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METHOD FOR ESTIMATING THE NUMBER OF ATOMS IN MAGNETO-OPTICAL TRAPS

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Abstract. A method for estimating the number of atoms in a magneto-optical trap and in the flux from a hot atom source is presented and examined in detail. The applicability of the method at various cooling stages is analyzed. The relative uncertainty of the method is calculated. The results of applying the described method on optical lattice clocks with cold strontium and ytterbium atoms are presented.

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1. INTRODUCTION

Laser cooling of atoms is used in laboratories worldwide for the development of devices such as optical lattice clocks (OLC) [1] and atomic interferometers (AI) [2,3]. Additionally, laser cooling is used in fundamental physical research [1]. In all these tasks, a magneto-optical trap (MOT) is used, among other things, for laser cooling.

The implementation of OLCs are carried out using two elements: stationary OLCs are being created and modernized using strontium-87 atoms [4] and transportable OLC development is being conducted using ytterbium-171 atoms [5]. It should be noted that in the research and development of OLCs using ytterbium atoms, isotope ^{174}Yb has been used to date. This isotope was used for initial OLC adjustment for several reasons: firstly, its relative abundance is more than 2 times higher ($31,8\%$ for ^{174}Yb and $14,3\%$ for ^{171}Yb) [6], secondly, due to a higher fluorescence level compared to ^{171}Yb - by approximately 3 times [7]. To develop an OLC, it is first necessary to prepare an ensemble of cold atoms for loading them into an optical lattice and ultimately probe the clock transition [1]. The probing result depends on how many atoms were successfully cooled at each stage. Accordingly, to optimize the setup operation, it is necessary to evaluate the number of atoms at each cooling stage. This is true not only for OLCs but also for other setups using cold atoms. To optimize cooling, a method for estimating the number of atoms in MOT was implemented using a series of atomic cloud images obtained with a CCD camera, similar to works [8, 9].

In OLC development, two-stage atom cooling in MOT is used. The first stage cooling is carried out on the transition $^1S_0$ - $^1P_1$. For further temperature reduction, a weaker transition $^1S_0$ - $^3P_1$ is used.

For first stage cooling of strontium atoms, transition at 461 nm wavelength with a natural linewidth of 30 MHz is used, and for second stage cooling, a transition with a wavelength of 689 nm with a natural linewidth of 7.4 kHz is used. Strontium atom cooling is implemented in a vacuum chamber, the model of which is shown in Fig. 1.

For atomic beam studies, the camera was placed on the collimation section (marked by a rectangle in Fig. 1), through which the fluorescence signal was collected onto the CCD camera matrix. To estimate the number of atoms in the MOT, the camera was placed at the side window (marked by a circle in Fig. 1), through which the fluorescence signal was collected onto the CCD camera matrix.

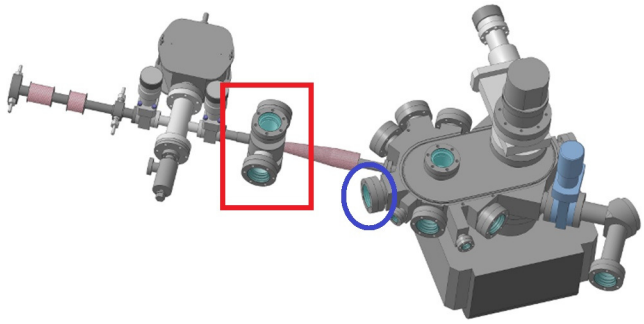

Fig. 1. 3D model of the general view of the optical spectroscope for cooling strontium atoms. The rectangle marks the collimation section, the circle marks the side window where the camera was placed to estimate the number of atoms in the MOT

For first stage cooling of strontium atoms, transition at 399 nm wavelength with a natural linewidth of 29 MHz is used, and for second stage cooling, a transition with a wavelength of 556 nm with a natural linewidth of 182 kHz is used. The cooling of ytterbium atoms is implemented in a vacuum chamber, the model of which is shown in Fig. 2.

