JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS, 2024, Vol. 165, No. 6, pp. 882–886

MODEL OF SOLITON TURBULENCE OF HIGH-FREQUENCY FLUCTUATIONS IN PARTIALLY MAGNETIZED PLASMA

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Received December 09, 2023
Revised January 21, 2024
Accepted January 24, 2024

Abstract. A theoretical consideration of high-frequency microfluctuations formed by electron current across the magnetic field has been conducted. The Ginzburg–Landau equation with a nonlocal term was obtained to describe the dynamics of electron-cyclotron drift fluctuations. The thresholds for transition to turbulent regime and the boundaries within which soliton turbulence regime can be realized were determined, depending on the parameters of this equation.

Keywords: *Plasma-dielectric waveguide, subterahertz radiation, relativistic electron beam, Cherenkov effect*

DOI: 10.31857/S004445102406e142

1. INTRODUCTION

Numerous active space experiments involving releases of high-velocity or explosion-generated plasma clouds have shown that at the plasma cloud front, a region forms with significant electron heating, increased ionization rate, and anomalous transport, where transverse electron transport greatly exceeds the level of classical and Bohm diffusion. This phenomenon in space plasma physics is known as CIV – Critical Ionization Velocity phenomenon (see review [1]). Active ionospheric experiments often focused primarily on studying this phenomenon. Experimental data were interpreted within this framework, for example, in the CRIT II experiment [2, 3]. At the time, several participants of the active space experiment CRIT II saw an analogy between these processes in space and laboratory plasma of magnetrons and Hall thrusters, and attempted to investigate this phenomenon in more detail under laboratory conditions [4,5]. It seems reasonable to use the extensive experimental material and theoretical developments made in the study of anomalous transport, heating, and ionization in laboratory plasma of magnetrons and Hall thrusters as applied to space plasma.

Generally, to explain electron transfer in these problems, one of the following hypotheses is invoked: electron-cyclotron drift waves, Simon-Howe instability, lower-hybrid, modified two-stream, or ion-acoustic instability. The main factors exciting instability are considered to be gradients of density, magnetic field, temperature, drift motion, and dissipation. Typically, theoretical considerations draw a clear line between fluid treatment of low-frequency long-wave perturbations and kinetic treatment of short-wave perturbations. In kinetic considerations, plasma is usually non-dissipative. In many cases, attempts are made to account for nonlinear effects. Including particle-in-cell methods that yielded electron-cyclotron wave structures. Several examples of nonlinear numerical models and calculations can be found in works [6–8]. Some authors associate anomalous transport with the existence of nonlinear structures (for example, [9]).

In ionospheric experiments, the electron concentration in plasma formations has the same orders of magnitude (10^{10}–10^{10} cm^{-3}). With this, the electric and magnetic fields are smaller by 2–3 orders (≈ 0.5 Gs and V/m respectively). Such small magnetic field values in the ionosphere (compared to laboratory ones) lead to a significant frequency shift in some of these instabilities. As a result, electron-cyclotron drift (ECD) turbulence appears to be the most preferable explanation for high-frequency turbulence in ionospheric experiments. There is no noticeable deviation of the electron distribution function from Maxwellian, due to significantly lower electric field values. The directional drift velocity of electrons is much less than the electron thermal velocity. The problem parameters go beyond the kinetic consideration conducted in work [10]. This suggests the possibility of fluid treatment of high-frequency turbulence with consideration of dissipation. Furthermore, the geometry of expanding plasma, in which the ion Larmor radius is comparable to the plasma front width, while the electron Larmor radius is much smaller, allows, unlike [11], the use of partially magnetized plasma models for constructing nonlinear structures.

In this work, an attempt is made to construct a model of small-scale, fast-moving high-frequency nonlinear wave structures of electrons forming in plasma in the presence of electron drift due to an electric field perpendicular to the magnetic field (in $E \times B$-plasma). Unlike reproducible laboratory plasma conditions, when considering the dynamics of plasma formations in ionospheric conditions, the main issues become plasma parameters corresponding to various modes of small-scale turbulence, conditions and time parameters of its development, and the effects produced by this turbulence. Therefore, to describe turbulence based on the system of fluid equations and Maxwell's equations using the small parameter expansion method, a one-dimensional complex Ginzburg-Landau equation (CGL) with an additional nonlocal (integral) nonlinear term was obtained as the simplest model of ECD turbulence near its threshold. The boundaries of transition to turbulence were investigated. A region of plasma parameters corresponding to soliton turbulence was obtained, which, according to the authors, is associated with the anomalous transport regime.

