

SIMULATION OF ZnWO_4 SANMARTINITE BY THE METHOD OF INTERATOMIC POTENTIALS

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Abstract. The structure and properties of ZnWO_4 have been simulated using the method of empirical interatomic potentials. The system of consistent interatomic potentials has been developed, which makes it possible to describe the structure, elastic and thermodynamic properties of zinc tungstate and provide the simulation of more complex composite media involving this component.

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INTRODUCTION

The mineral sanmartinite ZnWO_4 was first discovered in 1948 near San Martín, Argentina, and immediately attracted researchers' attention as a promising luminescent material [1]. It continues to be actively studied today, finding new applications.

ZnWO_4 undergoes congruent melting at $1216 \pm 5^\circ\text{C}$ [2] and does not experience polymorphic phase transitions below its melting point. Therefore, large ZnWO_4 crystals can be grown directly from the melt using the conventional Czochralski method [3–5]. ZnWO_4 crystals as large as 14 kg have been obtained [6]. Recent developments have made it possible to produce high-quality, large-volume zinc tungstate scintillator crystals with extremely low levels of radioactive contamination [7, 8].

Zinc tungstate is a multifunctional, non-toxic material with several unique properties, including a high refractive index, thermal and chemical stability, a high X-ray absorption coefficient, and a light yield higher than that of commercial $\text{Bi}_4\text{Ge}_3\text{O}_{12}$. It also boasts high density, short decay times, and low afterglow in luminescence [8–10]. Due to these properties, zinc tungstate is widely used as a scintillator [11], photocatalyst [12], and phosphor [13]. ZnWO_4 crystals are classified as anisotropic scintillators, exhibiting anisotropy in light output for heavy particles (protons, α -particles) as opposed to an isotropic response to β - and γ -radiation [14]. Consequently, zinc tungstate is of significant interest for researchers involved in double beta decay searches, dark matter detection, and cryogenic experiments aimed at identifying rare α - and β -decays [15].

Zinc tungstate crystals are effective nonlinear media and are utilized in the development of lasers based on stimulated Raman scattering [16]. ZnWO_4 -based phosphors doped with rare-earth elements play a crucial role in the creation of solid-state lasers emitting at various wavelengths [17, 18], as well as in the production of white LEDs for solid-state lighting applications [19, 20].

ZnWO_4 has a wolframite-type structure (monoclinic symmetry, space group $P2/c$) with two formula units per unit cell [21, 22]. The structure of ZnWO_4 is depicted in Fig. 1. Zinc and tungsten ions are coordinated octahedrally by oxygen (Fig. 1a). The ZnO_6 octahedron consists of two O_1 atoms and four O_2 atoms, while the WO_6 octahedron includes four O_1 atoms and two O_2 atoms. The structure comprises alternating layers of zigzag chains of ZnO_6 octahedra sharing edges and WO_6 octahedra also sharing edges, extending infinitely along the [001] direction (Fig. 1b). Each ZnO_6 octahedral chain is connected through shared oxygen vertices to four WO_6 octahedral chains, and vice versa, forming open channels along the [001] direction (Fig. 1a).

The structural and property simulation of zinc tungstate has been conducted *ab initio* in several studies using density functional theory (DFT) [23–26], employing various approaches. For example, in [23], to compute elastic constants and moduli, two now-standard numerical approximations were used to calculate exchange-correlation energy: the local-density approximation (LDA) [27] and the generalized gradient approximation (GGA) [28]. It was noted that GGA better describes inhomogeneous systems, particularly those with covalent bonding. In [24, 25], the generalized

gradient approximation (GGA) with the widely-used Perdew–Burke–Ernzerhof (PBE) functional was employed to describe the exchange–correlation potential [29]. In contrast to previous studies, [26] applied the DFT method using a linear combination of atomic orbitals (LCAO).

An alternative to *ab initio* calculations is atomistic simulation using empirical interatomic potentials, which has been successfully applied to describe various systems, including molybdates and tungstates, their solid solutions, intrinsic and impurity defects, and the

local environments of matrix and activator ions (e.g., [30–35]). However, sanmartinitite ZnWO_4 has not been studied using the interatomic potential method.