For atomic beam studies, the camera was placed on the collimation section (marked by a square in Fig. 2). To estimate the number of atoms in the MOT, the camera was placed at the side window (marked by a circle in Fig. 2). The fluorescence signal was collected onto the CCD camera matrix similarly to the experiment with strontium atoms.

2. METHOD FOR ESTIMATING THE NUMBER OF ATOMS

To optimize the parameters of first stage cooling of strontium atoms and start second stage cooling, it is necessary to know the number of atoms in the first stage MOT, as well as the number of atoms in the flux created by the hot atom source. The number of atoms in the first stage MOT can be determined by relating it to the number of photons scattered (reemitted) by the atom cloud. According to work [9], the number of photons N_0, observed during fluorescence of one atom, is determined by the expression:

$$N_0 = \Gamma_{sc} f T_0 \alpha \Delta t \tag{1}$$

Here $f = \Omega/4\pi = D^2/16b^2$ is the ratio of the solid angle captured by the imaging system to the total solid angle 4π, D – aperture diameter from which the light signal is collected (in our case D is determined by the lens in front of the CCD camera), b - distance from the atom cloud to the aperture (lens), $T_0 = P_t/P_i$ – optical

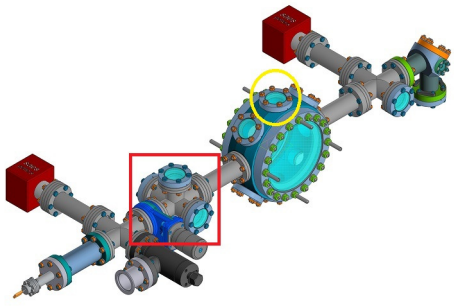

Fig. 2. 3D model of the vacuum chamber of the optical spectroscope for cooling ytterbium atoms. The square marks the collimation section, the circle marks the side window where the camera was placed to estimate the number of atoms in the MOT

transmission coefficient between the cloud and camera, determined by the ratio of transmitted optical power P_t to incident P_i, α – quantum efficiency of the detector, Δt – observation time (exposure of the measuring device), Γ_{sc} – scattering rate:

$$\Gamma_{sc} = \frac{\gamma}{2} \cdot \frac{I/I_s}{1 + I/I_s + (2\delta/\gamma)^2} \tag{2}$$

where γ – spectral linewidth of the transition from which fluorescence is observed, δ – frequency detuning from this transition, I – light field intensity, I_s – saturation intensity, determined by the expression $I_s = \pi hc\gamma/(3\lambda^3)$, h – Planck's constant, c – speed of light, λ – wavelength of radiation. The number of atoms N_{at} in MOT can be estimated through the ratio of total number of photons N_{ph} to the number of photons emitted by one atom N_0:

$$N_{at} = \frac{N_{ph}}{N_0} = \frac{N_{ph}}{\Gamma_{sc} f T_0 \alpha \Delta t} \tag{3}$$

In our case, the value N_{ph} is determined experimentally using a calibrated CCD camera (the calibration process is discussed in more detail below). Let

$$P = \tfrac{N_{ph} hc}{(\lambda \Delta t)}$$

be the optical power entering the camera. Then the number of photons hitting the CCD matrix of the camera during exposure time Δt, will be equal to:

$$N_{ph} = \frac{P\lambda\Delta t}{hc} \tag{4}$$

The same expression is valid for each pixel of the matrix individually ($N_{ph} = \sum_{px} N_{phpx}$):

$$N_{phpx} = \frac{P_{px}\lambda\Delta t}{hc} \tag{5}$$

where P_{px} - is the optical power per 1 pixel. Therefore, (3) takes the form:

