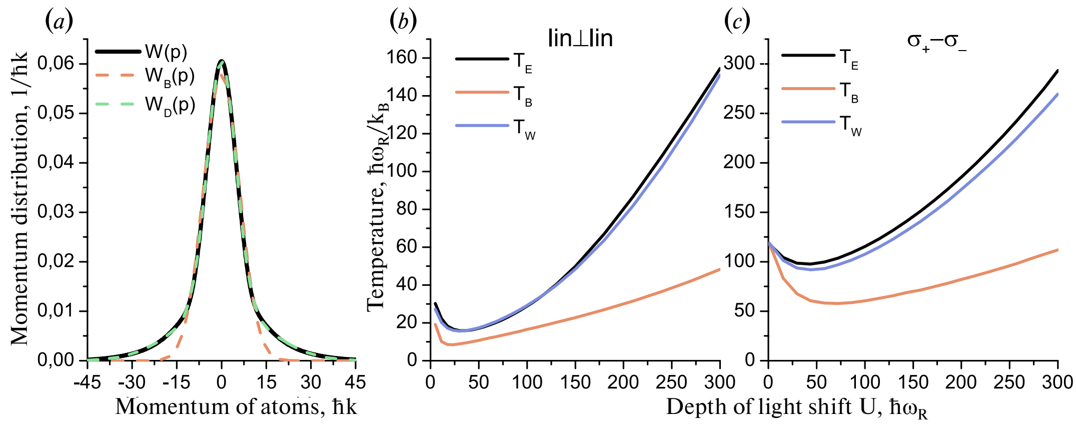

Fig. 2. (*a*) — Momentum distribution of the atomic ensemble $W(p)$ (black line) and its approximation with one Gaussian function $W_B(p)$ (dashed red line) and two Gaussian functions $W_D(p)$ (dashed green line) for the recoil parameter $\varepsilon_R = 10^{-2}$ in the field *lin⊥lin*-configuration with detuning $\delta = -2\gamma$ at $U = 240\hbar\omega_R$. Optical transition $F_g = 1 \to F_e = 2$. (*b, c*) — Temperature of cold atoms, defined as characteristic T_E (11), Boltzmann T_B (12) and weighted T_W (15) in fields (*b*) *lin⊥lin*- and (*c*) $\sigma_+ - \sigma_-$-configuration

Thus, the ensemble of atoms is divided into two fractions: "cold" — with lower temperature T_{cold}, characterizing the central part of the distribution, and "hot" — with higher temperature T_{hot}, characterizing the "substrate" of the distribution. Parameters N_{cold} and N_{hot} determine the proportions of atoms in the fractions, $N_{cold} + N_{hot} = 1$. For such distribution, a weighted temperature of "cold" and "hot" fractions can be introduced:

$$T_W = N_{hot}T_{hot} + N_{cold}T_{cold}. \tag{15}$$

Indeed, the two-temperature interpretation describes the momentum distribution of cold atoms significantly better (see Fig. 2 *a*). The weighted temperature T_W (15) better corresponds to the characteristic temperature T_E (Fig. 2*b*, *c*) and, thus, can be used to characterize laser cooling of atoms. The two-temperature distribution allows analyzing characteristics not only of the ensemble of cooled atoms as a unified system but also the proportions of "cold" and "hot" fractions depending on different cooling parameters. Maximizing the proportion of atoms in the "cold" fraction determines the efficiency of sub-Doppler laser cooling.

As is well known [28], sub-Doppler laser cooling of atoms occurs in fields with spatially inhomogeneous polarization, resonant with a closed optical transition of an atom $F_g \to F_e$ with angular momentum projection degenerate levels. Further, for comparative analysis of sub-Doppler laser cooling and the resulting two-temperature distribution of atoms, we will consider cooling within the framework of a model transition $F_g = 1 \to F_e = 2$, for which sub-Doppler cooling mechanisms are present in both light field configurations, *lin⊥lin* and $\sigma_+ - \sigma_-$.