This study focuses on developing a potential system within atomistic simulation that would describe the properties of ZnWO_4 and enable simulations involving this component in more complex systems, which are increasingly being applied [17–20, 36–39].

SIMULATION METHODOLOGY

Simulation was conducted using the interatomic potentials method with the GULP 4.0.1 (General Utility Lattice Program) software [40], which is based on minimizing the energy of interatomic interactions.

The atomistic approach relies on empirically determined potentials that describe interactions between ions in the crystal. The pair potential U_{ij} for ions i and j with charges q_i and q_j was defined as follows:

$$U_{ij}(R_{ij}) = q_i q_j e^2 / R_{ij} + A_{ij} \exp(-R_{ij}/\rho_{ij}) - C_{ij} / R_{ij}^6, \quad (1)$$

where R_{ij} is the interatomic distance, A_{ij} , ρ_{ij} , C_{ij} are empirical parameters of short-range potentials. In this study, the interaction range was set at 15 Å for oxygen–oxygen contacts and 12 Å for other contacts. The covalent nature of the bonding was accounted for by introducing the effective charge of the ions.

The initial model adopted the structure of ZnWO_4 with cell parameters and atomic coordinates according to [22]. For zinc, a charge of 1.26 e was used, as in ZnO [41]. The charges for tungsten and oxygen were varied. The potential parameters for ZnWO_4 were determined through an iterative “fitting relax” procedure [40], under isotropic changes in the unit cell parameters.

DISCUSSION OF RESULTS

The best agreement with experimental values for unit cell parameters and atomic coordinates of ZnWO_4 was achieved using the atomic charges and interatomic potential parameters listed in Table 1. These parameters were used to determine a range of properties of zinc tungstate. The results are presented in Tables 2–5, compared with existing literature data.

Table 2 shows the unit cell parameters, cell volume, and atomic coordinates. The structure of ZnWO_4 has been studied in many works with varying levels of reliability and accuracy. The most precise, detailed, and reliable experimental data from three studies [22, 42, 43] are provided in Table 2. As can be seen, the results from these studies are in good agreement, as are the simulation results from this study. The unit cell parameters and volume reproduce the experimental data with an accuracy within tenths of a percent. In most cases, atomic positions are localized quite accurately

