COSMIC RAY INTENSITY FORECAST FOR THE CURRENT CENTURY

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Abstract. To diagnose and forecast the state of the heliosphere, as well as space weather and climate, it is necessary to know the temporal changes of galactic cosmic rays flux at the Earth's orbit. The aim of the work is to forecast the cosmic ray flux for the next century based on the relationship between the modulation of galactic cosmic rays and the characteristics of solar activity. For a long-term forecast, one parameter models of solar activity were used that determines the modulation of galactic cosmic rays - the number of sunspots or the potential of cosmic rays solar modulation. As a result, a long-term forecast of the cosmic ray flux was obtained based on the analysis of a dozen models of solar activity behavior for the next century. The analysis suggests that, contrary to earlier forecasts, the probability of a large solar minimum at the end of the 21st century is small. This is shown by the majority of long-term solar activity forecasts by various authors which was analyzed by us. An almost twofold increase in the level of solar activity is expected by the middle of the century and a subsequent transition to approximately current level at the end of the century. Reduced intensity of galactic cosmic rays is expected at the Earth's orbit by mid-century.

Keywords: *solar activity, sunspot number, forecast, modulation, intensity of galactic cosmic rays*

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1. INTRODUCTION

Galactic cosmic rays (CR) affect the Earth's atmosphere, exerting a significant influence not only on space weather but also on the Earth's climate [Morner, 2015; Vinos, 2022]. Understanding the future development of Earth's climate requires long-term and accurate forecasts of solar cycles and the resulting predictions of galactic cosmic ray flux. Forecasting is mainly conducted only for the nearest solar cycle. Changes in solar activity on the scale of a single cycle is a relevant task and has both scientific interest (since understanding solar activity determines the accuracy of its forecast) and practical significance (for long-term planning of activities in space).

Changes in solar activity on a long-term scale lead to fundamental consequences for Earth's climate and for all life on Earth [Gray et al., 2010; Vinos, 2022; Xepapadeas, A., 2024]. Therefore, understanding changes in solar activity in the past, present, and future is the focus of many studies, being an urgent task.

A number of mechanisms of solar impact on the Earth's climate are considered. The most obvious mechanism of solar influence is associated with changes in the heating of the Earth's atmosphere, occurring due to changes in the total solar radiation flux across the entire spectrum . According to direct measurements, the changes observed during the solar cycle amount to only 1 W/m^2, and they have remained virtually unchanged over the past 9000 years [Gulev et al., 2023].

Another mechanism of the Sun's influence on Earth's climate involves the direct impact of observed changes in solar ultraviolet radiation on stratospheric ozone, with corresponding temperature changes due to the emergence of temperature and wind gradients.

Solar modulation of galactic cosmic rays is also considered as a mechanism of solar activity influence on Earth's climate through the ability of cosmic ray flux to affect Earth's cloud cover. Ionization of air by secondary cosmic rays leads to the occurrence of many atmospheric processes. These include the formation of clouds [Palle et al., 2004; Vinos, 2022] and precipitation [Kniveton, 2004], the formation of aerosols as condensation nuclei [Lushnikov, 2014], processes of baric system evolution in the lower atmosphere [Veretenenko et al., 2012; Yanchukovsky, 2024] and the emergence or development of cyclonic circulation [Tinsley, 2012], as well as changes in atmospheric transparency [Kudryavtsev et al., 2011] and parameters of the global electrical circuit [Tinsley et al., 2006; Tinsley, 2012]. The contribution of the modulated cosmic ray flux to tropospheric ionization is continuous. This indicates that ionization of the atmosphere by cosmic rays is one of the links in solar-atmospheric connections.

Long-term trends in solar activity have been actively considered by other researchers. Until the last decade, there were several forecasts of long-term solar activity [for example, Bonev et al., 2004; Abreu et al., 2010; Barnard et al., 2011; Lockwood, 2010; Lockwood et al., 2011; Usoskin, 2017; Biswas at al., 2023]. All these studies are mainly based on the instrumental series of sunspot numbers spanning 400 years, and less frequently on the reconstructed series of millennial solar activity obtained from cosmogenic radionuclides (^{10}Be and ^{14}C) [Knudsen et al., 2009].

In our discussed work, we highlighted only those long-term forecasts of solar activity that were based on the observed 400-year series of sunspot numbers and on the 9400-year series of solar activity indices reconstructed from cosmogenic radionuclide data. Such works were mainly carried out over the last decade.

