CRYSTALLOGRAPHY REPORTS, 2025, Vol. 70, No. 1, pp. 51–59

PHYSICAL PROPERTIES OF CRYSTALS

ACCOUNTING FOR THE IMPERFECTION OF THE SPECTROPHOTOMETRIC COMPLEX OPTICAL ELEMENTS WHEN MEASURING TRANSMISSION SPECTRA OF GYROTROPIC UNIAXIAL CRYSTALS.

II. SAMPLES ARE CUT PARALLEL TO THE OPTICAL AXIS

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Received November 29, 2024
Revised November 29, 2024
Accepted December 11, 2024

Abstract. A theoretical and experimental study of the effect of imperfections of the polarizer, analyzer and photomultiplier tube (PMT) on the measurement results of the transmission coefficient spectra of the langasite family crystals cut *parallel* to the optical axis has been carried out. It is shown that in the absence of an analyzer, oscillations appear on the transmission spectra, the amplitude of which depends on the rotation of the crystal. These oscillations are associated with linear birefringence and appear due to the imperfection of the PMT, which plays the role of a partial analyzer. From the obtained spectra, the parameters of the PMT are calculated depending on the wavelength. The calculation of the birefringence of the studied crystals was carried out and an estimate of the error of such calculation was obtained. It is shown that the imperfection of optical elements does not lead to additional errors in the calculation of birefringence. Thus, when the plate is cut perpendicular to the optical axis, circular birefringence is manifested, when parallel, there is linear birefringence.

DOI: 10.31857/S00234761250107e5

INTRODUCTION

In the first part of the work [1] it is shown that the imperfection of the optical elements of the spectrophotometric complex has a significant effect on the results of measuring the spectral dependences of light transmission for $Ca_3TaGa_3Si_2O_{14}$ samples cut *perpendicular* to the optical axis z (z-cut). The most noticeable errors in spectrophotometric measurements are associated with the imperfection of the photomultiplier tube (PMT). In particular, it is shown that the imperfection of the PMT leads to a difference between the spectral dependences of transmission measured at p- and s-polarizations of incident light. The obtained spectra have special points at wavelengths of $\lambda = 720$ and 1050 nm, associated with the change of optical elements in this particular spectrophotometer. It is shown how the imperfection of the optical elements affects the results of calculating the rotation of the plane of light polarization (circular birefringence) from the measured transmission spectra.

In this paper, a theoretical and experimental study of the imperfection of the optical elements of a spectrophotometer is carried out using the example of measurements of the transmission spectra of some crystals of the langasite and quartz families, cut *parallel* to the optical axis (x-cut), at different angles between the optical axis of the crystal and the directions of greatest transmission of the polarizer and analyzer.

Accurate measurements of the spectra of light transmission coefficients are necessary for subsequent determination of optical parameters from experimental data using methods presented, for example, in [2–4]. Optical activity (circular birefringence) for an x-cut plate manifests itself very weakly, but linear birefringence of the crystal can be determined on such a plate. An assessment of the errors in calculating the birefringence values from the obtained experimental transmission spectra is carried out.

MEASUREMENT AND CALCULATION OF LIGHT TRANSMISSION SPECTRA TAKING INTO ACCOUNT THE NON-IDEALITY OF THE SPECTROPHOTOMETRIC COMPLEX

To calculate the spectral dependences of transmission, it is convenient to use the Mueller matrix method [5]. For a plate of a uniaxial absorbing crystal cut parallel to the optical axis, the Mueller matrix has the form [6]:

$$M = e^{-\zeta}\begin{pmatrix} \mathrm{ch}\delta & \mathrm{sh}\delta & 0 & 0 \\ \mathrm{sh}\delta & \mathrm{ch}\delta & 0 & 0 \\ 0 & 0 & \cos\Delta & -\sin\Delta \\ 0 & 0 & \sin\Delta & \cos\Delta \end{pmatrix}. \qquad (1)$$

Here the optical axis is parallel to the x-axis, $\Delta = 2\pi d\left(n_e - n_o\right)/\lambda$, $\zeta = 2\pi d\left(\kappa_o + \kappa_e\right)/\lambda$, $\delta = 2\pi d\left(\kappa_e - \kappa_o\right)/\lambda$, n_o, n_e are the principal refractive indices, κ_o, κ_e are the absorption indices of the crystal.

The optical elements in the case under consideration include a polarizer, an analyzer, and a PMT. All of them may be non-ideal. Let p_1 be the polarizer transmission in the direction of greatest transmission, and p_2 be the polarizer transmission in the perpendicular direction [5]. In the ideal case, $p_2 = 0$. For the PMT, we introduce the parameters f_1 and f_2, where f_1 characterizes the registration of p-polarized radiation, f_2 – s-polarized radiation, and in the ideal case, $f_1 = f_2$. The Mueller matrix of a non-ideal PMT has the same form as the matrix of a non-ideal polarizer [5, 7].

Measurements of the transmission spectra of the crystals were carried out in the range of 200–2500 nm with a step of 1 nm on a Cary-5000 spectrophotometer with a universal measurement accessory UMA [8] without a polarizer in naturally polarized light, with one polarizer without an analyzer for s- and p-polarizations of light incident on the crystal, with a polarizer and analyzer in parallel and crossed positions.

