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# TWO-PHOTON GRAVITON CONVERSION ON BOUND ATOMIC STATES

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**Abstract**. A quantum field approach is developed, combining relativistic electrodynamics and linearized quantum gravity in application to the problem of electromagnetic graviton conversion on bound atomic states. A hydrogen atom is used as an example, and the process of inelastic graviton scattering on an atomic electron with subsequent re-emission of two photons is considered. Expressions for the process cross-section and angular correlations are obtained. The prospects for experimental detection of two-photon graviton conversion using optical amplification of weak signals are discussed.

**Keywords**: *linearized quantum gravity, quantum electrodynamics, S-matrix, hydrogen atom*

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## 1. INTRODUCTION

Under the influence of the fundamental principles of quantum field theory, modern gravitational physics suggests that quantum gravity effects should be observable at extremely high energies and incredibly small length scales [1,2]. This area, known as the Planck scale, includes particle energies reaching GeV, or lengths of the order of m. Access to this range presents significant difficulties, leading to very limited prospects of achieving the necessary technological solutions in the coming decades [3]. The current level of achieved energy range in collisions at the Large Hadron Collider is still many orders of magnitude below the energies associated with the Planck scale. Nevertheless, some hypotheses suggest potentially stronger gravitational interaction at elevated energies, explaining this, in particular, by the presence of additional dimensions [4,5]. Despite ongoing efforts, due to the enormous technological obstacles associated with achieving extremely high energies in particle accelerators, convincing evidence for such extensions of the standard model has not yet been found. In contrast, at lower energies, laboratory experiments with quantum systems with masses significantly exceeding atomic ones [6, 7] are gradually becoming available in laboratory conditions [8]. Within this approach, existing theories suggest that gravity influences the dynamics of quantum states. Recently, a number of ideas have been proposed regarding the manifestation of various aspects of quantum gravity, such as superpositions of gravitational fields [9], gravity-induced wave function collapse through self-gravity, or decoherence [10] caused by external gravitational fields.

It is also encouraging that many of these proposals seem experimentally and technologically achievable in the near future.

Another approach to studying the quantum properties of gravity consists in astrophysical observations of the Universe. In particular, paper [11] noted that the quantum nature of gravity may manifest itself in supernova bursts. Furthermore, a crucial aspect in modern cosmology is understanding how the quantized gravitational field affects the early stages of Universe formation. Although characteristic signatures could have disappeared during the later expansion of the Universe, the detection of primordial gravitational waves could provide insight into quantum gravity effects shortly after the Big Bang [12]. A recent paper [13] also discussed that specific features of the cosmic microwave background may provide additional information about this epoch. Moreover, various astrophysical tests have been proposed, such as studying light from distant quasars . However, no reliable evidence of the quantum nature of gravity has been detected so far.

It should be noted that the detection of individual gravitons, which are a key indicator of gravity quantization, has historically presented significant difficulties [16-18]. Despite this, research into innovative approaches to this problem still continues [19]. Considerable attention is being paid to studying elementary processes related to graviton scattering, particularly in application to astrophysical studies of the early Universe [20], as well as laboratory methods for detecting signs of the quantum nature of gravity. In this regard, the development of appropriate theoretical methods and approaches for investigating this problem becomes extremely important.

Given the relevance of the subject matter in modern theoretical physics, the aim of this study is to apply a quantum field approach that combines relativistic quantum electrodynamics and linearized quantum gravity theory to the problem of graviton scattering on bound atomic states. Using the hydrogen atom as an example, we consider the process of inelastic graviton scattering on an atomic electron with subsequent conversion into two photons. Applying the apparatus of relativistic quantum field theory for bound states, we provide a consistent derivation of the expression for the process cross-section, including angular correlations between incident and scattered particles. We discuss the prospects for experimental detection of two-photon graviton conversion and optical amplification of weak signals.

The paper is structured as follows. Section 2 provides a brief derivation of linearized Einstein equations, discusses their quantization and solution in the form of graviton wave functions in coordinate space. Then, in Section 3, within the framework of the contour line method and *S*-matrix for bound states, a consistent derivation of the cross-section for inelastic graviton scattering on a hydrogen atom is given. Discussion of the obtained expressions as applied to the problem of detecting individual gravitons is given in Section 4. Throughout the paper, relativistic units are used in which $\hbar = c = 1$. The gravitational constant $G$ in these units can be found from the definition of the Planck length

$$l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \cdot 10^{-35} m.$$

## 2. LINEARIZATION OF EINSTEIN EQUATIONS

The simplest quantum field model of gravity with gravitons as a quantum particle with spin 2 can be obtained from considering the linearized Einstein equations for gravity [1]. Following the standard approach to describing this problem [21], we will consider small perturbations $h_{\mu\nu}$ of the Minkowski metric tensor, $\eta_{\mu\nu} = (1, -1, -1, -1)$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{1}$$