$$N_{at} = \frac{\lambda \sum_{px} P_{px}}{\Gamma_{sc} hc f T_0 \alpha} = \frac{2\tau\lambda(1 + S_0 + (4\pi\tau\delta)^2) \sum_{px} P_{px}}{hc f T_0 \alpha S_0} \tag{6}$$

where $S_0 = I/I_s$ – is the ratio of laser radiation intensity to saturation intensity, τ is the lifetime of the upper state. To determine P_{px} the equation obtained from the description of SDU-R285 camera parameters [10, 11] is used:

$$P_{px} = \frac{\alpha I_{px} k_{grad} \Delta t_{grad}}{\Delta t \cdot 2^{u/128}} \tag{7}$$

where I_{px} – pixel brightness (from 0 to 255), k_{grad} is the camera calibration coefficient linking I_{px} with P_{px} and determined experimentally, Δt_{grad} is the camera exposure during calibration, u is the camera gain when imaging the atom cloud to determine atom number. It should be noted that when using other detectors, (7) may take a different form.

During calibration, the gain is set to zero. During the calibration process, laser radiation of the corresponding wavelength and known power is directed onto the CCD camera matrix. The beam is cut off by a diaphragm positioned in front of the camera so that the intensity distribution in the beam cross-section is uniform. Only those pixels from the rectangular matrix that received laser radiation are taken into account. After completing the measurements, a graph is plotted using the obtained data showing the power per pixel, versus pixel brightness in the image. In our case, this dependence is linear. The slope coefficient $k_{g\,rad}$ is determined from the graph.

Thus, the final equation for estimating the number of atoms trapped in MOT from the image obtained with the CCD camera is derived:

$$N_{at} = \frac{2\tau\lambda(1 + S_0 + (4\pi\tau\delta)^2) k_{grad} \Delta t_{grad} \sum_{px} I_{px}}{hc f T_0 S_0 \Delta t \cdot 2^{u/128}} \tag{8}$$

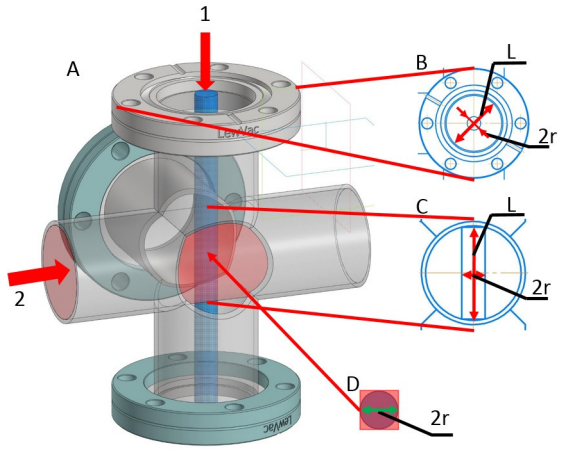

Fig. 3. Diagram of laser beam interaction with atomic flux. 1 - laser beam, 2 - atomic flux, A - six-way cross with schematic propagation of atomic flux and laser radiation, B - ratio of laser beam cross-section to internal vacuum tube size, C - ratio of laser radiation and flux areas, D - area ratios for determining average transit time

Using the obtained expression (8) it is possible to estimate the number of atoms in the image, knowing all the necessary parameters.

The number of atoms per time unit , F in the flux is determined by expression: $F = N_{at}/\Delta t$, however in this case, to calculate N_{at} using (8) for the observation time, instead of Δt it is necessary to use the time atoms take to pass through the detecting laser beam t_{fl}. The assumption that all atoms pass through the laser beam along all trajectories in the same time is only possible in the case of a square cross-section of the laser beam. Due to the fact that the beam cross-section is circular (see Fig. 3, element D), causing atoms with different trajectories to pass through the beam along different paths and, consequently, in different times, it is necessary to introduce a correction coefficient derived from the ratio of the laser beam cross-sectional area to the square area. Thus, the transit time will be determined as:

$$t_{fl} = \frac{\pi r}{2v_{beam}} \tag{9}$$

where r – is the radius of the laser beam cross-section with Gaussian intensity distribution, $v_{beam} = \sqrt{3k_BT/M}$ – is the velocity of atoms ejected from the source [12] (k_B is the Boltzmann constant, T is the temperature of the atom source in Kelvin, M is the mass of atoms). It should be noted that interaction with atoms occurs only in the part of the vacuum volume of the collimating section where laser radiation propagates (see Fig. 3, elements B,C). Therefore, only part of the flux is illuminated by laser radiation, which actually has a cross-section equal to that of the vacuum tube. Thus, to calculate the number of atoms per time unit, F for the total flux, the obtained result must be multiplied by the ratio of the laser beam volume to the atomic flux volume $l^2/4r^2$, where l is the length of the interaction region with atoms. Accordingly, using expression (8) the atomic flux F is determined as:

$$F = \frac{l^2\tau\lambda(1 + S_0 + (4\pi\tau\delta)^2)k_{grad}\Delta t_{grad}\sum_{px} I_{px}}{2r^2\Delta thcfT_0S_0t_{fl}\cdot 2^{u/128}} \tag{10}$$

It should be noted that the theoretical calculations did not use any approximations related to the choice of a specific cooling method and, consequently, there are no limitations on the applicability of the presented method only to the first stage MOT and atomic flux. The fluorescence signal can be obtained in both the second stage MOT and the optical lattice - it is necessary to add a detecting beam illuminating the atoms. In the case of using more precise detectors, calibration must also be performed taking into account the specifics of the equipment used.

Table 1. Results of uncertainty calculation for the method of estimating the number of strontium atoms in the first stage MOT

	u_i	$Value$	%
λ, nm	$5.8 \cdot 10^{-6}$	$4.6 \cdot 10^{2}$	$\frac{2u_C}{N_{at}}$=4
$P_i(front)$, W	$1.0 \cdot 10^{-4}$	$2.0 \cdot 10^{-3}$	
$P_i(left)$, W	$3.4 \cdot 10^{-4}$	$6.7 \cdot 10^{-3}$	
$P_i(right)$, W	$3.4 \cdot 10^{-4}$	$6.8 \cdot 10^{-3}$	
δ, MHz	5.8	34.0	
P_{grad}, W	$1.0 \cdot 10^{-5}$	$2.0 \cdot 10^{-4}$	
Δt_{grad}, s	$1.0 \cdot 10^{-8}$	$2.5 \cdot 10^{-5}$	
b, mm	$5.8 \cdot 10^{-2}$	$2.0 \cdot 10^{2}$	
D, mm	$5.8 \cdot 10^{-2}$	35.0	
r, mm	$5.8 \cdot 10^{-2}$	$1.2 \cdot 10^{2}$	
$P_t(front)$, W	$1.4 \cdot 10^{-4}$	$2.8 \cdot 10^{-3}$	
$P_t(left)$, W	$4.7 \cdot 10^{-4}$	$9.4 \cdot 10^{-3}$	
$P_t(right)$, W	$4.8 \cdot 10^{-4}$	$9.6 \cdot 10^{-3}$	
t, s	$1.0 \cdot 10^{-8}$	$1.0 \cdot 10^{-2}$	
l, mm	$5.8 \cdot 10^{-2}$	36.6	$\frac{2u_C}{F} = 35$
T, K	2.9	723.0	

3. UNCERTAINTY ASSESSMENT OF THE PROPOSED METHODS FOR STRONTIUM AND YTTERBIUM ATOMS

The uncertainty of estimating the number of atoms, u_C was calculated by determining the contribution from each measured parameter:

$$u_C = \sqrt{\sum_i C_i^2 u_i^2} \tag{11}$$

where u_i – uncertainties related to the resolution of measuring instruments, C_i – weight coefficient determined by the value of the partial derivative of the estimation equation with respect to the measured value.