Without polarizer

Unlike z-cut samples, for which nothing changes when they are rotated around the normal to the surface, for a sample cut parallel to the optical axis, the transmission spectra will depend on the sample rotation φ. Let the initial position of the sample φ = 0 correspond to the case when its optical axis is parallel to the x axis.

The transmission spectra of $Ca_3TaGa_3Si_2O_{14}$ and $La_3Ga_5SiO_{14}$ crystals in unpolarized light for different angles φ are shown in Fig. 1. It is evident that the curves have discontinuities at λ = 720 and 1050 nm. At λ = 720 nm, a "lattice change" occurs, and at 1050 nm, a channel change occurs in the device detector (transition from Si to InGaAs [9]). At angles φ not equal to 0° and 90°, oscillations are observed, the amplitude of which is maximum at φ = 45°. For the $Ca_3TaGa_3Si_2O_{14}$ crystal, the oscillation frequency is noticeably higher than for $La_3Ga_5SiO_{14}$.

Oscillations similar to those obtained in Fig. 1 were observed in [10] for irradiated samples of $La_3Ga_{5.5}Ta_{0.5}O_{14}$ langatate and were associated with the presence of a damaged layer on the surface. However, in the case under study, the samples are not irradiated, so there should be no damaged layer.

Let us write down the formula for the intensity of the transmitted light taking into account the non-ideality of the PMT. If unpolarized light with the Stokes vector $S_0 = \{1, 0, 0, 0\}$ falls on the crystal, we obtain:

$$I = I_0 e^{-\zeta}\left(\mathrm{ch}\delta + \frac{f_1 - f_2}{f_1 + f_2}\cos 2\varphi\,\mathrm{sh}\delta\right). \qquad (2)$$

It is evident that in this approximation the non-ideality of the PMT does not lead to the appearance of oscillations.

Let us assume that the incident light is not actually unpolarized. We consider that the incident light has an arbitrary elliptical polarization $S_0 = \{1, \cos 2\omega\cos 2\chi, \cos 2\omega\sin 2\chi, \sin 2\omega\}$. Then for the intensity of the transmitted light we obtain:

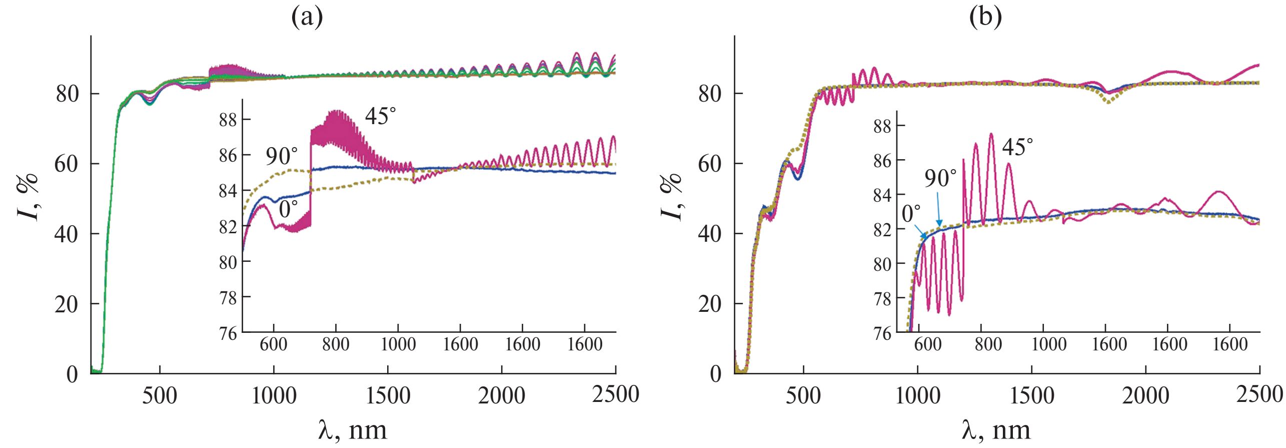

Fig. 1. Transmission spectra in unpolarized light at different sample rotations φ: a – katangasite $Ca_3TaGa_3Si_2O_{14}$, φ changes from 0° to 105° in 15° increments, inset – spectra for φ = 0°, 45° and 90° on an enlarged scale; b – langasite $La_3Ga_5SiO_{14}$, φ = 0°, 45° and 90°, inset – the same spectra on an enlarged scale.

$$I = \frac{I_0 e^{-\zeta}}{2\left(f_1 + f_2 + \left(f_1 - f_2\right)\cos 2\omega\cos 2\chi\right)}(2\left(f_1 + f_2\right) + 2\left(f_1 - f_2\right)\sin 2\omega\sin\Delta\sin 2\varphi + \\ +2 + \left(f_1 - f_2\right)\cos 2\omega\cos 2\chi\left(\cos 2\varphi^2 + \cos\Delta\sin 2\varphi^2\right) - \left(f_1 - f_2\right)\cos 2\omega \times \left(-1 + \cos\Delta\right)\sin 4\varphi\sin 2\chi). \tag{3}$$

At $\varphi = 45°$

$$I = \frac{I_0 e^{-\zeta}}{2\left(f_1 + f_2 + \left(f_1 - f_2\right)\cos 2\omega\cos 2\chi\right)} \times \\ \times(2\left(f_1 + f_2\right) + 2\left(f_1 - f_2\right)\sin 2\omega\sin\Delta + \\ + 2\left(f_1 - f_2\right)\cos 2\omega\cos 2\chi\cos\Delta). \tag{4}$$

Thus, at $\varphi \neq \pi n/2$, a dependence on birefringence appears, which explains the presence of oscillations (Fig. 1). The jumps in the curves are associated with a change in the polarization of the incident light at 720 nm and a change in the parameters of the PMT at 1050 nm.