Here we assume that $h_{\mu\nu}$ represents small additions defining gravitational interaction. In this approximation, the Ricci tensor is given by the expression

$$R_{\mu\nu} = \frac{1}{2}(\partial_\sigma \partial_\mu h^\sigma_\nu + \partial_\sigma \partial_\nu h^\sigma_\mu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu}) + O(h^2), \tag{2}$$

where the notations $h = \eta^{\mu\nu} h_{\mu\nu}$ and $\Box = \eta^{\mu\nu}\partial_\mu \partial_\nu$ are introduced.

In the linearized quantum theory of gravity, the potential gauge for the electromagnetic potential $h_{\mu\nu}$, satisfying the equation

$$\partial_\mu h^\mu_\nu = \frac{1}{2}\partial_\nu h^\rho_\rho, \tag{3}$$

is called harmonic [21] (this is analogous to the Lorentz gauge $\partial_\mu A^\mu = 0$ for the electromagnetic potential $A^\mu$). Taking into account expression (3), the linearized Einstein equation reduces to the following equality:

$$h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^{\rho}_{\rho}\right) \tag{4}$$

where $T_{\mu\nu}$ is the energy-momentum tensor.

Now let us proceed to discuss the secondary quantization procedure for the tensor field $h_{\mu\nu}$. For this, it is necessary to write down the general solution of the linearized equation of motion in the absence of matter ($T_{\mu\nu} = 0$), which takes the form

$$h_{\mu\nu} = 0. \tag{5}$$

The solution to equation (5) represents a free gravitational wave. Based on these linear equations of motion for metric perturbations, one can construct a consistent theory of free quantum field for $h_{\mu\nu}$. Taking into account the form of equation (5), the quantization of the gravitational field can be performed in complete analogy with the quantization of the electromagnetic field. This leads to the following decomposition for the potential:

$$\hat{h}_{\mu\nu}(x) = \sum_{\lambda_g}\int \frac{d^3\mathbf{k}_g}{(2\pi)^3}\frac{1}{\sqrt{2|\mathbf{k}|}}\varepsilon_{\mu\nu}^{(\lambda_g)} \times \left(\hat{a}^{\dagger}_{\mathbf{k},\lambda_g} e^{-ikx} + \hat{a}_{\mathbf{k},\lambda_g} e^{ikx}\right) \tag{6}$$

Here $\hat{a}^{\dagger}_{\mathbf{k},\lambda_g}$ and $\hat{a}_{\mathbf{k},\lambda_g}$ represent creation and annihilation operators respectively, which follow the canonical commutation relations for bosons [22]; $k = (k_0, \mathbf{k}_g)$ is the four-dimensional momentum vector of the graviton, $\mathbf{k}_g$ is the corresponding three-dimensional wave vector, is $x = (t, \mathbf{r})$ the four-dimensional spatial vector, $\varepsilon_{\mu\nu}^{(\lambda_g)}$ is the polarization tensor with $\lambda_g = 0,1,2,3$.

The harmonic condition given by equation (3) still ambiguously determines the choice of inertial reference frame. Obviously, a new coordinate transformation of the form $h_{\mu\nu}$ where the parameters

$$h'_{\mu\nu}(x) = h_{\mu\nu}(x) - \partial_{\mu}\xi_{\nu}(x) - \partial_{\nu}\xi_{\mu}(x), \tag{7}$$

can be performed on the field $\xi_{\nu}(x)$ satisfy the condition $\xi_{\nu} = 0$.

Since $h_{\mu\nu}$ is a solution to the wave equation (5), the four-dimensional vector $k$ also satisfies the equality $k^2 = k_{\mu}k^{\mu} = 0$. Then the harmonic condition for the polarization tensor, following from equation (3), takes the form [21]

$$k^{\mu}\varepsilon_{\mu\nu} = \frac{1}{2}k_{\nu}\varepsilon_{\mu\mu}. \tag{8}$$

Setting $\xi_{\nu}(x) = i\xi_{\nu}e^{-ikx}$ in equation (7), we can obtain the following relation:

$$\varepsilon'_{\mu\nu} = \varepsilon_{\mu\nu} - k_{\mu}\xi_{\nu} - k_{\nu}\xi_{\mu}. \tag{9}$$