The results of calculating the uncertainty of the number of atoms in MOT and flux for strontium atoms are presented in the table. Values P_i и P_t represent the laser power incident and transmitted respectively, coupled into the MOT in three directions (front, right, and left). Similar calculations were performed for ytterbium atoms.

Thus, the relative expanded uncertainty of estimating the number of atoms in MOT was determined to be 4%, and the relative expanded uncertainty of estimating the number of strontium atoms in the flux was 35% (see table 1). All values indicated in the table are used to calculate the relative expanded uncertainty of estimating the number of atoms in the flux. To estimate the same parameter in MOT, the last two values from the table are not used.

Similarly, using (11), the relative uncertainty of estimating the number of ytterbium atoms in the flux and MOT was calculated. Thus, the relative expanded uncertainty of estimating the number of atoms in MOT was determined to be 5%, and the relative expanded uncertainty of estimating the number of ytterbium atoms in the flux was 32%.

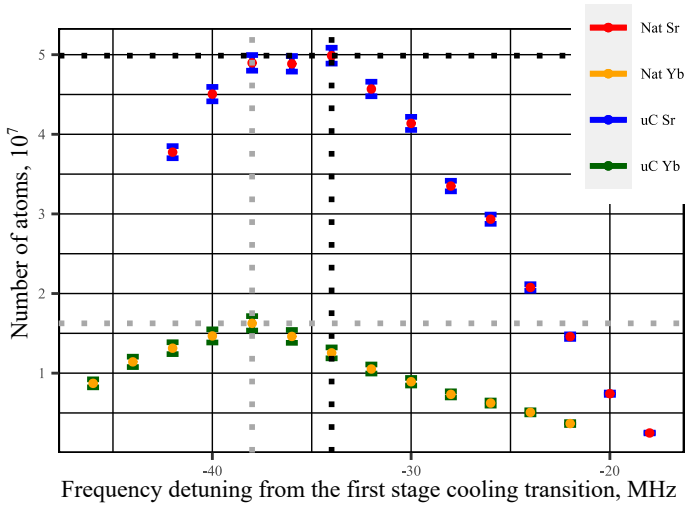

Fig. 4. Dependencies of the number of atoms ^{87}Sr and ^{174}Yb, trapped in the MOT, on the frequency detuning of MOT radiation from the resonance of the first stage cooling transition. The optimum is marked with dashed lines. The given uncertainties show the expanded uncertainty of the estimation of the number of atoms in the MOT

4. RESULTS OF THE FIRST STAGE COOLING OF STRONTIUM-87 ATOMS

In the strontium atom experiment, a Zeeman slower, two repumping laser systems with wavelengths of 679 and 707 nm, and collimating laser radiation were used to increase the number of atoms in the MOT. The first study conducted was an assessment of the number of atoms in the flux created by the hot strontium atom source. The atom source used in this experiment had a length of 32 cm and two heating sections: a reservoir with metallic strontium and a nozzle. The reservoir with metallic strontium was located 9 cm away from the nozzle. The nozzle had a cylindrical shape with a radius of 0.1 cm and length of 1.2 cm. The heating elements were located directly on the source outside the vacuum part. The reservoir was heated to a temperature of about 450 ^{o}C, the nozzle was heated to 700 ^{o}C. The source was located 90 cm from the working zone and 33 cm from the collimator section. To study the flux, a CCD camera with the used lens was placed on the collimator section shown in Fig. 1. Laser radiation from the collimator section was used to excite the atomic transition from which fluorescence was observed. The number of atoms in the flux was estimated from a series of images obtained with the CCD camera.

As a result, the number of atoms in the flux was estimated using (10), with uncertainty calculated using (11). The number of atoms in the flux per second was $(1.0 \pm 0.4) \cdot 10^{11}$.

The second study conducted was an assessment of the dependencies of the number of atoms ^{87}Sr in the first stage MOT on various cooling parameters. The considered parameters are the frequency detuning of radiation from the cooling transition and the magnitude of the magnetic field gradient. The radiation causing atom fluorescence was the MOT laser radiation itself.