Expressions (3), (4) are written for simplicity without taking into account the linear dichroism δ. When considering the oscillations in Fig. 1, taking into account the linear dichroism does not give any new effects, but makes the formulas more cumbersome.

However, when considering the difference between the crystal positions $\varphi = 0°$ and 90°, it is necessary to take into account linear dichroism (otherwise the obtained intensities of the transmitted light should be equal). Let us write the expressions for the intensities as:

$\varphi = 0$:

$$I = I_0 e^{-\zeta}\left(\operatorname{ch}\delta + \frac{f_1 - f_2 + \left(f_1 + f_2\right)\cos 2\omega\cos 2\chi}{f_1 + f_2 + \left(f_1 - f_2\right)\cos 2\omega\cos 2\chi}\operatorname{sh}\delta\right), \tag{5}$$

$\varphi = 90°$:

$$I = I_0 e^{-\zeta}\left(\operatorname{ch}\delta - \frac{f_1 - f_2 + \left(f_1 + f_2\right)\cos 2\omega\cos 2\chi}{f_1 + f_2 + \left(f_1 - f_2\right)\cos 2\omega\cos 2\chi}\operatorname{sh}\delta\right). \tag{6}$$

Since δ is a small quantity and is also a small quantity, we can approximately write

$$I\left(0\right) - I\left(90\right) = 2I_0 e^{-\zeta}\cos 2\omega\cos 2\chi\,\operatorname{sh}\delta. \tag{7}$$

This formula explains the jump in the transmitted light intensities at 720 nm (the polarization of the incident light and the values of ω and χ change). At 1050 nm, the jump in intensity is also present, but it is not as pronounced as at 720 nm. Most likely, at 1050 nm, there is also a jump in polarization, but it is smaller. This is confirmed by the baselines of polarizers considered in [1].

With polarizer without analyzer, ***p****- and* ***s****-polarization of incident light*

Experimental transmission spectra for *p*- and *s*-polarizations of incident light for crystals of catangasite $Ca_3TaGa_3Si_2O_{14}$, langasite $La_3Ga_5SiO_{14}$, langatate $La_3Ga_{5.5}Ta_{0.5}O_{14}$ and quartz SiO_2 for sample rotation angles $\varphi = 0°$ and 45° are shown in Fig. 2 (left part). The right part of Fig. 2 shows the transmission spectra calculated using the formulas described below. At $\varphi = 45°$, oscillations associated with birefringence occur. Thus, if the polarization direction of incident light is parallel or perpendicular to the optical axis ($\varphi = 0$, 90°), oscillations are absent, if at an angle, oscillations appear. Fig. 2 also shows the averaged spectra at $\varphi = 45°$ (half-sum of the spectra for *p*- and *s*-polarizations). It is clear that averaging does not lead to the disappearance of oscillations.

Let us calculate the intensities of transmitted light for *p*- and *s*-polarizations taking into account the non-ideal polarizer and PMT:

$$I_p = \frac{I_{0p} e^{-\zeta}}{2\left(f_1 p_1 + f_2 p_2\right)}\left(\left(f_1 + f_2\right)\left(p_1 + p_2\right) + \\ +\left(f_1 - f_2\right)\left(p_1 - p_2\right)\left(\cos 2\varphi^2 + \cos\Delta\sin 2\varphi^2\right)\right), \tag{8}$$

$$I_s = \frac{I_{0s} e^{-\zeta}}{2\left(f_2 p_1 + f_1 p_2\right)}\left(\left(f_1 + f_2\right)\left(p_1 + p_2\right) - \\ -\left(f_1 - f_2\right)\left(p_1 - p_2\right)\left(\cos 2\varphi^2 + \cos\Delta\sin 2\varphi^2\right)\right). \tag{9}$$

It is evident that the presence of oscillations is explained by the non-ideality of the PMT (if $f_1 = f_2$, the non-ideality of the polarizer only leads to the appearance of a certain factor without changing the general appearance of the curve).

We will consider only the non-ideality of the PMT and rewrite formulas (8) and (9) as follows:

$$I_p = \frac{I_{0p} e^{-\zeta}}{2}\left(\left(1 + f_2 / f_1\right)\operatorname{ch}\delta + 2\cos 2\varphi\operatorname{sh}\delta + \\ +\left(1 - f_2 / f_1\right)\left(\cos 2\varphi^2\operatorname{ch}\delta + \cos\Delta\sin 2\varphi^2\right)\right), \tag{10}$$

$$I_s = \frac{I_{0s} e^{-\zeta}}{2}\left(\left(1 + f_1 / f_2\right)\operatorname{ch}\delta - 2\cos 2\varphi\operatorname{sh}\delta + \\ +\left(1 - f_1 / f_2\right)\left(\cos 2\varphi^2\operatorname{ch}\delta + \cos\Delta\sin 2\varphi^2\right)\right). \tag{11}$$

At $\varphi = 0$, we obtain:

$$I_p = I_{0p} e^{-\frac{4\pi d\kappa_o}{\lambda}}, \quad I_s = I_{0s} e^{-\frac{4\pi d\kappa_e}{\lambda}}.$$