Both of these equations imply that the tensor components $\varepsilon_{\mu\nu}$ cannot be arbitrary: they must satisfy the constraints given by equations (8) and (9). To determine which polarizations are permissible, let us consider a gravitational wave moving along the axis $z$, i.e., choose the four- dimensional momentum vector in the form $k = \omega_g(1,0,0,1)$. Then, according to equation (9), it is possible to choose the function $\xi_{\mu}$ in such a way that the polarization tensor components $\varepsilon_{00}$, $\varepsilon_{13}$, $\varepsilon_{23}$ and $\varepsilon_{33}$ are eliminated. Consequently, only two independent polarization tensors remain for the gravitational wave, which can be written in matrix form as follows:

$$\varepsilon^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \varepsilon^{(2)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

where the coefficient $1/\sqrt{2}$ is chosen to satisfy the normalization condition

$$\sqrt{Tr(\varepsilon\cdot\varepsilon^{T})} = 1.$$

The representation of gravitational wave polarization tensors in the form of equations (10) corresponds to the so-called Transverse Traceless gauge. With this choice, the transversality condition given by equation (8) reduces to

$$k^{\mu}\varepsilon_{\mu\nu} = 0. \tag{11}$$

Taking into account that the timelike components of the polarization tensors are zero, for (10) the parameterization in spherical coordinates is permissible for an arbitrary direction of vector $\mathbf{k}_g$, which is defined by spherical angles $\theta$ and $\phi$ [23]:

$$e^{(1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos^2\theta\cos^2\varphi - \sin^2\varphi & \cos^2\theta\sin\varphi\cos\varphi + \sin\varphi\cos\varphi & -\sin\theta\cos\theta\cos\varphi \\ \cos^2\theta\sin\varphi\cos\varphi + \sin\varphi\cos\varphi & \cos^2\theta\sin^2\varphi - \cos^2\varphi & -\sin\theta\cos\theta\sin\varphi \\ -\sin\theta\cos\theta\cos\varphi & -\sin\theta\cos\theta\sin\varphi & \sin^2\theta \end{pmatrix} \quad (12)$$

$$e^{(2)} = \frac{1}{\sqrt{2}}\begin{pmatrix} -2\cos\theta\sin\varphi\cos\varphi & \cos\theta\cos^2\varphi - \cos\theta\sin^2\varphi & \sin\theta\sin\varphi \\ \cos\theta\cos^2\varphi - \cos\theta\sin^2\varphi & 2\cos\theta\sin\varphi\cos\varphi & -\sin\theta\cos\varphi \\ \sin\theta\sin\varphi & -\sin\theta\cos\varphi & 0 \end{pmatrix} \quad (13)$$

Thus, in linearized theory, all angular correlations in the gravitational wave scattering process are completely determined by the matrices of two possible polarizations (12), (13).

## 3. S-MATRIX APPROACH FOR DESCRIBING GRAVITON SCATTERING ON BOUND STATES

The expression for the cross-section of the two-photon conversion process on a bound electron can be obtained by considering the $S$-matrix element corresponding to the process of atomic electron excitation by a graviton, followed by emission of two photons, see Fig. 1. According to Feynman rules in momentum space for linearized gravity theory, see [24, 25], the corresponding interaction vertex of quantized gravitational field with spinor field is given by the expression [26, 27]

$$\Gamma^{\mu\nu} = -\frac{i\kappa}{8}[(p_1 + p_2)^\mu\gamma^\nu + \gamma^\mu(p_1 + p_2)^\nu - 2\eta^{\mu\nu}(\hat{p}_1 + \hat{p}_2 - 2m)]. \quad (14)$$

Here $\kappa = \sqrt{32\pi G}$, where $G$ is the gravitational constant; $\gamma_\mu = (\gamma_0, \gamma)$ are Dirac matrices; $p_1$ and $p_2$ are momenta of incoming and outgoing spinor particles respectively). The interaction of spinor and photon fields is given by the standard vertex $-ie\gamma^\rho$. Taking into account the conservation law $p_2 = p_1 - k_1$ at the fermion-graviton vertex, the transversality condition (11), properties of the graviton polarization tensor in $TT$-gauge, the $S$-matrix element corresponding to the diagram in Fig. 1 can be written in the coordinate representation as

$$\hat{S}_{if}^{(3)} = (-ie)^2\left(-\frac{ik}{2}\right)\int d^4x_3 d^4x_2 d^4x_1 \overline{\psi}_f(x_3)\times$$
$$\times \gamma^{\mu_3}A^*_{\mu_3}(x_3)S(x_3,x_2)\gamma^{\mu_2}A^*_{\mu_2}(x_2)S(x_2,x_1)\times$$
$$\times(-i\partial^{\nu_1})\psi_i(x_1)\gamma^{\mu_1}h_{\mu_1\nu_1}(x_1). \quad (15)$$