The aim of the work is to make a long-term forecast of the cosmic ray (CR) flux for the next century based on available data on the relationship between galactic CR modulation and predicted solar activity. Objectives: 1) to analyze published long-term forecasts of sunspot numbers and solar CR modulation potential for the current century; 2) to build a pair model of CR variations as a function of sunspot numbers, as well as to use the force field approximation to relate potential and CR flux; 3) to forecast the intensity of galactic cosmic rays for the next hundred years for various scenarios of solar activity prediction.

The scientific novelty consists in a critical review of available forecasts of the solar activity index for the next century and the solar CR modulation potential, and conducting on their basis, using a unified methodology, an assessment of CR fluxes from the perspective of application for Earth's climate prediction.

2. FORECAST DATA OF SOLAR ACTIVITY PARAMETERS

When forecasting solar activity for hundreds of years, it is necessary to involve millennial data from past epochs. Currently, the only way to obtain such a long series of solar activity data is the cosmogenic isotope method, based on natural archives of carbon content in tree rings or beryllium in ice cores . As a result of calibrating the content of radionuclides, a series of sunspot numbers or solar CR potential is formed.

Solar cycles differ in duration, shape, and amplitude, and also periodically enter periods of almost inactive state, called grand minima of solar activity. But these cycle characteristics cannot be obtained from cosmogenic isotope data, as the dating capabilities of radionuclide content are limited by a temporal resolution of ~10 years and only from 1510 - annual resolution [Stuiver et al., 1998]. All solar activity forecast data are provided for average values or for 11- or 22-year smoothed values.

Forecasts of solar activity for the 21st century, obtained by various methods, are shown in Table 1. The most commonly used is spectral information contained in past solar activity. Some authors employ a model based on the theory of perturbation of the solar dynamo, founded on the interaction of planets with the Sun. Also popular is the model of superposition of observed low-frequency periodicities (11, 22, 53 years, modulated by the 88-year Gleissberg cycle, the 208-year de Vries cycle, or even the 2300-year Hallstatt cycle). Below, Table 1 lists the data used and forecasting methods. For all forecasts listed in Table 1, graphical material can be found in the "Forecast_2100" folder on Yandex Disk (https://disk.yandex.ru/d/mKHMM2dztqNoHw), folder Forecast_2100. Below are the results of forecasts for both sunspot numbers and cosmic ray solar modulation potential for the next 100 years.

3. FORECASTS OF SUNSPOT NUMBERS

The number of sunspots is the most easily observed indicator of solar activity level and the source of the longest recorded data series on the history of solar activity.

For assessments, nine long-term solar activity forecasts for which digital data could be obtained were analyzed in this work. In the cited works, along with the duration of the solar cycle, either the maximum number of sunspots in the cycle [Clilverd et al., 2006; Hiremath, 2008; Salvador, 2013; Barnard et al., 2011; Karak et al., 2017] or both maximum and minimum numbers of sunspots [Nasirpour et al., 2021; Herrera et al., 2021; Vinos, 2022], less frequently – the average number of sunspots in the cycle [Rigozo et al., 2010] were forecasted. After converting all data to the same scale, all forecasts and a brief description of forecasting methods are summarized in Table 1, and a graphical comparison is shown in Fig.1 (upper panel). For the sunspot number forecast [Barnard et al., 2011], an error corridor is also provided. The forecast errors are large, for example, for the rightmost time point of 2080 R_z □84□76, i.e., about 100%.

Three forecasts can be identified [Herrera et al., 2021; Nasirpour et al., 2021; Vinos, 2022], made in the last few years. They have similar temporal patterns; moreover, the works

[Nasirpour et al., 2021; Vinos, 2022] quite accurately predict the maximum of solar cycle 25. In the following, we will use only these forecasts (they are shown separately in the bottom panel of Fig.1). The figure shows that our adopted forecasts initially agree in phase, but then by the end of the century, the forecast [Vinos, 2022] gradually shifts in phase by approximately 4 years. This shift is related to the difficulty in predicting the duration of the solar cycle .

4. FORECASTS OF THE SOLAR MODULATION POTENTIAL OF COSMIC RAYS

Forecasts of other solar activity indices are also published, for example, the solar modulation potential of cosmic rays; however, this index is meaningful exclusively in relation to cosmic ray fluxes in the heliosphere, and direct comparison of such a forecast with the sunspot number forecast is not possible. The reconstruction of cosmic ray intensity is performed based on the force-field model (Section 6).

We examined two forecasts of the solar modulation potential of cosmic rays, which were based on a 9300-year record of 25-year average values of the solar modulation potential [Steinhilber et al., 2008] according to cosmogenic isotope and neutron monitor data, but performed using different methodologies (see Table 1).