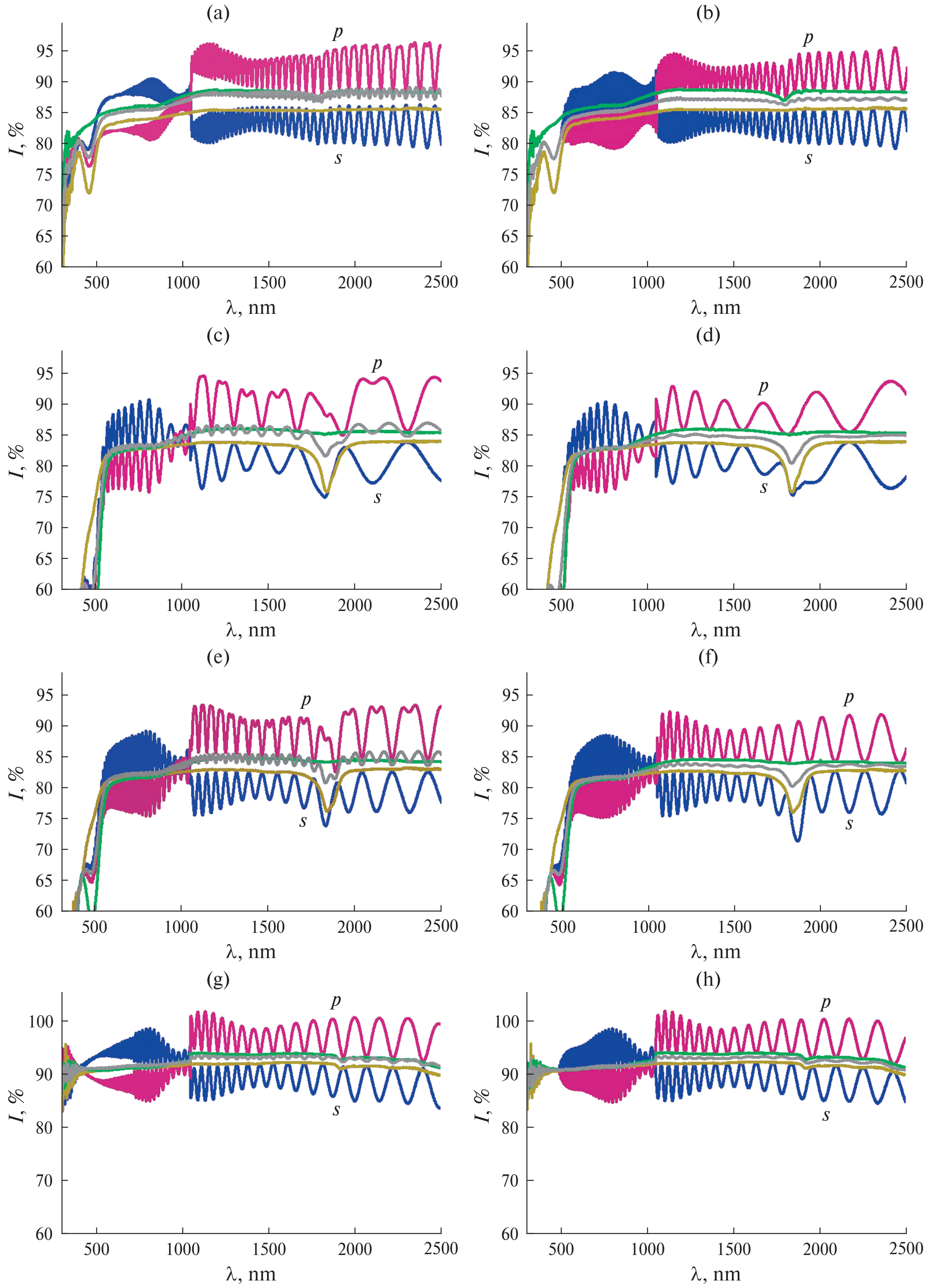

Fig. 2. Experimental at φ = 0°, 45° (left) and calculated at φ = 45° with a non-ideal PMT (right) transmission spectra of *x*-cut $Ca_3TaGa_3Si_2O_{14}$, $d = 1$ mm (a, b), $La_3Ga_5SiO_{14}$, $d = 1$ mm (c, d), $La_3Ga_{5.5}Ta_{0.5}O_{14}$, $d = 1$ mm (e, f) and quartz, $d = 3.1$ mm (g, h) samples in the range of 300–2500 nm, *p*- and *s*-polarization. Oscillating curves in the middle are the average values of $(I_p(\varphi = 45°) + I_s(\varphi = 45°))/2$.