In expression (15) it is taken into account that the momentum operator entering $\Gamma^{\mu\nu}$, in coordinate representation transforms into operator $-i\partial_\rho$; $\psi_a(x) = \psi_a(\mathbf{r})e^{-iE_a t}$, $\psi_a(\mathbf{r})$, is the solution of the Dirac equation for bound electron, is the Dirac energy, $\overline{\psi}$ denotes Dirac conjugation, $E_a S(x_1,x_2)$ is the electron propagator, $A_\mu(x)$ is the photon wave function in coordinate representation [28],

$$A_\mu(x) = \frac{1}{\sqrt{2\omega_\gamma}}e_\mu^{(\lambda_\gamma)}e^{-ikx}. \quad (16)$$

In equation (16) $k$ is the photon momentum 4-vector $(\omega_\gamma, \mathbf{k}_\gamma)$, $\mathbf{k}_\gamma$ – photon wave vector, $\omega_\gamma = |\mathbf{k}_\gamma|$ –photon energy, $x^\rho$ is the spatial 4-vector. For components of the photon polarization 4-vector, the notation is introduced $e_\mu^{(\lambda_\gamma)}$ $(\lambda_\gamma = 0,1,2,3)$, where $A_\mu$ corresponds to the absorbed photon, and $A^*_\mu$(complex

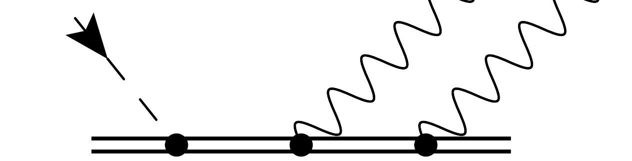

**Fig. 1.** Feynman diagram describing the conversion of an incident graviton (dashed line) into two photons (wavy lines) on a bound atomic electron in the Furry picture (double solid line)

conjugate function) to emitted photon. In complete analogy with (16), the graviton wave function $h_{\mu\nu}(x)$ in expression (15) is written as

$$h_{\mu\nu}(x) = \frac{1}{\sqrt{2\omega_g}} \epsilon_{\mu\nu}^{(\lambda_g)} e^{-ikx}, \qquad (17)$$

where $\omega_g = |\mathbf{k}_g|$ is graviton energy, $\epsilon_{\mu\nu}^{(\lambda_g)}$ are polarization tensor components, see equations (12) and (13). The expansion of electron propagator $S(x_1,x_2)$ in equation (15) over one- electron eigenstates of Dirac Hamiltonian for hydrogen atom can be conveniently written as follows [28]:

$$S(x_1,x_2) = $$
$$= \frac{i}{2\pi}\int_{-\infty}^{\infty} d\Omega\, e^{-i\Omega(t_1-t_2)} \sum_n \frac{\psi_n(\mathbf{r}_1)\bar{\psi}_n(\mathbf{r}_2)}{\Omega - E_n(1-i0)}, \qquad (18)$$

where summation over $n$ implies summation over the entire Dirac spectrum for electron in the Coulomb field of nucleus, including continuum.

The description of real photons also implies the transversality condition $k^{\mu}\epsilon_{\mu} = 0, k^2 = k_{\mu}k^{\mu} = 0$. Then for a photon wave propagating along the $z$, caxis, there are two independent polarizations:

$$\epsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \epsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad (19)$$

For an arbitrary direction of the wave vector $\mathbf{k}_g$ , defined by spherical angles $\theta$ and $\phi$, the spatial part of these polarization vectors can be found by rotating the axis $z$ in the direction $\mathbf{k}_g$. This leads to the following parameterization:

$$\epsilon^{(1)} = \begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix} \quad \epsilon^{(2)} = \begin{pmatrix} \sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \qquad (20)$$

Note that the two graviton polarization tensors given by equations (12) and (13) can be obtained from expressions (20) as a linear combination of tensor products:

$$\epsilon^{(1)} = \frac{1}{\sqrt{2}}(\epsilon^{(1)} \otimes \epsilon^{(1)} - \epsilon^{(2)} \otimes \epsilon^{(2)}), \qquad (21)$$

$$\epsilon^{(2)} = \frac{1}{\sqrt{2}}(\epsilon^{(1)} \otimes \epsilon^{(2)} + \epsilon^{(2)} \otimes \epsilon^{(1)}). \qquad (22)$$