The presented results of the "cold" fraction ratio in Fig. 3 show that the choice of light field configuration fundamentally affects the thermodynamic state of atoms. Thus, for lin *lin⊥lin*-lin configuration in the case of extremely small recoil parameters $\varepsilon_R \ll 10^{-3}$ (Fig. 3 *a*) the fraction of "cold" atoms weakly depends on detuning, and for $U > 100\hbar\omega_R$ there is a region of parameters where the fraction equals one. In this case, the momentum distribution is close to the classical Maxwell-Boltzmann distribution and can be described with a single temperature, which corresponds to the results in Fig. 1. However, for $\sigma_+ - \sigma_-$-configuration, even under the condition of extremely small recoil parameter (Fig. 3 *c*) and high cooling field intensity, the fraction of "cold" atoms tends to $1/2$. Consequently, for $\sigma_+ - \sigma_-$-field configuration for extremely small recoil parameters $\varepsilon_R \ll 10^{-3}$, the steady state of the ensemble of cooled atoms has a vividly expressed two-temperature distribution, which was also observed experimentally in work [27]. Meanwhile, for larger recoil parameter $\varepsilon_R = 10^{-2}$ (Fig. 3 *b*, *d*) the transition to classical Maxwell-Boltzmann distribution does not occur even for lin *lin⊥lin*-configuration of the field. A strong dependence on field detuning appears to grow with increasing *U*, and the fraction of "cold" atoms does not reach a constant value, but, on the contrary, begins to decrease.

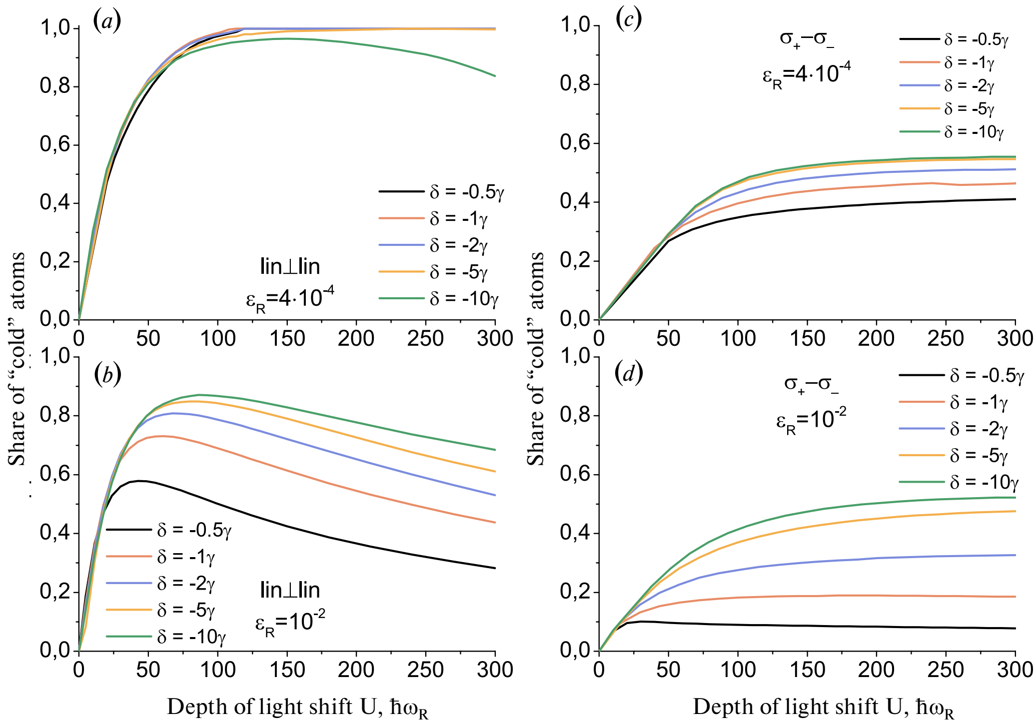

Fig. 3. Dependencies of "cold" atoms fraction on light shift U in $\varepsilon_R = 4 \cdot 10^{-4}$ (a, c) in and $\varepsilon_R = 10^{-2}$ (b, d) in field (a, b) *lin⊥lin*- and (b, d) $\sigma_+ - \sigma_-$-configuration with different detunings δ