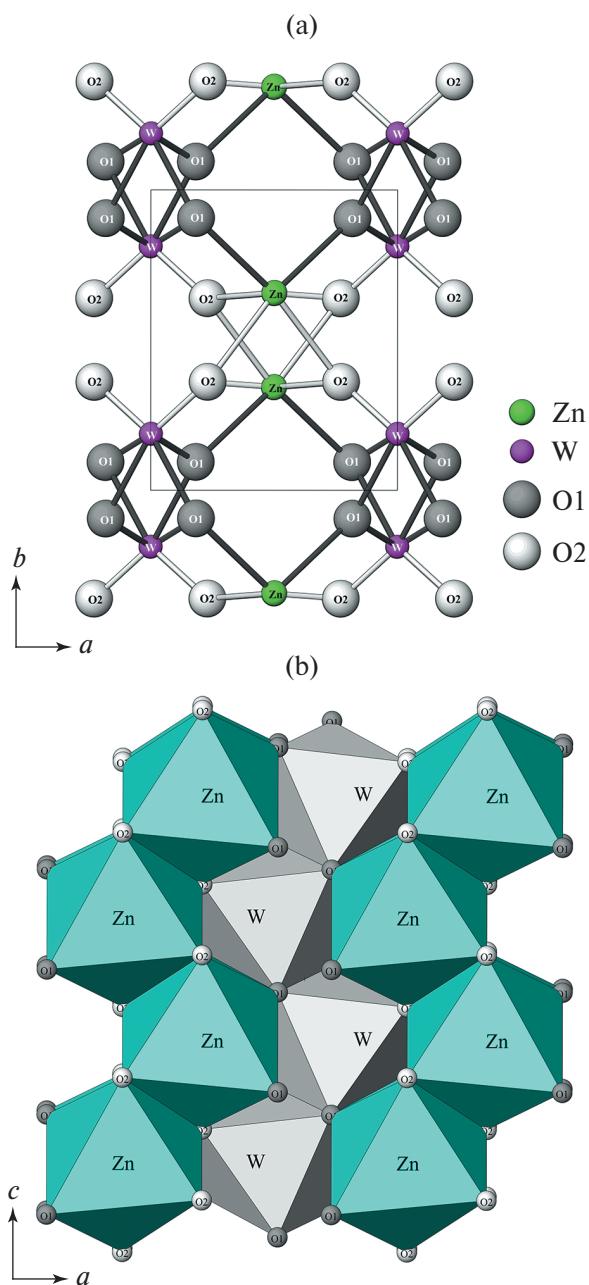


Fig. 1. Structure of zinc tungstate ZnWO_4 , projection onto the plane: a – ab, b – ac.

(the deviations of atomic coordinates from experimental values do not exceed 5%). The least accurately determined positions are those of oxygen atoms along the x -axis, which likely contributes the most to the error in determining interatomic distances.

Table 3 presents a comparison of calculated interatomic distances with experimental data. For the ZnO₆ and WO₆ octahedra, the Zn–O and W–O bond pairs are shown. Additionally, the average bond lengths within the octahedra (R_{av}) — which determine the size of the polyhedron — and the bond length dispersion (ΔR) — the difference between the largest and smallest bond lengths, characterizing the degree of polyhedral distortion — were evaluated. The greatest deviations from

experimental values were observed for the Zn–O1 and W–O2 bond lengths, while the other distances agreed well with experimental results, particularly considering the variability of experimental data reported in different studies. According to the simulation results, as well as experimental data, the ZnO₆ octahedron is larger than the WO₆ octahedron (with a higher R_{av}). However, the WO₆ octahedron is more distorted (with a higher ΔR). Thus, the simulation results for ZnWO₄ provide a good description of the structural characteristics of this compound.

Tables 4 and 5 present the results of assessing the elastic properties of ZnWO₄ in comparison with literature data. Table 4 lists the elastic constants (C_{ij}).

Table 1. Parameters of interatomic interaction potentials obtained in the work

Interaction	Potential parameters			Atom	Charge, e
	A , eV	ρ , Å	c , eV·Å ⁶		
Zn–O	98686.612014	0.171609	0.0	Zn	1.26
W–O	1005.782073	0.352777	0.0	W	5.1
O–O	2433.647679	0.269041	49.853817	O	−1.59

Table 2. Unit cell parameters and atomic coordinates in comparison with experimental data

Parameters, coordinates	This work	[42]	[43]	[22]
a , Å	4.6806	4.69263(5)	4.6986(8)	4.6902(1)
b , Å	5.7052	5.72129(7)	5.7293(8)	5.7169(1)
c , Å	4.9167	4.92805(5)	4.9367(11)	4.9268(1)
β , degrees	90.626	90.6321(9)	90.615(25)	90.626(1)
V , Å ³	131.29	132.300(2)	132.89(4)	132.14(1)
Zn y/b	0.6563	0.6833(4)	0.6840(2)	0.6838(4)
W y/b	0.1876	0.1823(5)	0.18258(6)	0.1820(4)
O1 x/a	0.1825	0.2171(3)	0.2169(10)	0.2171(3)
O1 y/b	0.9056	0.8955(3)	0.1051(9)	0.8953(2)
O1 z/c	0.4499	0.4360(3)	−0.0637(9)	0.4373(3)
O2 x/a	0.2299	0.2547(3)	0.2565(10)	0.2557(3)
O2 y/b	0.3604	0.3772(3)	0.3777(10)	0.3751(3)
O2 z/c	0.3902	0.4005(3)	0.3996(10)	0.3999(3)