Substituting expressions (18), (17), and (16) into equation (15) and performing integration over time variables and electron propagator frequencies, we obtain

$$\hat{S}_{if}^{(3)} = -2\pi i\,\delta(E_f - E_i + \omega_2 + \omega_1 - \omega_g) U_{if}^{(3)}, \qquad (23)$$

where the process amplitude is denoted as

$$U_{if}^{(3)} = \frac{\kappa e^2}{4\sqrt{2\omega_g \omega_1 \omega_2}} \times \sum_{n_1,n_2} \Bigg[\frac{\langle f|\alpha e_2^{*(\lambda_2)} e^{ik_2 r}|n_2\rangle\langle n_2|\alpha e_1^{*(\lambda_1)} e^{ik_1 r}|n_1\rangle}{E_f + \omega_2 - E_{n_2}} +$$
$$+ \frac{\langle f|\alpha e_1^{*(\lambda_1)} e^{ik_1 r}|n_2\rangle\langle n_2|\alpha e_2^{*(\lambda_2)} e^{ik_2 r}|n_1\rangle}{E_f + \omega_1 - E_{n_2}}\Bigg] \times$$
$$\times \frac{\langle n_1|p_i \alpha_j \epsilon_{ij}^{(\lambda_g)} e^{ik_g r}|i\rangle}{E_i + \omega_g - E_{n_1}}. \qquad (24)$$

Here we omitted the infinitesimal part in the denominators of equation (18) for brevity, and indices $n_{1(2)}$, $i$ and $f$ and imply a set of all quantum numbers describing intermediate, initial, and final atomic states, respectively. The second term in square brackets of expression (24) arises when considering permutations of photon vertices in Fig. 1 among themselves. The remaining four terms, arising from the complete set of all six gauge-invariant Feynman diagrams and also taking into account permutations of photon vertices with the graviton, represent small non- resonant corrections to the resonant scattering amplitude [28, 29] . Thus, amplitude (24) is written in the resonant approximation [28, 29].

If we transition to the non-relativistic limit using the Foldy–Wouthuysen transformation, which in the first order implies $\psi^{+}\alpha\psi \approx \phi^{+}\frac{\mathbf{p}}{m}\phi$, where $\phi$ is the solution of the Schrödinger equation with the corresponding non-relativistic eigenvalue $E$

,and considering that $\mathbf{kr} \ll 1$, then equation (24) simplifies to

$$U_{if}^{(3)} = \frac{\kappa e^2}{4\sqrt{2\omega_g \omega_{\gamma_1} \omega_2}} \times$$

$$\times \sum_{n_1,n_2} \left[ \frac{\left\langle f \left| \mathbf{e}_2^{*(\lambda_2)} \mathbf{p} \right| n_2 \right\rangle \left\langle n_2 \left| \mathbf{e}_1^{*(\lambda_1)} \mathbf{p} \right| n_1 \right\rangle}{E_f + \omega_2 - E_{n_2}} + \right.$$

$$\left. + \frac{\left\langle f \left| \mathbf{e}_1^{*(\lambda_1)} \mathbf{p} \right| n_2 \right\rangle \left\langle n_2 \left| \mathbf{e}_2^{*(\lambda_2)} \mathbf{p} \right| n_1 \right\rangle}{E_f + \omega_1 - E_{n_2}} \right]$$

$$\times \frac{\left\langle n_1 \left| \varepsilon_{ij} p_i p_j \right| i \right\rangle}{E_i + \omega_g - E_{n_1}}. \quad (25)$$

Here, unlike equation (24), it is assumed that the matrix elements are calculated with non- relativistic wave functions and non-relativistic energy spectrum.

Since we are interested in a specific scattering process

$$1s + g \rightarrow 3d \rightarrow 1s + \gamma_1 + \gamma_2,$$

It is necessary to set $i = f = 1s$ and $n_1 = 3d$. Then according to the selection rules for matrix elements of the dipole operator, it immediately follows that only transitions to intermediate states with $n_2 = np$. are allowed. Using the known commutation relation $p_i = i[H_S, r_i]$ and applying the completeness condition

$$\sum_n |n\rangle\langle n| = 1$$

in the matrix element involving only spatial components of the operator $\varepsilon_{ij} p_i p_j$, equation (25) can be rewritten in the “length form” as follows:

$$U_{if}^{(3)} = \frac{\kappa e^2 \omega_2 \omega_1 \omega_g^2}{4\sqrt{2\omega_g \omega_{\gamma_1} \omega_2}} \mathcal{M}(\omega_1, \omega_2). \quad (26)$$