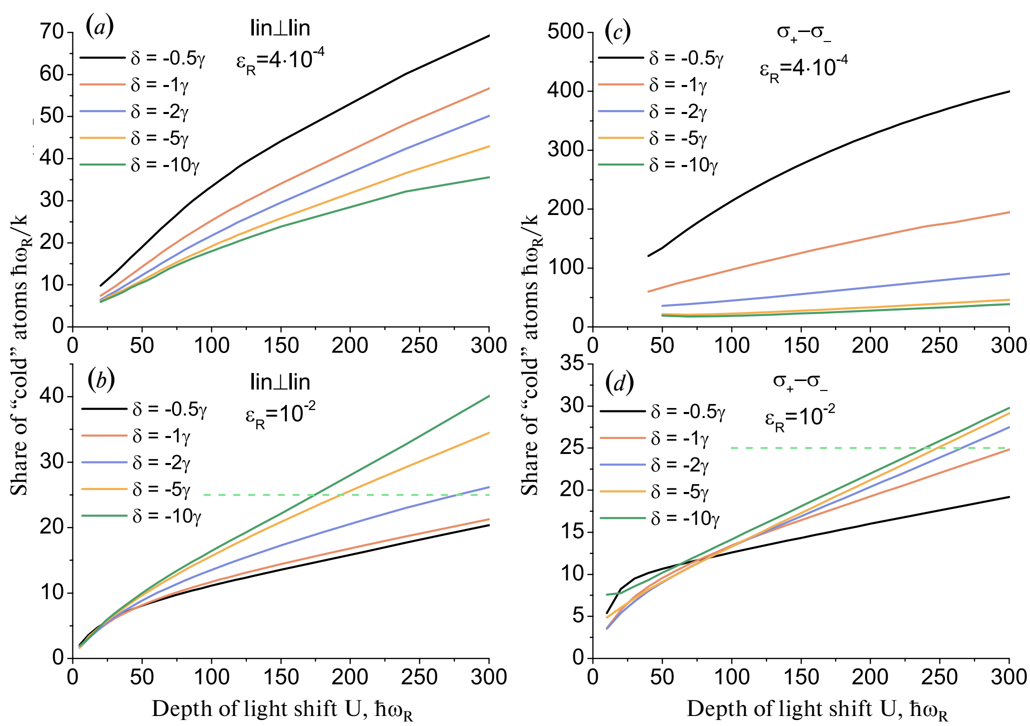

Fig. 4. Dependencies of "cold" atoms temperature on light shift U for $\varepsilon_R = 4 \cdot 10^{-4}$ (a, c) in and $\varepsilon_R = 10^{-2}$ (b, d) in field (a, b) *lin⊥lin*- and (c, $г$) $\sigma_+ - \sigma_-$-configuration with different detunings δ. Dashed lines indicate the Doppler limit