Table 3. Interatomic distances in ZnO_6 and WO_6 octahedra

Distances	This work	[42]	[43]	[22]	[44]	[45]	[46]
$\text{Zn}-\text{O1}, \text{\AA}$	2.286	2.026(2)	2.025(5)	2.025(2)	2.0606		
$\text{Zn}-\text{O2}, \text{\AA}$	2.165 2.223	2.090(2) 2.227(3)	2.094(5) 2.226(5)	2.088(1) 2.234(2)	2.1387 2.1326		
R_{av}	2.225	2.114	2.115	2.115	2.209		
ΔR	0.122	0.201	0.201	0.209	0.078		
$\text{W}-\text{O1}, \text{\AA}$	1.794 2.065	1.915(2) 2.133(3)	1.915(5) 2.140(5)	1.908(1) 2.134(2)	1.8554 2.1838	1.84(1) 2.13(1)	1.8938 2.0310
$\text{W}-\text{O2}, \text{\AA}$	1.609	1.790(2)	1.797(5)	1.784(2)	1.8156	1.84(1)	1.8370
R_{av}	1.823	1.946	1.951	1.942	1.952	1.936	1.920
ΔR	0.456	0.343	0.343	0.350	0.368	0.29	0.194

Table 4. Elastic constants of ZnWO_4

C_{ij}	This work	Experiment	Calculation		
		[47]	[23] DFT–LDA	[23] DFT–GGA	[24] DFT–GGA + PBE
C_{11}	198.65	240.23	252.25	196.88	199.1
C_{22}	164.54	214.93	233.91	150.88	164.7
C_{33}	317.72	287.96	314.59	258.50	247.8
C_{44}	70.58	69.65	77.51	63.65	53.97
C_{55}	118.79	70.01	94.96	65.27	61.3
C_{66}	84.67	24.93	39.49	12.73	15.2
C_{12}	108.34	108.94	125.17	75.91	89.98
C_{13}	142.46	102.21	123.68	94.09	104.96
C_{15}	28.80		16.04	16.04	13.10
C_{23}	66.82	112.99	122.46	93.80	93.7
C_{25}	6.74		13.12	19.65	3.89
C_{35}	76.87		15.03	6.49	11.3
C_{46}	–24.27		–7.93	1.11	8.55

Table 5. Elastic modules of ZnWO₄

Modules		This work	Experiment		Calculation		Calculation method
<i>K</i> , GPa		136.66	153	[47]	179.03 ± 1.94	[23]	DFT–LDA
					125.75 ± 5.25		DFT–GGA
			161(3)	[22]	102.37	[24]	DFT–GGA + PBE
					140		DFT–GGA + PBE
			145(6)	[48]	257	[26]	DFT–LCAO
<i>G</i> , GPa		69.03	<i>G_z</i> = 69.86	[47]	—	[23]	—
<i>E_i</i> , GPa	x	93.95	176		171.23		DFT–LDA
	y	104.52	147		146.80		DFT–GGA
	z	187.59	217		156.98		DFT–LDA
			183 ± 21	[49]	103.15		DFT–GGA
					231.93		DFT–LDA
					185.41		DFT–GGA

The elastic properties of zinc tungstate have been relatively underexplored. There is only one experimental study from 1988 [47], where the elastic constants of ZnWO₄ were determined via ultrasonic phase velocity measurements. In [47], the estimation of C_{ij} was conducted under the approximation of orthorhombic symmetry, resulting in an incomplete set of elastic constants (Table 4). A complete matrix has not yet been determined experimentally.

Regarding the simulation of elastic constants of zinc tungstate, they were evaluated in [23–26] using various first-principles simulation approaches. It is evident that there is a significant spread in the results, with the largest deviation observed in simulations using the DFT–LCAO method [26].

Table 5 presents the elastic moduli: bulk modulus *K*, shear modulus *G* and Young's modulus *E_i*. The bulk and shear moduli are given according to the Hill approximation. Young's modulus indicates the anisotropy of elastic properties. ZnWO₄ exhibits the highest elasticity along the [001] direction, which coincides with the direction of open channels in its structure. Table 5 also includes literature data on elastic moduli. Experimental results for the bulk modulus were obtained in [22, 47, 48] using various methods. In [47], as mentioned earlier, the bulk modulus of zinc tungstate was determined based on the speed of ultrasound propagation in crystals. In [22], the bulk modulus was estimated from a neutron diffraction experiment examining the temperature dependence of ZnWO₄ lattice parameters, while in [48], assessments were based on an X-ray diffraction experiment determining ZnWO₄ lattice parameters under applied pressure. In [49],

resonance methods based on laser Doppler vibrometry were used to obtain data on Young's modulus for ZnWO₄ nanowires. The crystals were grown along the [001] direction, with lengths of several hundred micrometers and thicknesses ranging from 67 to 120 nm. The value presented in the table refers to the maximum diameter of the nanocrystals and should correspond to values for bulk single crystals, making it comparable to other results. The calculated literature values of elastic moduli are provided in Table 5, along with the respective calculation methods.