Here and further, the following notation is used for the process amplitude:

$$\mathcal{M}(\omega_1, \omega_2) = \sum_{n, m_p, m_d} \times$$

$$\times \left[ \frac{\left\langle 1s \left| \mathbf{e}_2^{*(\lambda_2)} \mathbf{r} \right| np, m_p \right\rangle \left\langle np, m_p \left| \mathbf{e}_1^{*(\lambda_1)} \mathbf{r} \right| 3d, m_d \right\rangle}{E_{1s} + \omega_2 - E_{np}} + \right.$$

$$\left. + \frac{\left\langle 1s \left| \mathbf{e}_1^{*(\lambda_1)} \mathbf{r} \right| np, m_p \right\rangle \left\langle np, m_p \left| \mathbf{e}_2^{*(\lambda_2)} \mathbf{r} \right| 3d, m_d \right\rangle}{E_{1s} + \omega_1 - E_{np}} \right]$$

$$\times \frac{\langle 3d, m_d \,|\, \varepsilon_{ij}^{(\lambda_g)} r_i r_j \,|\, 1s\rangle}{E_{1s} + \omega_g - E_{3d} + i\frac{\Gamma_{3d}}{2}}, \quad (27)$$

where $m_p$ and $m_d$ are magnetic quantum numbers of states with $p$- and $d$-orbital moments respectively. It is important to note that in the nonrelativistic expression (25), the contribution arising from summation over the negative spectrum exactly cancels out when transitioning from the velocity form to the “length form” (27) when using the aforementioned commutation relation. A detailed discussion of the corresponding transformations can be found in book [30] (see equation (29.37) in chapter 29.8, as well as chapter 35) and works [31, 32].

The differential scattering cross-section can be obtained from the squared modulus of amplitude (23) with subsequent multiplication by the corresponding phase volumes of two emitted photons $\frac{d^3\mathbf{k}_1}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3}$. If the polarizations of particles in initial and final states are considered unknown, then it is also necessary to perform additional summation over all possible polarizations of each particle participating in the considered process. This leads to the following expression for the cross-section:

$$\frac{d\sigma}{d\omega_1 d\Omega_1 d\Omega_2} = \frac{\alpha^2 l_p^2}{2\pi^2} (\omega_1 \omega_g)^3 (\omega_g - \omega_1)^3 \times$$

$$\times \sum_{\lambda_1, \lambda_2, \lambda_g} \left| \mathcal{M}(\omega_1, \omega_g - \omega_1) \right|^2, \quad (28)$$

where it is taken into account that $\kappa^2 = 32\pi G = 32\pi l_p^2$ and $e^2 = 4\pi\alpha$.

If we perform summation over polarizations in equation (28) and projections of angular momenta of intermediate states, using the Wigner–Eckart theorem [33], and also perform angular integration in matrix elements, then the radial and angular dependencies of the scattering cross-section are factorized as follows:

$$\frac{d\sigma}{d\omega_1 d\Omega_1 d\Omega_2} = \frac{\alpha^2 l_p^2}{2\pi^2}(\omega_1\omega_g)^3(\omega_g - \omega_1)^3 \times$$

$$\times \left|\sum_n \left(\frac{D_{1s;np}D_{np;1s}}{E_{1s} + \omega_1 - E_{np}} + \frac{D_{1s;np}D_{np;1s}}{E_{1s} + \omega_g - \omega_1 - E_{np}}\right)\right|^2 \times$$

$$\times \frac{Q^2_{3d;1s}}{(E_{3d} - E_{1s} - \omega_g)^2 + \frac{\Gamma^2_{3d}}{4}} A(\theta_1,\varphi_1,\theta_2,\varphi_2,\theta_g,\varphi_g). \quad (29)$$

The multiplier determining the correlation between the directions of the incident graviton and emitted photons is given by the expression

$$A(\theta_1,\varphi_1,\theta_2,\varphi_2,\theta_g,\varphi_g) = \frac{1}{28800}[8\sin^2\theta_1\sin^2\theta_g \times$$

$$\times \cos 2(\varphi_1 - \varphi_g) + 8\sin 2\theta_1 \sin 2\theta_g \cos(\varphi_1 - \varphi_g) +$$