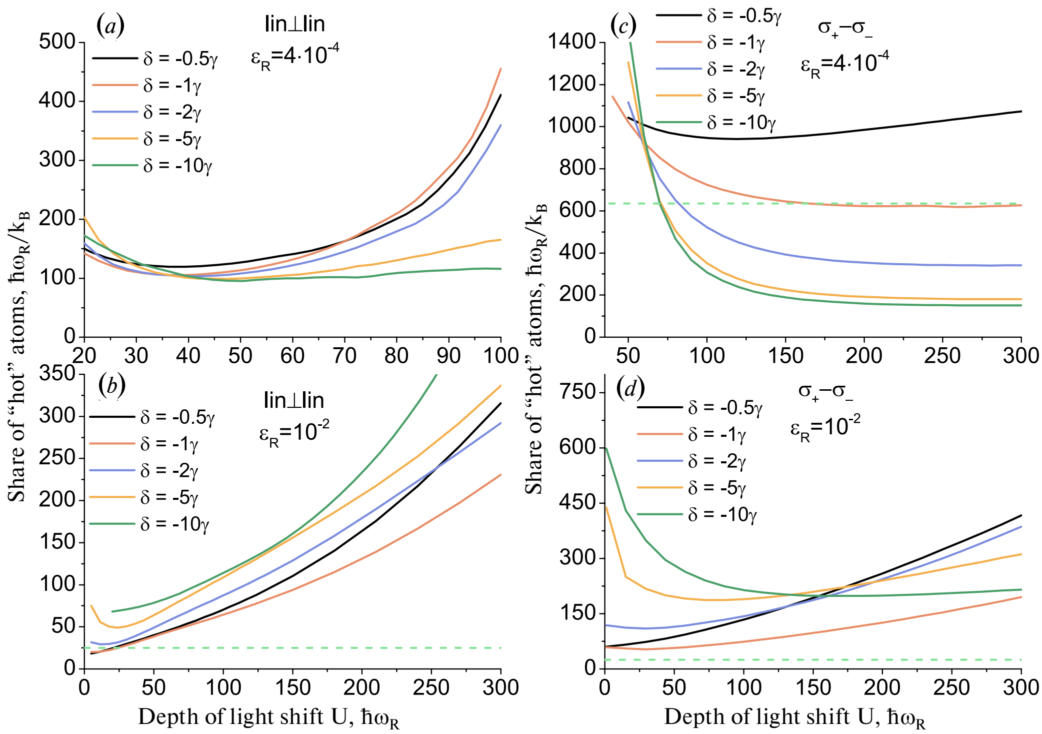

Fig. 5. Dependencies of "hot" atoms temperature on light shift U for $\varepsilon_R = 4 \cdot 10^{-4}$ (a, c) in and $\varepsilon_R = 10^{-2}$ (b, d) in fields (a, b) *lin⊥lin*- and (c, d) $\sigma_+ - \sigma_-$-configuration with different detunings δ. Dashed lines indicate the Doppler limit

Analysis of the temperature of the "cold" atomic fraction is presented in Fig. 4. The temperature T_{cold} increases with increasing cooling field intensity, which is consistent with known theories of sub-Doppler laser cooling [14, 28, 29, 26]. For atoms with extremely small recoil parameters $\varepsilon_R \ll 10^{-3}$ the temperature of the "cold" fraction is below the Doppler limit. However, with an insufficiently small recoil parameter $\varepsilon_R \gtrsim 10^{-2}$ (13), an inverse dependence on the detuning value is observed (Figs. 4 b, d): the lowest temperatures are achieved at the lowest detunings δ. The same effect is observed for the "hot" fraction (Fig. 5). In the cooling mode with an insufficiently small recoil parameter (13) at small parameter values U the temperature of the "cold" fraction is below the Doppler limit (Figs. 4 b, d), while the temperature of the "hot" fraction is, conversely, higher (Figs. 5 b, d). Furthermore, the proportion of the "cold" fraction also decreases with increasing recoil parameter (Figs. 3 b, d). This means that for atoms with $\varepsilon_R \gtrsim 10^{-2}$ it is the proportion and temperature of the "hot" fraction that determine the weighted temperature T_W. Nevertheless, for the *lin⊥lin*-configuration, one can identify a parameter region where the temperatures of both "cold" and "hot" fractions are below the Doppler limit (Fig. 5 b).

Thus, the data presented in Figs. 3, 4, allow selecting light field intensities (parameter U) at a chosen detuning δ for atoms with a given value ε_R, enabling maximization of the "cold" fraction and/or minimization of temperature (either of the "cold" fraction or weighted).