2. BASICS OF THE MODEL

For high-frequency small-scale fluctuations, plasma can be considered partially magnetized, as the fluctuation scales are much smaller than the ion Larmor radius. For laboratory plasma, several authors associate the growth of instability with non-Maxwellian electron velocity distribution; however, in the present work, a simpler model of distribution distortion is adopted in the form of adding drift velocity to the Maxwellian distribution. The factors determining the growth of instability and its nonlinear stabilization are taken to be the plasma density gradient and electron collisions and diffusion.

In our consideration, we used a system of fluid equations for electron motion in the plane perpendicular to the magnetic field. Temperature changes at the scale of small-scale high- frequency turbulence are not taken into account, and the gyroviscosity tensor (inapplicable at such scales) is not considered. At the same time, it seems necessary to take into account electromagnetic corrections, since at electron temperatures of $T_e \geqslant 1$ eV observed in experiments, purely electrostatic waves do not exist.

The system of equations assumes oscillatory motion of electrons only against the background of stationary ions. Additional consideration of ion motion leads to accounting for the ponderomotive force, which introduces a correction to the nonlinear term. Electron drift occurs along the x axis with velocity u_d. The magnetic field is directed along the z axis. A wave perturbation propagating along the drift direction (axis x) perpendicular to the magnetic field is sought. The system of equations includes

1. electron motion equations along the axes x and y:

$$m_e n_e \frac{dV_e}{dt} = en_e\left(E + \frac{1}{c}H\right) - \nabla p_e; \tag{1}$$

continuity and Poisson equations

$$\begin{gathered} \frac{\partial n_e}{\partial t} + \nabla(n_e V_e) = -eD\nabla^2 n_e, \\ \frac{\partial E_x}{\partial x} = n_{i0} - n_e; \end{gathered} \tag{2}$$

2. two Maxwell equations introducing electromagnetic corrections:

$$\begin{gathered} \text{rot} \quad H = \frac{4\pi}{c} j, \\ \text{rot} \quad E = -\frac{1}{c}\frac{\partial H}{\partial t}. \end{gathered} \tag{3}$$

In quasi-one-dimensional consideration, all quantities depend only on the coordinate x and time t taking into account changes in electron velocities along the axes x and y, magnetic field components H_z and electric field E_x and E_y. Additionally, non-uniformity along the axis x of electron gas density is assumed

$$\gamma_n = \frac{\partial \ln(n_e(x))}{\partial x}.$$

The modeling is based on expansion in a small parameter of the system of equations in dimensionless form. Variables are normalized to corresponding constants: time t to inverse plasma frequency ω_{pe}^{-1}, spatial coordinate – to Debye radius r_D, velocities — to electron thermal velocity. The Krylov–Bogolyubov–Mitropolsky expansion method [12, 13] allows obtaining the dispersion equation and Ginzburg-Landau equation for the amplitude of the electric field wave perturbation.

In paper [14], when considering two-fluid plasma in the 5-moment approximation, it was found that the ECR dispersion relation has two asymptotic solutions. One of them is similar to the ion-sound mode, while the other agrees with the dispersion relation obtained in our expansion. This mode is defined in this work as a relation for Doppler-shifted "hybrid" waves. The authors suggest that the waves arise at the intersection of these two branches of the dispersion curve, forming the resulting nonlinear response. The system of equations used in the present work is supplemented so that the dispersion relation obtained in this work includes two corrections. One is determined by accounting for electromagnetic perturbations (two Maxwell equations). The other correction is introduced as a dissipative term to account for the role of dissipation in the formation of nonlinear wave structure. This dissipative term is similar to the term in equation (25) of paper [15] – the continuity equation. This is a diffusion term $D\nabla_{\perp}^2 n$, where the diffusion coefficient $D = \nu\rho_e$ depends on the electron Larmor radius ρ_e and their collision frequency ν with both ions and the neutral component. The focus on diffusion when considering plasma formation dynamics is also related to the fact that in paper [9], when analyzing numerical simulation results, diffusion leading to the smearing of resonances is assigned the main role in forming the frequency spectrum.