$$+ 3\cos 2(\theta_1 - \theta_g) + 3\cos 2(\theta_1 + \theta_g) +$$

$$+ 2\cos 2\theta_1 + 2\cos 2\theta_g + 22]\times$$

$$\times [8\sin^2\theta_2\sin^2\theta_g \cos 2(\varphi_2 - \varphi_g) +$$

$$+ 8\sin 2\theta_2 \sin 2\theta_g \cos(\varphi_2 - \varphi_g) +$$

$$+ 3\cos 2(\theta_2 - \theta_g) + 3\cos 2(\theta_2 + \theta_g) +$$

$$+ 2\cos 2\theta_2 + 2\cos 2\theta_g + 22], \quad (30)$$

where $\theta_g,\varphi_g$ are spherical angles determining the direction of the graviton incident on the atom, $\theta_i,\varphi_i$ ($i = 1,2$) are spherical angles determining the direction of emission of the radiated photons. The summation over $n$ in expression (29) implies both summation over discrete states and integration over momentum of continuous states of radial solutions of the Schrödinger equation for the hydrogen atom. The dipole and quadrupole radial matrix elements in (29) are defined as follows:

$$D_{n'l';nl} = \int_0^\infty dr r^3 R_{n'l'}(r)R_{nl}(r), \quad (31)$$

$$Q_{n'l';nl} = \int_0^\infty dr r^4 R_{n'l'}(r)R_{nl}(r). \quad (32)$$

The structure of the obtained graviton scattering cross-section on a hydrogen atom is analogous to the structure of photon scattering cross-section during absorption in the line $1s \to 3d$ with subsequent two-photon re-emission in the decay $3d \to 1s + \gamma(E1) + \gamma(E1)$ [34]. The only difference is that the angular correlation, determined by expression (30), has a different angular dependence due to the tensor structure of graviton polarization. It should also be taken into account that in a graviton scattering experiment, such a transition could also be caused by "parasitic" blackbody radiation inevitably acting on the atomic system. Therefore, in addition to different angular distributions in these two processes, it is important to understand to what temperature the walls of the proposed setup should be cooled to unambiguously suppress such influence. The estimate of thermal photon absorption rate in the quadrupole transition $1s - 3d$, denoted hereinafter as $W^{E2,abs}_{1s3d}$, should be compared with the corresponding graviton absorption $W^{g}_{1s3d}$ in the same line. The absorption rate $W^{E2,abs}_{1s3d}$ relates to spontaneous, $W^{E2,spon}_{3d1s} = 594\,\mathrm{s}^{-1}$ [35], and induced, $W^{E2,ind}_{3d1s}$, emission rates according to the relations

$$W^{E2,abs}_{1s3d} = \frac{g_{1s}}{g_{3d}} W^{E2,ind}_{3d1s} =$$

$$= \frac{g_{1s}}{g_{3d}} W^{E2,spon}_{3d1s} \frac{1}{e^{\frac{\omega_{3d1s}}{k_B T}} - 1}, \quad (33)$$

where the level population $g_a = 2(2l_a + 1)$, $k_B$ — Boltzmann constant, $T$ is temperature in Kelvin. At $T = 300$ K (room temperatures) the probability of thermal photon absorption per unit time

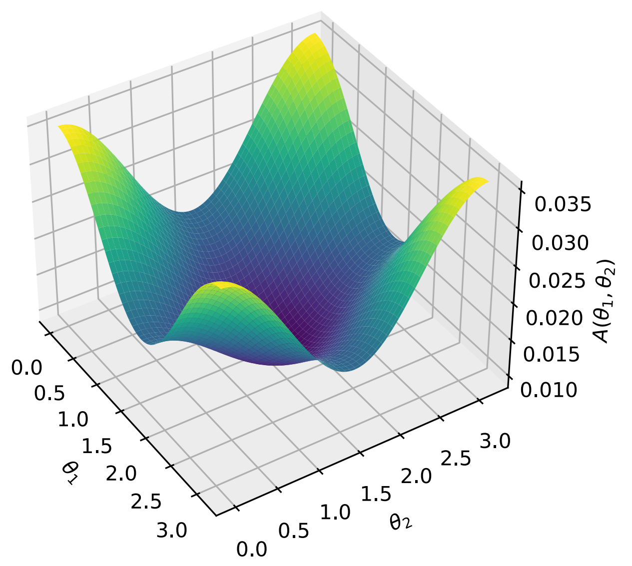


**Fig. 2.** (In color online) Angular distribution function $A(\theta_1,\theta_2,\theta_g=0,\varphi_g=0)$ of scattered photons for the case of graviton propagation along the $z$ axis, see expression (34)

$W^{E2,abs}_{1s3d} \sim 10^{-200}\ s^{-1}$ and, consequently, can be considered negligibly small, since $W^{E2,ind}_{3d1s} = W^{g}_{1s3d}$. Taking this into account, graviton scattering in the line $1s - 3d$ is more preferable than thermal photon scattering in the same transition at typical room temperatures.