Let's examine in detail the influence of the recoil parameter value ε_R on the characteristics of the two-temperature distribution of the atomic ensemble. In the case of lin *lin⊥lin*-polarization (Fig. 6 a, b, c) it can be noted that for extremely small recoil parameters $\varepsilon_R \ll 10^{-3}$ with increasing parameter U for "cold" atoms rapidly grows to one (Fig. 6a), i.e., at small U the energy of the entire ensemble is determined by the temperature of the "hot" fraction of atoms, and at large U — by the temperature of the "cold" fraction. For $\varepsilon_R \geq 8 \cdot 10^{-3}$ there is an optimum in U for the fraction of "cold" atoms. At the same time, for $\sigma_+ - \sigma_-$-polarization at large values of parameter U the fraction of "cold" atoms reaches a certain constant value, close to 1/2 at extremely small ε_R (Fig. 6 d).

As we showed earlier in paper [26], the influence of quantum recoil effects for atoms with $\varepsilon_R \gtrsim 10^{-2}$ reduces the efficiency of sub-Doppler laser cooling

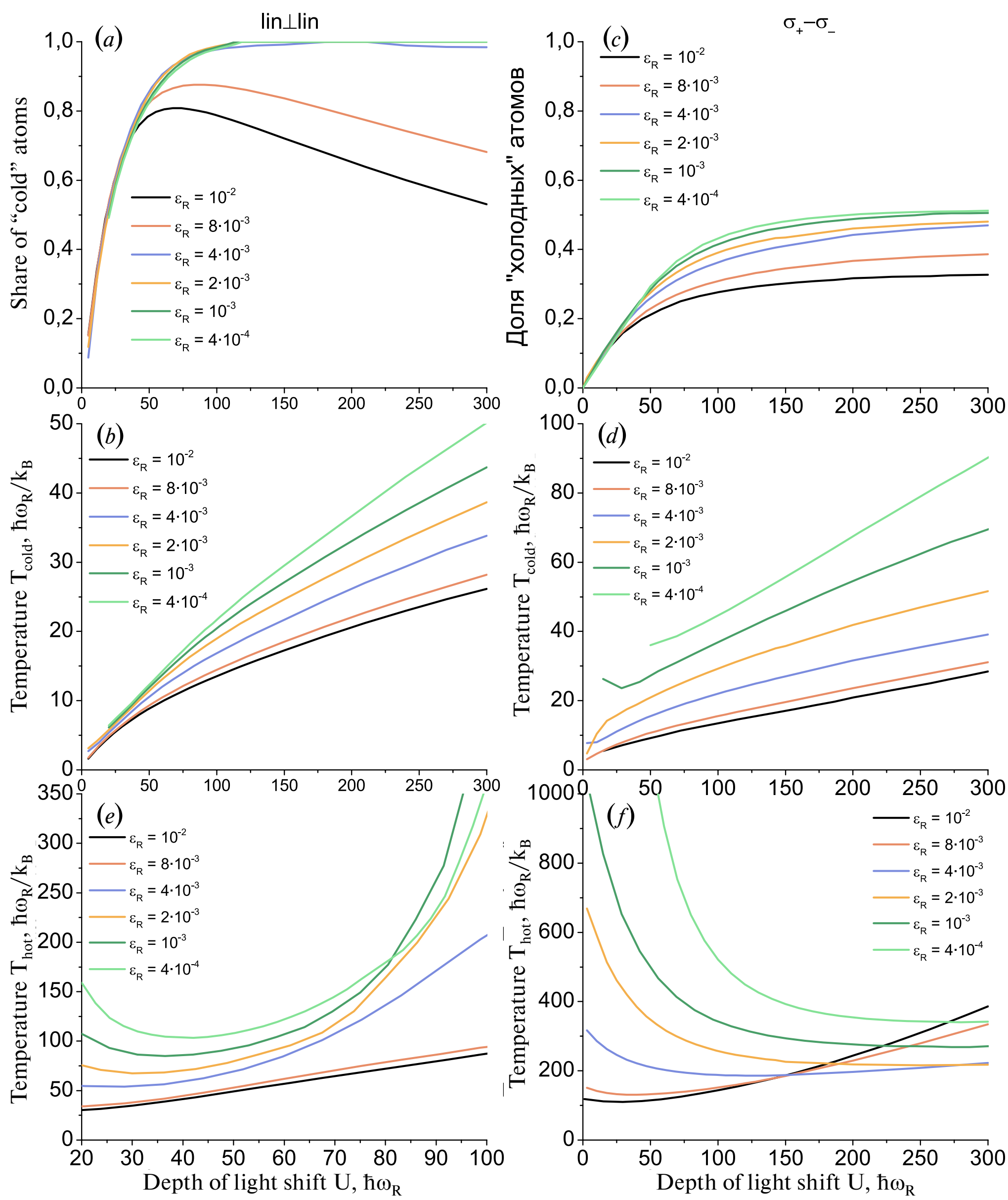

Fig. 6. Dependencies of the fraction of "cold" atoms (a, d), temperature of "cold" (b, e) and "hot" (b, f) atoms on the light shift U in fields (a, b, c) *lin⊥lin*- and (d, e, f) $\sigma_+ - \sigma_-$-configuration with detuning $\delta = -2\gamma$ for different recoil parameters ε_R

mechanisms. The temperature of the "cold" fraction (Figs. 6 b, f) can be seen to remain below the Doppler limit, however, their fraction decreases (Figs. 6 a, d). Thus, the weighted temperature T_W is mainly determined by the "hot" fraction, whose temperature significantly decreases (Figs. 6 c, f).

4. CONCLUSIONS

Temperature is one of the key characteristics used to describe laser cooling of atoms. Its definition for specific thermodynamic systems is fundamental. The classical definition of temperature using the Maxwell-Boltzmann distribution (12) describes a classical system of non-interacting particles. However, in the laser cooling problem, the interaction of atoms with single photons of the field leads to the fact that the particle system is not in thermodynamic equilibrium and, strictly speaking, cannot be described using the Maxwell-Boltzmann distribution, i.e., the classical definition of temperature may be inapplicable.