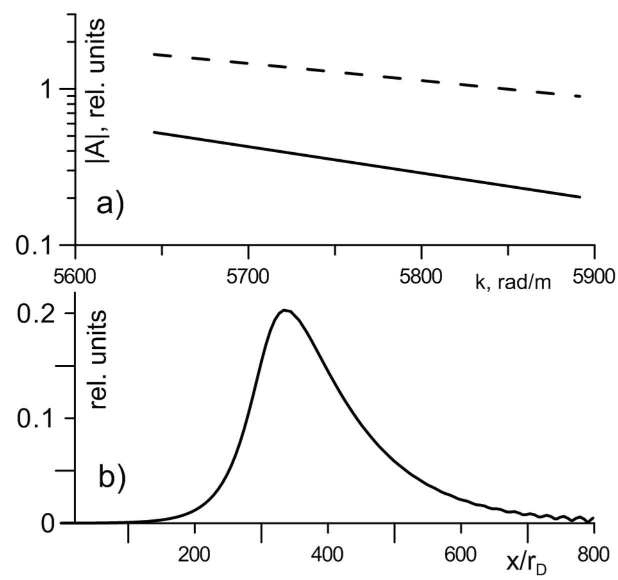

Fig. 1. *a* — Amplitude of the exact solution (solid line) and analytical estimate (dashed line). *b* — Example of soliton shape lence.

The real frequency in dimensionless variables is found from equation (4):

$$\mathrm{Disp} = -(\omega - ku_d)^2 + k^2 + 1 + \\ + \frac{\omega_{ce}^2 / \omega_{pe}^2}{1 + (V_{Te}^2 / c^2) / k^2} + \Delta D' = 0. \quad (4)$$

The imaginary frequency component λ determines the correction

$$\Delta(D') = 3\lambda^2 + 2\lambda D' k^2,$$

associated with the diffusion coefficient D' (in dimensionless form).

The resulting equation can be considered as a nonlinear Schrödinger equation or complex Ginzburg–Landau equation with additional terms with coefficients $c_{1'}$, $c_{2'}$, the second of which represents the coefficient of the nonlocal integral term:

$$iA_t + PA_{xx} + Q|A|^2 A = i\delta' A + i\beta' A_{xx} + \\ + i\varepsilon' |A|^2 A - c_{1'} A_x - c_{2'} A \int_{-\infty}^{x} e^{\gamma(x'-x)} |A|^2 dx'. \quad (5)$$

The reduction A_{xx} corresponds to $\partial^2 A / \partial x^2$; A_t corresponds to $\partial A / \partial t$ and so on. The coefficient P equals