A notable feature is the significant discrepancy among the literature data regarding the elastic properties of ZnWO₄. For instance, estimates of elastic constants can differ by more than 100%. The values of the bulk modulus vary from 102 to 257 GPa, while differences in Young's modulus in some cases exceed 50%. The results of interatomic potential simulation generally fall within the range defined by the literature data or are comparable to them.

Fig. 2 presents the results of simulation the temperature dependence of the heat capacity at constant volume *C_v* compared to experimental data for the heat capacity at constant pressure *C_p*. Data from studies on the variation of *C_p* in the range of 5–520 K [50], 5–550 K [51], and 81–301 K [52] align well with each other, almost merging due to the closeness of their values, making them difficult to distinguish in Fig. 2. The figure shows good agreement between the simulation results and experimental estimates, especially since *C_p* is typically slightly higher than *C_v*. However, the results from [16] in the range of 293–573 K (shown by the dashed line), which indicate a significant, nearly linear

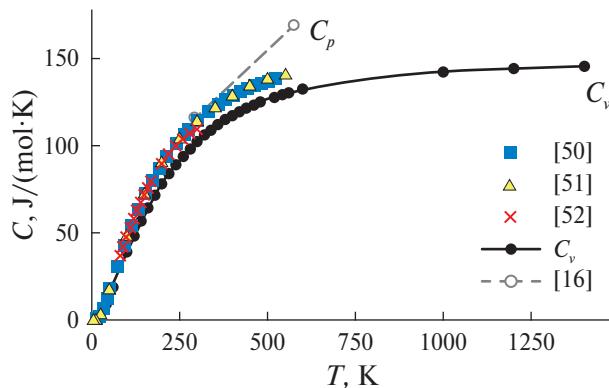


Fig. 2. Temperature dependence of the heat capacity of ZnWO_4 : C_v – this work, C_p – [50–52], [16] (dashed line).

increase in heat capacity in this range, are not confirmed by the simulation results. Fig. 3 shows the temperature dependence of entropy compared to experimental results [50], which also demonstrate good agreement.

Thus, the crystal structure simulations successfully reproduced the temperature dependences of heat capacity and entropy, showing reasonable agreement with available experimental data. Furthermore, they enabled reliable extrapolation of these thermodynamic properties to higher temperature ranges.

CONCLUSION

The empirical interatomic potential method was used to model ZnWO_4 crystals. The lattice parameters, atomic coordinates, interatomic distances, elastic constants and moduli, and temperature dependencies of heat capacity and entropy were evaluated. In most cases, the obtained results are consistent with and complement existing literature data. The developed system of interatomic potentials enables simulations of more complex composite media, solid solutions containing ZnWO_4 , as well as for identifying compositions with optimized properties.

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CONFLICT OF INTERESTS

The authors of this work declare that they have no conflicts of interest.

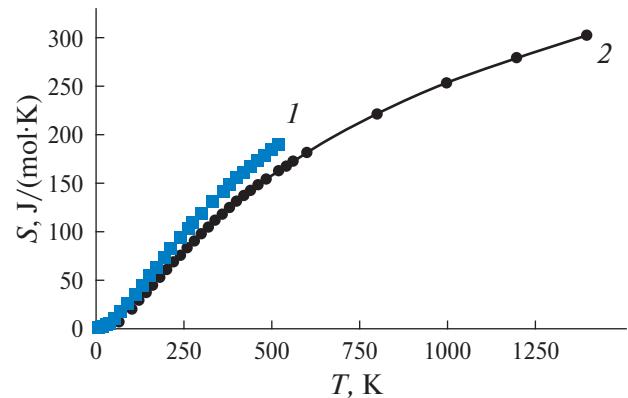


Fig. 3. Temperature dependence of the entropy of ZnWO_4 : 1 – [50], 2 – this work.

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