Using formulas (10), (11) and experimental transmission spectra at $\varphi = 0°$ and 45°, we can approximately estimate the dependence of f_1/f_2 on the wavelength. To select the values of f_1/f_2, we used the expressions (ignoring linear dichroism, $\delta = 0$):

$$I_p(\varphi = 45°)/I_p(\varphi = 0°) = ½(1 + f_2/f_1 + (1 - f_2/f_1)\cos\Delta),$$

$$I_s(\varphi = 45°)/I_s(\varphi = 0°) = ½(1 + f_1/f_2 + \\ +(1 - f_1/f_2)\cos\Delta). \qquad (12)$$

When $\cos\Delta = 1$ both ratios are equal to one, and when $\cos\Delta = -1$, we obtain

$$I_p(\varphi = 45°)/I_p(\varphi = 0°) = f_2/f_1,$$

$$I_s(\varphi = 45°)/I_s(\varphi = 0°) = f_1/f_2. \qquad (13)$$

Thus, considering the extrema of the ratios (12), we can find the values of f_1/f_2 at individual points. The values of f_1/f_2 calculated using (12), (13) from experimental data at different wavelengths are shown in Fig. 3. It is evident that at $\lambda = 1050$ nm there is a jump-like change in f_1/f_2. The dispersion dependences of f_1/f_2 are calculated from these points using polynomial approximation. At those wavelengths when f_1/f_2 cannot be estimated from the experiment due to noise, we take $f_1/f_2 = 1$.

The examples of the obtained functions $f_1/f_2(\lambda)$ for the katangasite crystal:

$$\lambda \leqslant 518 \text{ nm, f1/f2} = 1;\ 518 < \lambda < 1050 \text{ nm},$$

$$f_1/f_2 = -346.0 - 1.82 \times 10^7/\lambda^2 + 1.27 \times 10^5/\lambda +$$

$$+ 0.47\lambda - 3.13 \times 10^{-4}\lambda^2 + 8.22 \times 10^{-8}\lambda^3;$$

$$\lambda > 1050 \text{ nm},$$

$$f_1/f_2 = 91.28 + 2.28 \times 10^7/\lambda^2 - 7.27 \times 10^4/\lambda - 0.055\lambda +$$

$$+ 1.62 \times 10^{-5}\lambda^2 - 1.88 \times 10^{-9}\lambda^3.$$

For langasite crystal:

$$\lambda \leqslant 500 \text{ nm}, f_1/f_2 = 1;\ 500 < \lambda < 1050 \text{ nm},$$

$$f_1/f_2 = -91.75 - 4.34 \times 10^6/\lambda^2 + 3.17 \times 10^4/\lambda +$$

$$+ 0.135\lambda - 9.59 \times 10^{-5}\lambda^2 + 2.68 \times 10^{-8}\lambda^3;$$

$$\lambda \geqslant 1050 \text{ nm},$$

$$f_1/f_2 = 29.0 + 8.50 \times 10^6/\lambda^2 - 2.50 \times 10^4/\lambda - 0.015\lambda +$$

$$+ 3.87 \times 10^{-6}\lambda^2 - 3.84 \times 10^{-10}\lambda^3.$$

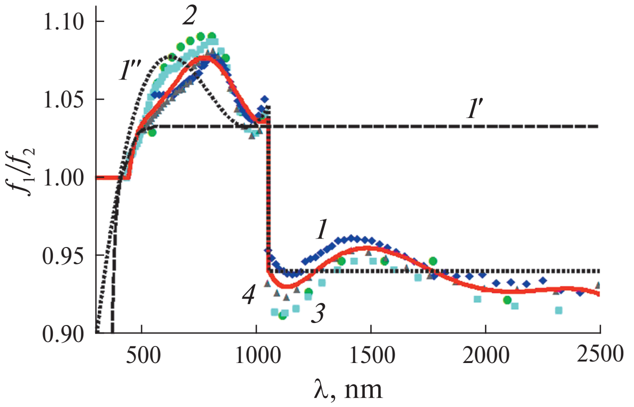

Fig. 3. Calculated values of f_1/f_2: *1* – katangasite, *2* – langasite, *3* – langatate, *4* – quartz. The solid line is an approximation based on the points obtained for all four crystals. The dotted line shows the values of f_1/f_2 calculated in [1] for *z*-cut katangasite samples with a thickness of 1 mm (*1′*) and 10 mm (*1″*).

To find the f_1/f_2 values correctly, there should be enough extremes on the $I_p(\lambda)$, $I_s(\lambda)$ curves. But if there are too many extremes, the measurement accuracy may not be enough to correctly determine their amplitude. The dotted line in Fig. 3 shows the f_1/f_2 values obtained in [1] for thin (1 mm) and thick (10 mm) *z*-cut katangasite samples. It is evident that to correctly determine f_1/f_2 on *z*-cut samples, a large thickness of these samples is required. For *x*-cut, a thickness of 1 mm is quite sufficient.

The values of f_1/f_2 depend only on the parameters of the device and should be the same for all crystals. The difference between the obtained values of f_1/f_2 (Fig. 3) is probably due to the presence of calculation errors associated with the inaccuracy of determining the position and amplitude of the extremes, as well as with linear dichroism, imperfection of the samples, inaccuracy of their orientation, etc.

If we approximate the points obtained for all four crystals with one curve (solid line in Fig. 3), we obtain:

$$\lambda \leqslant 440 \text{ nm}, f_1/f_2 = 1,$$

$$440 < \lambda < 1050 \text{ nm},$$

$$f_1/f_2 = -62.68 - 2.74 \times 10^6/\lambda^2 + 2.11 \times 10^4/\lambda +$$

$$+ 0.0943\lambda - 6.79 \times 10^{-5}\lambda^2 + 1.90 \times 10^{-8}\lambda^3,$$

$$\lambda \geqslant 1050 \text{ nm},$$

$$f_1/f_2 = 63.82 + 1.66 \times 10^7/\lambda^2 - 5.18 \times 10^4/\lambda -$$

$$- 0.037\lambda + 1.06 \times 10^{-5}\lambda^2 - 1.18 \times 10^{-9}\lambda^3.$$