$$P = \frac{1}{2} \frac{\partial V_g}{\partial k}.$$

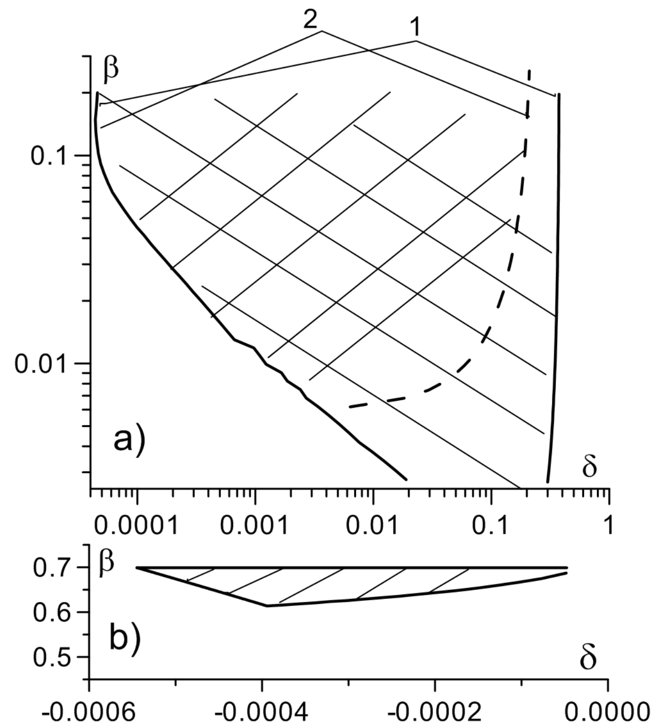

Fig. 2. Boundaries of parameter regions where soliton turbulence is realized: *a* – at plasma concentration 5×10^{11} cm^{-3} and two drift velocity values; *b* – at plasma concentration 5×10^{9} cm^{-3}:

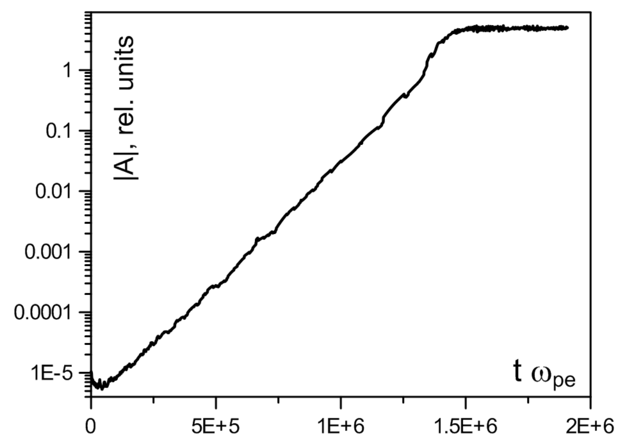

Fig. 3. Time dependence of average oscillation amplitude |A|

The coefficient δ' is determined by the difference of terms proportional to the density gradient and electron collision frequency; coefficients β' and ε' are determined by the diffusion coefficient; coefficient c_1 is determined by the electron collision frequency. All these coefficients depend on the wave vector. The exact form of the expressions defining the coefficients is not provided since some of them are quite cumbersome. Further consideration is conducted only for wave vector values corresponding to the anomalous dispersion of the equation.

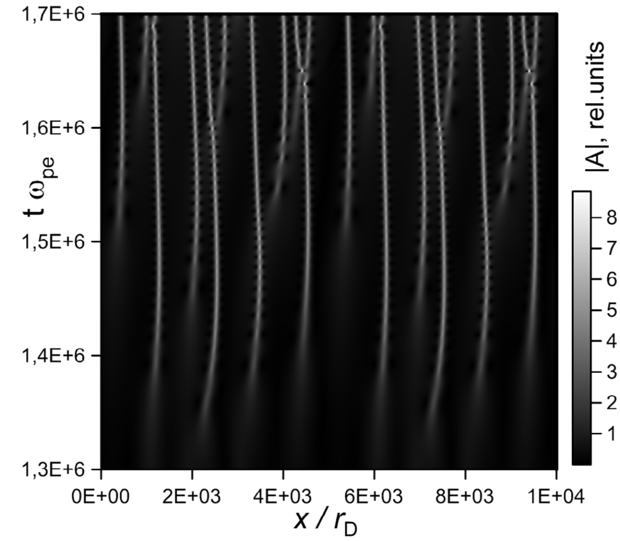

Fig. 4. Numerical solution of system (6)

An equation of this type is the simplest equation describing the system near the turbulence threshold. For the wave vector region corresponding to anomalous dispersion, the equation describes the subcritical bifurcation area, where above a certain plasma density gradient threshold, the fluctuation regime changes abruptly, and turbulence emerges (see [16] and literature cited therein). In the area near the bifurcation threshold, the formation of soliton turbulence is possible.