As a result, the optimal values of radiation frequency detuning and magnetic field gradient magnitude were determined to obtain the maximum possible number of atoms in the first stage MOT.

For a visual representation of the data, by fixing one of the parameters at the optimal value and scanning the values of the other, we constructed corresponding separate two-dimensional dependencies of the number of trapped atoms. For example, the dependence of the number of atoms ^{87}Sr, trapped in the first stage MOT on the frequency detuning relative to the cooling transition is shown in Fig. 4. The magnetic field gradient in this case is at the previously determined optimal value of 18 G/cm. Thus, the optimum is achieved at a frequency detuning of -34 MHz and a magnetic field gradient of 18 G/cm. The corresponding number of atoms in the MOT was $(2.1 \pm 0.1) \cdot 10^7$, with uncertainty calculated using (11).

To confirm the assumptions made in Section 2 about the applicability of the presented method at various stages of cooling and trapping atoms, a study was conducted on the number of strontium atoms trapped in the second stage MOT and loaded into the optical lattice. The result of the conducted studies was $(5.1 \pm 0.2) \cdot 10^6$ atoms trapped in the second stage MOT, and $(1.2 \pm 0.1) \cdot 10^4$ atoms loaded into the optical lattice.

5. RESULTS OF THE FIRST STAGE COOLING OF YTTERBIUM-174 ATOMS

In addition to studying the atomic flux and number of atoms trapped in the MOT for strontium atoms, similar experiments were conducted for ytterbium. It should be noted that in the experiment with ytterbium atoms, unlike the experiment with strontium atoms, only MOT radiation was used. The isotope ^{174}Yb was studied. The ytterbium atomic source used in the work had a length of 17 cm. The reservoir with metallic ytterbium was located 7 cm from the capillaries. The capillaries are metal tubes 1 cm long with an internal diameter of 0.05 cm. The number of tubes is 20. The heating sleeve was attached to the reservoir inside the vacuum volume. It had only one heating section, maintained at a temperature of 400 ^{o}C. The source was located at a distance of 56 cm from the working zone and 26 cm from the collimator section. To study the flux, the CCD camera with the used lens was placed on the collimator section shown in Fig. 2.

As a result, the number of atoms in the flux was evaluated using (10), with uncertainty calculated using (11). The value obtained was $(8.9 \pm 2.9) \cdot 10^{11}$ atoms per second. The measurement stages sequence exactly repeated the one described for the experiment with ^{87}Sr.

For visual data representation, similarly to the experiment with strontium atoms, by fixing one parameter at its optimal value and scanning the values of another, we constructed corresponding separate two-dimensional dependencies of the number of trapped atoms. For example, the dependence of the number of ^{174}Yb, atoms trapped in the first stage MOT on the magnetic field gradient change is shown in Fig. 4. Thus, the optimum is achieved at a frequency detuning of -38 MHz and a magnetic field gradient of 25 G/cm. The corresponding number of atoms in the MOT was $(1.7 \pm 0.1) \cdot 10^7$,with uncertainty calculated using (11).

6. CONCLUSIONS

The method that allows estimating the number of atoms in the flux from the hot atom source and the number of atoms trapped in the MOT has been developed and described in detail. The proposed method was tested on the OLC setup with cold strontium atoms and applied to the developing transportable OLC with cold ytterbium atoms. Additional studies of the method conducted on the OLC setup with cold strontium atoms showed the possibility of its application in estimating the number of atoms trapped in the second stage MOT, as well as those loaded into the optical lattice. The application of the presented method allows for more precise optimization of laser cooling parameters at each stage. The proposed method, after appropriate calibrations, can be used with other register devices. Since the presented method was tested on two elements (strontium and ytterbium), it can be said that it is applicable to other elements after incorporating corresponding parameters for the atoms used.

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