The calculated intensities of transmitted light at $\varphi = 45°$ are shown in Fig. 2 (right side). The experimental transmission values at $\varphi = 0°$ were adopted as the quantities $I_{0p}e^{-\xi}$, $I_{0s}e^{-\xi}$ in formulas (10), (11). The experimental refractive indices of katangasite [11], langasite, langatate [12] and quartz [13, 14] were used in the calculation. These figures also show the experimental transmission spectra at $\varphi = 0°$, which should be the envelopes of the spectra at $\varphi = 45°$. The experimental and calculated dependences at $\varphi = 45°$ are in fairly good agreement, especially at $\lambda > 1050$ nm. At $\lambda < 1050$ nm the similarity is worse, which is associated with a less accurate determination of the f_1/f_2 ratio in this region due to frequent oscillations and an insufficient experimental step for an accurate determination of the extremum values.

With polarizer and analyzer

Let us consider the transmission spectra in the presence of both a polarizer and an analyzer. The polarizer is set at a fixed angle α between the direction of greatest transmission and the optical axis of the crystal, the position of the analyzer (angle β) changes. The most visual spectra are obtained at wavelengths where the birefringence is not very large (Fig. 4).

If the crystal is oriented so that the optical axis is parallel to the x axis ($\varphi = 0°$), taking into account the non-ideal PMT, we obtain:

$$I = I_0 e^{-\zeta} \frac{f_1 + f_2 + (f_1 - f_2)\cos 2\beta}{f_1 + f_2 + (f_1 - f_2)\cos 2\alpha} \times$$
$$\times\left(\cos^2(\alpha - \beta) - \sin 2\alpha \sin 2\beta \sin^2\frac{\Delta}{2}\right). \qquad (14)$$

In real measurements, the values $\alpha = 0°$ and $90°$ are usually used, so in this case there will be no oscillations.

For $\alpha = 0°$, $90°$ and crystal rotation by angle φ:

$$\alpha = 0°,$$
$$I = \frac{I_0}{2f_1} e^{-\zeta}\left(f_1 + f_2 + (f_1 - f_2)\cos 2\beta\right) \times$$
$$\times\left(\cos\beta^2 + \sin^2\frac{\Delta}{2}\sin\left[2(\beta - \varphi)\right]\sin 2\varphi\right), \qquad (15)$$

$$\alpha = 90°,$$
$$I = \frac{I_0}{2f_2} e^{-\zeta}\left(f_1 + f_2 + (f_1 - f_2)\cos 2\beta\right) \times$$
$$\times\left(\sin\beta^2 - \sin^2\frac{\Delta}{2}\sin\left[2(\beta - \varphi)\right]\sin 2\varphi\right). \qquad (16)$$

The normalization was chosen so that the transmittance of parallel polarizers ($\beta = \alpha$) without a sample was equal to 100 %. Figs. 4a, 4b, 4d, 4e show the experimental transmission spectra of katangasite and langasite crystals for the case of parallel and crossed polarizers at $\varphi = 45°$, Figs. 4c, 4f – the calculation according to formula (15) with the previously obtained values of f_1/f_2. It is evident that the non-ideality of the PMT leads to different values of the intensity maxima for parallel and crossed polarizers. At a wavelength of $\lambda = 1050$ nm, according to formulas (14)–(16), an insignificant shift is observed in the curves at angles $\beta \neq \alpha$. The non-ideality of the polarizers is manifested in the fact that the intensity minima in the IR region become less than zero (Fig. 4d, 4e); This is due to the fact that the transmission of crossed polarizers is greater than zero and this value is considered zero during normalization [1].

Thus, in this case, the non-ideal PMT, in contrast to the case of only a polarizer, does not affect the appearance of the curves and manifests itself only in some change in the intensity value.

Calculation of birefringence from measured transmission spectra

The calculation of the crystal birefringence Δn can be carried out using the maxima or minima of the obtained spectral dependences of transmission. For the calculation, it is possible to use the dependences obtained at any angles between the polarizer and the analyzer, provided that $\varphi \neq 0°$, $90°$ and $\beta - \varphi \neq 0°$, $90°$. However, it is better to use such angles β and φ at which the value of $\sin[2(\beta - \varphi)]\sin 2\varphi$ is close to unity (the greater the amplitude of oscillations, the more accurately the positions of the maxima and minima can be determined). If λ_1, λ_2 are the positions of adjacent maxima or minima, the value of Δn can be calculated using the formula ($\lambda_2 > \lambda_1$):

$$\Delta n = \frac{\lambda_1 \lambda_2}{d(\lambda_2 - \lambda_1)}. \qquad (17)$$

If we use the maximum and the minimum adjacent to it, we can write

$$\Delta n = \frac{\lambda_1 \lambda_2}{2d(\lambda_2 - \lambda_1)}. \qquad (18)$$

These expressions do not take into account the difference between the values of Δn at points λ_1 and λ_2. Let us calculate Δn for three samples – $Ca_3TaGa_3Si_2O_{14}$ and $La_3Ga_5SiO_{14}$ with a thickness of 1 mm, SiO_2 with a thickness of 3.1 mm. Table 1 shows a comparison of the results of the calculation of Δn, carried out using theoretical (formulas (15), (16)) and experimental curves $I(\lambda)$. The calculation error was determined using the formulas:

$$\delta(\Delta n)_{calc} = (\Delta n_{calc} - \Delta n_{exp})/\Delta n_{calc},$$
$$\delta(\Delta n)_{meas} = (\Delta n_{meas} - \Delta n_{exp})/\Delta n_{meas}. \qquad (19)$$

Here Δn_{calc} and Δn_{meas} are calculated from the theoretical and measured curves $I(\lambda)$, respectively, the experimental values Δn_{exp} are taken from the variances given in [11–14], at the point $\lambda_1 = \lambda_{calc}$. The calculations given in Table 1 were obtained using formula (17).