The analysis of CGL regimes is typically carried out for the equation form normalized so that

$$P = \pm\frac{1}{2}, \quad Q = \pm 1.$$

Under standard normalization transformation, the equation is reduced to a form depending on fewer parameters:

$$iA_t + A_{xx} + |A|^2 A = i\delta A + i\beta A_{xx} +$$
$$+ i\varepsilon |A|^2 A - c_1 A_x - c_2 A \int_{-\infty}^{x} e^{\gamma(x'-x)} |A|^2 \, dx'. \quad (6)$$

In this form, the main factors affecting the solution become δ and β. Many authors have studied the equation in this form. The stabilizing role of nonlocal (integral) terms of various types in CGL has been analyzed in recent years in works [19–21]. Additional dissipative and nonlocal terms in our derived equation suggest the possibility of soliton turbulence formation. To estimate the amplitudes of emerging solitons, works [22,23] were used with a somewhat different integral term $A\int_{-\infty}^{x}|A|^2dx$ in the auto-soliton generation mode [24,25]. The exact semi-analytical solution was obtained using the method proposed in work [26]. Comparing it with the analytical estimate shows that such an estimate gives several times overestimated soliton amplitudes and does not account for their shape distortion. However, these solutions allow estimating the range of plasma parameters where stable solitons exist (soliton stability was evaluated using the method described in works (27, 28), and estimating their amplitudes and wave vectors. The stable soliton regime is only realized for fluctuations with wave vectors $k > k_s$, where $k_s^2 \sim \delta/\beta$. Thus, this solution applies only to short-wave turbulence.

Comparison of analytically estimated amplitudes and amplitudes of the exact solution is shown in Fig. 1*a*. The solid line represents exact solution amplitudes, the dashed line represents analytical estimate. Fig. 1*b* shows an example of soliton shape. The exponential in the integrand of the integral included in equation (6) leads to soliton shape distortion and frequency shifts.

The stability limits determine the lower boundary of soliton wave vectors in Fig. 1*a*. The asymmetric shape of solitons (see Fig.1*b*) is determined by the density gradient value.

Fig. 2 shows the regions of soliton turbulence existence depending on two defining parameters δ and β. In Fig. 2*a* these regions are shown for plasma typical for magnetrons, with large magnetic and electric fields and high ionization level. In this case, the plasma has the following parameters: $n_e = 5\cdot10^{11}$ cm^{-3}; $u_d / v_{Te} = 0.02$ (shaded area within boundaries 1) and $u_d / v_{Te} = 2.0$ (shaded area within boundaries 2). To the right of this region, chaotic turbulence forms. The main conclusion that can be drawn from this graph: soliton turbulence occurs only with small deviations of from zero, and with increasing electric field, the region of soliton turbulence implementation decreases and partially transitions into chaotic. Figure δ shows the region of soliton turbulence realized under parameters corresponding to space experiment conditions. In this case $n_e = 5\cdot10^9$ cm^{-3}, $u_d / v_{Te} = 0.002$. Under these conditions, soliton turbulence is realized at $\delta < 0$. Under these conditions, soliton turbulence is realized at. Solitons have negative group velocity.

3. NUMERICAL CALCULATIONS

To confirm the possibility of forming a sequence of solitons in the system described by the above equation, numerical calculation was performed to reach stable solutions from initial chaotic perturbation of small amplitude at system parameter values corresponding to the region of stable solitons, in particular, at point $\delta = 0.0773$, $\beta = 0.0176$, lying in the region of soliton turbulence on the plane of Fig. 2*a*. Equation (6) was solved on a segment of length $L / r_D = 10^4$ Equation (6) was solved on a segment of length $-0.765\cdot10^{-2}$.For the approximation of spatial derivatives, a compact differencing scheme was used (see, for example, [29]) with periodic boundary conditions. A small initial perturbation was set as random oscillations with frequencies within the Nyquist frequency. For time integration, a 6th order Runge–Kutta method was used. Figure 3 shows the time dependence of the average oscillation amplitude on the considered segment. During the shown time $t\omega_{pe} \approx 10^6$ the amplitude of forming solitons stabilizes. The dependence $|A(x,t)|$ obtained from solving system (6) is shown in Fig. 4. Formation of soliton structures with limited amplitude is observed.

4. CONCLUSION

Thus, the paper shows the possibility of formation of small-scale wave structures of ECD. This consideration allows finding the parameter region where soliton turbulence and associated anomalous plasma transport are formed. Obtaining these criteria for soliton turbulence can help both in numerical modeling of collisionless plasma expansion in the ionosphere and in analyzing experimental data on such expansion.

FUNDING

The work was carried out within the State Assignment No. 122032900184-8.

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