The obtained angular dependencies (30) are significantly simplified if we assume that the incident graviton propagates along the $z$, axis, i.e., setting $\theta_g = 0$ and $\varphi_g = 0$. Then the orientation of the two emitted photons does not depend on either of the azimuthal angles $\varphi_1$ and $\varphi_2$, which simplifies the correlation factor in equation (30) to the following expression:

$$A(\theta_1,\theta_2,\theta_g=0,\varphi_g=0) = \\ = \frac{1}{450}(3+\cos(2\theta_1))(3+\cos(2\theta_2)). \quad (34)$$

The corresponding graphical representation of the angular distribution is shown in Fig. 2.

## 4. CONCLUSION

Considering the hydrogen atom as an example and applying the quantum electrodynamic approach combined with linearized quantum gravity theory, we obtained expressions for the cross-section of the two-photon conversion process and corresponding angular correlations. In the considered scenario, any massive astrophysical object can serve as a source of gravitons without loss of generality. In particular, in recent years there have seendiscussions in literature regarding the emission of gravitons from binary Primordial Black Holes (PBH) in the early Universe or those arising from PBH evaporation [36]. Such gravitons can contribute to the stochastic background of gravitational waves. The spectrum of such radiation depends on both the mass and spin of black holes, as well as their redshift [37]. The same applies to the situation considered in work [38], where perturbations of hydrogen atoms by gravitons in the primordial plasma were investigated. In [38], gravitons emitted by hydrogen and helium in the early Universe during recombination were considered as a possible source of high-frequency gravitational waves. The calculations showed that the most notable contribution is given by the transition $3d - 1s$of singly ionized helium $He^+$, which leads to a peak in the Planck distribution at a frequency of 1013 Hz. However, as shown by the study authors, the corresponding energy density is too small for detection.

In the scenario proposed here, where a hydrogen atom serves as a detector of incident gravitons, the analysis of angular correlations in the inelastic scattering process could serve as a means of determining the nature of the particle that acts as the source of atomic excitation. However, the practical implementation of such a detector under laboratory conditions is hindered due to the extremely small cross-section of the process.

This circumstance, apparently, does not seem so hopeless if we apply an experimental scheme with laser amplification of a weak signal, similar to that previously discussed in works [39–41]. Following [40], let's consider a system of three-level atoms. These atoms are assumed to be in a specially constructed optical cavity containing a resonant signal mode at the transition frequency $1s$–$3d$, as well as two resonant modes at the transition frequency $3d$–$4p$. In the absence of population at levels $3d$ and $4p$, the latter two modes are frequency degenerate and not coupled to each other. One of these two modes (pump mode $3d$–$4p$) is excited, i.e., contains a strong monochromatic pump field specified by an external source (laser). The second mode is intended for output signal excitation — we will further refer to it as output $4p$–$3d$. In the initial state, i.e., before the signal arrival, it is not excited and contains no fields. The detector's operating principle is as follows. Before the signal arrives in the signal mode, the atom, not being in resonance with the pump field and practically not interacting with it, remains in the ground state $1s$. After the signal arrives at frequency $\omega_{1s3d}$, some population appears at level $3d$. At this point, under the influence of the strong pump field, transitions begin between levels $3d$ and $4p$, and an oscillating dipole moment appears at frequency $\omega_{3d4p}$, which excites the field in the output mode. The task is to demonstrate that the output signal can be significantly larger than the input and determine the characteristic time of output signal increase. The corresponding estimates carried out in work [42] showed that with 100 photons in the pump mode and one photon in the signal mode, in the considered scheme, under the influence of a weak signal, the excitation level of the output mode reaches the pump mode excitation level within $\tau = 10^{-8}$ s, and consequently, has high sensitivity [43]. The effectiveness of such a scheme in case the signal mode is excited by a graviton requires separate consideration. It is important to note that the number of gravitons in a classical low-frequency gravitational wave can be very large (on the order of $10^{36}$cm, see, for example, [19]).

Despite the limitations we have considered in detecting a single graviton, it should be emphasized that the approach developed in this work to describe angular correlations can be applied to any other hypothetical tensor particle with spin 2 and a higher-order coupling constant. Although the graviton scattering cross-section is proportional to the square of the Planck length, research of this kind continues. In particular, it was recently proposed to increase the sensitivity of graviton detection in laboratory experiments using perturbations of a massive ensemble of atoms [19]. The estimates of graviton absorption probabilities in various media provided in [19] give hope for the implementation of the approach proposed by the authors.

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