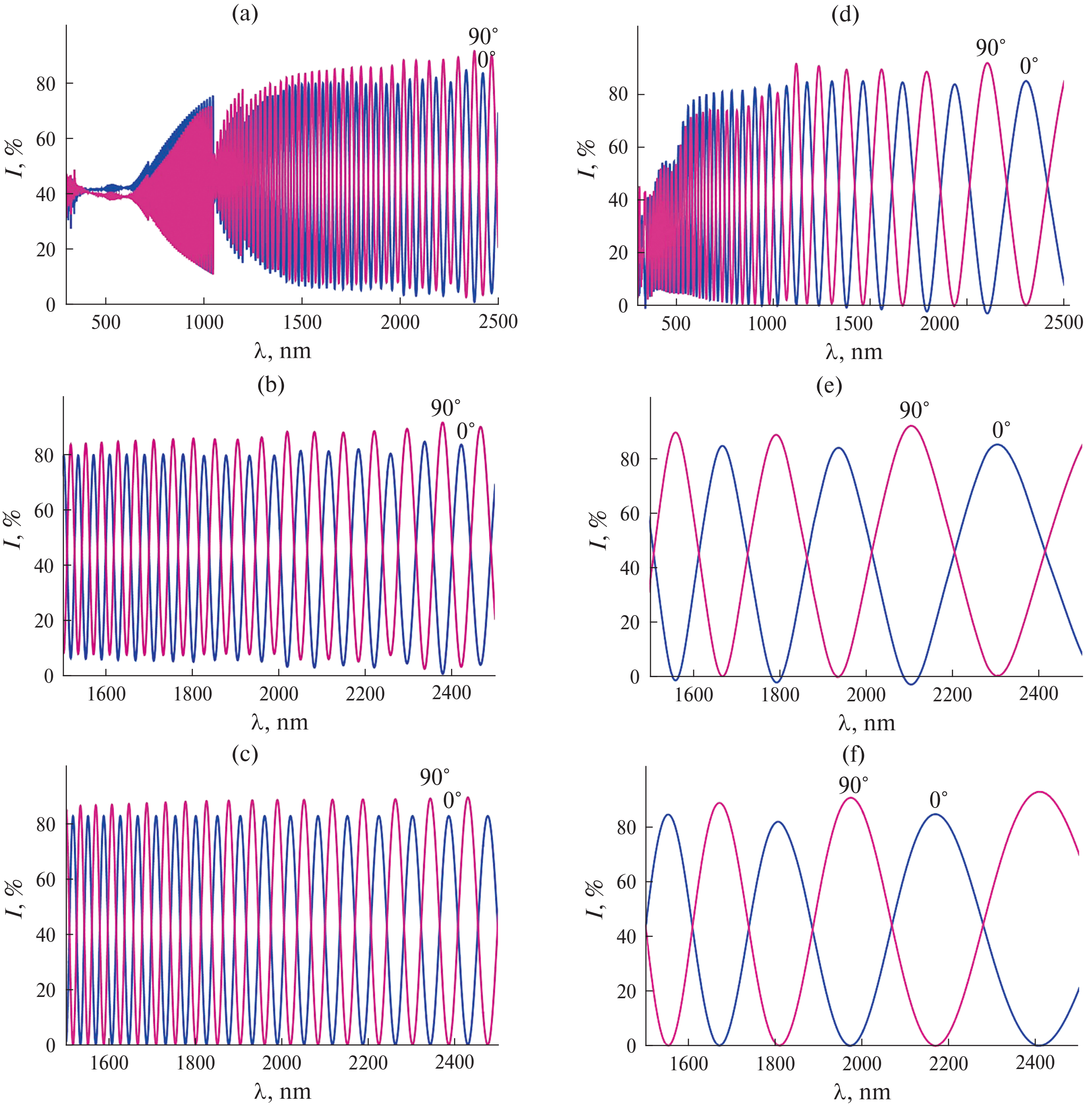

Fig. 4. Transmission spectra for samples of katangasite (a, b, c) and langasite (d, e, f) with parallel and crossed polarizers ($\alpha = 0$, $\beta = 0$, 90°), the angle between the optical axis and the direction of greatest transmission of the polarizer $\varphi = 45°$. The experimental spectra are given in the ranges of 300–2500 nm (a, d) and 1500–2500 nm (b, e), the calculated ones – in the range of 1500–2500 nm (c, f).

The difference between the calculation results using formulas (17), (18) is insignificant (when calculated using theoretical data, no more than 0.1–0.2 % for all cases considered).