Within this work, a significant discrepancy was shown between the classical Boltzmann temperature

T_B and the characteristics of the ensemble of "cold" atoms. It was demonstrated that a two-temperature distribution characterized by the proportions of "cold" and "hot" atomic fractions and their temperatures can be used to describe the ensemble of "cold" atoms. The introduced concept of "weighted temperature" T_W (15) can be used for quantitative description of laser cooling of the entire atomic ensemble.

When considering the problem of laser cooling of atoms in an optical molasses with different recoil parameters ε_R we discovered that the temperatures of "cold" and "hot" fractions depend not only on the cooling laser field parameters but also on its chosen configuration. For atoms with an extremely small recoil parameter $\varepsilon_R \ll 10^{-3}$ for *lin*$\perp$*lin*-configuration, the fraction of "cold" atoms with increasing U tends to unity, i.e., it is effectively described by a single-temperature distribution, while for $\sigma_+ - \sigma_-$-configuration, the fraction of "cold" atoms tends to 1 / 2. Thus, even in the case of an extremely small recoil parameter value $\varepsilon_R \ll 10^{-3}$, which is detailed within well-known quasi-classical approaches, for $\sigma_+ - \sigma_-$-configuration of the light field, the thermodynamic state of the ensemble is substantially non-equilibrium and can be described in terms of a two-temperature distribution. This is particularly important considering that the standard method of laser cooling used in experiments includes cooling in a magneto-optical trap, which is formed by such fields. Meanwhile, optimizing the proportion of the "cold" fraction and its temperature is a separate task for implementing effective laser cooling. Without such optimization, the efficiency of sub-Doppler cooling may be reduced because most of the cooled atoms will end up in the "hot" fraction, perceived as a "substrate," since its temperature is an order of magnitude higher than the temperature of the "cold" fraction (on the order of and greater than the Doppler limit temperature).

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