The most accurate calculation based on theoretical curves was obtained for the langasite crystal. The calculation error for langasite and katangasite in the visible region of the spectrum is greater than in the IR range; for quartz, the calculation errors in different regions are comparable. The greatest difference between the Δn values calculated based on theoretical and experimental transmission spectra was obtained for the langasite crystal. This may be due to both the inaccuracy of determining the positions of the maxima and minima and the difference in the birefringence of the sample under study from the data presented in the literature. For quartz and katangasite samples, the difference between Δn_{calc} and Δn_{meas} is not as great and is smaller than the difference between Δn_{calc} and Δn_{exp} (this is especially noticeable for quartz).

Table 1. Results of calculation of birefringence of crystals at different wavelengths

Crystal	λ_{calc}, nm	λ_{meas}, nm	Δn_{exp} [11–14]	Δn_{calc}	Δn_{meas}	$\delta(\Delta n)_{calc}$, %	$\delta(\Delta n)_{meas}$, %
$Ca_3TaGa_3Si_2O_{14}$	580	581	0.0731	0.0927	0.0850	21	14
	901	898	0.0676	0.0750	0.0742	10	8.9
	1515	1517	0.06516	0.0676	0.06727	3.6	3.1
$La_3Ga_5SiO_{14}$	587	589	0.01116	0.01181	0.01178	5.5	5.3
	913	906	0.01095	0.01120	0.01200	2.2	8.7
	1552	1560	0.01086	0.01094	0.01196	0.7	9.2
SiO_2	577	577	0.00912	0.00994	0.00995	8.2	8.3
	912	917	0.00882	0.00936	0.00934	5.8	5.6
	1546	1572	0.00848	0.00938	0.00936	9.6	9.4

Note: λ_{calc} – positions of extrema on theoretical $I(\lambda)$ curves, λ_{meas} – positions of extrema on measured $I(\lambda)$ curves.

Thus, the main error in determining Δn is not associated with errors in measuring transmission spectra, but with the lack of consideration of the dependence of Δn on λ.

To calculate the value of Δn it is also possible to use the spectral dependences of transmission measured with only one polarizer at $\varphi = 45°$ (Fig. 2a, 2c, 2e, 2g). The calculation is carried out using the same formula (17), λ_1, λ_2 are the positions of adjacent maxima or minima. The results obtained: for quartz $\Delta n_{meas}(578\ \text{nm}) = 0.00998$, $\Delta n_{meas}(918\ \text{nm}) = 0.00936$, $\Delta n_{meas}(1570\ \text{nm}) = 0.00915$, for katangasite $\Delta n_{meas}(896\ \text{nm}) = 0.0739$, $\Delta n_{meas}(1526\ \text{nm}) = 0.0700$. From a comparison with the data presented in Table 1, it is evident that the calculation error is comparable with the corresponding error when using standard transmission spectra measured with a polarizer and an analyzer.

Thus, the imperfection of the PMT (as well as the polarizer and analyzer) does not in any way affect the calculation of birefringence based on the maxima and minima of the dependences of the transmission coefficients.

CONCLUSION

A theoretical and experimental study of the spectral dependences of the transmission coefficients of $Ca_3TaGa_3Si_2O_{14}$, $La_3Ga_5SiO_{14}$, $La_3Ga_{5.5}Ta_{0.5}O_{14}$ and SiO_2 crystals was carried out taking into account the non-ideality of the optical elements of the spectrophotometric complex for samples cut parallel to the optical axis. With such an orientation, the optical activity (circular birefringence) of the crystal manifests itself very weakly, but the influence of linear birefringence and dichroism is maximally expressed.

It is shown that during measurements without an analyzer, the non-ideality of the PMT leads to the appearance of oscillations in the transmission spectra associated with birefringence, the amplitude of which depends on the rotation of the crystal. The greatest amplitude of oscillations is observed at an angle of $\varphi = 45°$ between the optical axis and the direction of the greatest transmission of the polarizer. Thus, the PMT plays the role of a partial analyzer. From the obtained spectra, the PMT parameters are calculated depending on the wavelength.

The birefringence Δn of the crystals under study was calculated from the transmission spectra measured with a polarizer and analyzer. For comparison, the Δn value was also calculated from the spectra measured with one polarizer without an analyzer at $\varphi = 45°$. The calculation accuracy by both methods is comparable. It is shown that the main measurement error of Δn is associated with the lack of consideration of the dependence of Δn on the wavelength. The imperfection of the polarizer, analyzer, and PMT does not lead to additional errors in calculating Δn, since it does not affect the phase difference Δ.

Thus, the imperfection of the PMT has a significant effect on the type of spectral dependences of transmission, especially at $\varphi \neq 0°, 90°$. The imperfection of the polarizers does not affect the type of dependences, but can lead to the appearance of negative intensity values in those areas where the transmission of crossed polarizers is not zero. At the same time, the indicated imperfections do not affect the results of calculating the birefringence of crystals from the measured spectra.

ACKNOWLEDGMENTS

The authors express their gratitude to JSC Fomos-Materials and personally to O.A. Buzanov for providing crystal samples.

FUNDING

The work was carried out within the framework of the state assignment of the National Research Center "Kurchatov Institute" in terms of calculating the optical parameters of crystals and taking into account the imperfection of the optical elements of the spectrophotometer. Measurements of the transmission spectra of crystals were carried out at the MUIL PPMiD "Single crystals and blanks based on them" with the financial support of the Ministry of Education and Science of Russia within the framework of the state assignment to universities FSME-2023-0003.

CONFLICT OF INTERESTS

The authors of this work declare that they have no conflicts of interest.

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