The first forecast of the solar modulation potential of cosmic rays for the period 2000-2080 was performed in [Barnard et al., 2011] using the polynomial extrapolation method (Fig.2). The error corridor, for example, for the rightmost point in time $\phi = 370 \pm 320$ MV, i.e., about 100%.

Another forecast of the cosmic ray (CR) solar modulation potential for the period 2000-2500 was performed in [Steinhilber et al., 2013] using two methods: Fast Fourier Transform (FFT) and wavelet decomposition combined with autoregressive model (WTAR) of the CR solar modulation potential data from the past. The result of the CR solar modulation potential forecast is shown in Fig. 2. Both methods predict a solar activity minimum by the end of the 21st century, which will be comparable to the Dalton minimum. It should be noted immediately that this is not confirmed by the forecasts shown above in Fig. 1. Unfortunately, the 21st century range of interest is obtained with low resolution, and the presented data only indicates a rapid decrease in the CR solar modulation potential by the end of the current century. The forecasts from [Barnard et al., 2011] and [Steinhilber et al., 2013], performed on the same experimental material [Steinhilber et al., 2008] but using different methods, are consistent with each other.

5. MODEL OF COSMIC RAY VARIATIONS

If the solar activity forecast is made for the number of sunspots, then to estimate the expected CR variations, it is necessary to: 1) obtain the expected variations of νCR in interplanetary space (global spectrographic method - GSM) using data from the ground-based network of CR detectors, and 2) build a model of long-term variations of either CR flux intensity as a function of the number of sunspots R_z(model of long-term variations *LTV).* LTV).

1) The estimation using the GSM method of the expected CR variations in the isotropic approximation in interplanetary space ν was carried out using data from the World Network of CR detectors [Belov et.al., 2018; Belov et al., 2018]. If a spectrum of primary particles $J(R)$ falls on the magnetosphere and atmosphere, then the count rate of the N^i i th ground-based detector is determined as

$$N^i = \int_{R_c^i}^{R_U} m(h_0^i, R) \cdot J(R) dR \quad (i \approx 1 \div 50),$$

where $m(h_0^i, R)$ is the integral generation multiplicity, i.e., the number of particles of a certain type that are formed in the atmosphere from one primary particle and registered by the detector, R_c^i is the geomagnetic cutoff rigidity. By varying the latter expression and converting to relative values, we get

$$\frac{\delta N^i}{N^i} = \int_{R_c^i}^{R_U} \frac{m(h_0^i, R) J}{N^i} \cdot \frac{\delta J}{J}(R) \cdot dR = \int_{R_c^i}^{R_U} W^i(R_c^i, h_0^i, R) \cdot \frac{\partial J}{J}(R) \cdot dR \ ,$$

where $W^i(R_c^i, h_0^i, R)$ are coupling functions (determined experimentally or by calculation), $\delta J/J$ is the spectrum of primary CR variations (protons, nuclei).

Effective rigidities of detector particles (stratospheric sounding, neutron monitors, and muon telescopes) from ~0.5 GV to 70 GV, which allows studying the energy characteristics of variations. Finally, the observed secondary variations v^i are related to the spectrum of primary variations $\delta J/J$ of cosmic rays by a system of Fredholm integral equations of the first kind:

$$v^i = \int_{R_c^i}^{R_U} W^i(R_c^i, h_0^i, R) \frac{\partial J}{J}(R) dR \ .$$

where the coupling function $W^i(R_c^i, h_0^i, R)$ serves as the kernel of the integral equation, and the unknown function is the variation spectrum $\delta J/J$, i.e., the energy dependence of cosmic ray variations.

In our case, an important circumstance is that the sought variation spectrum $\nu = \delta J / J$ can be found in the form of an analytical function with a certain number of parameters. We considered a three-parameter spectrum $\delta J / J = a \cdot (R_0 + R)^{-\gamma}$. For long-term variations, γ = 1÷2 and changes with solar activity, R_0 depends on the polarity of the Sun's magnetic field, a is the amplitude of the variation spectrum, which is shown in Fig.1.

2) The found variation spectrum is used to construct the simplest one-parameter LTV model, i.e., a model of linear pairwise correlation between cosmic ray variations $\nu(R)$ in the heliosphere and the number of sunspots R_z

$$\nu^i = a_0 + a_1 \cdot R_z^i \pm \sigma^i , \quad (1)$$

the solution of which for the period 1985-2023

$$\nu_{model} = -(1.56 \pm 0.30) - (0.078 \pm 0.001)\, R_z [\%].$$

Fig.3 compares the observed and model values of cosmic ray variations with rigidity of 10 GV and their residuals. Variations are calculated relative to the base period of 2009, which is also indicated in the figure. For such a simple model, the residual is several percent, however, in certain periods it can be significantly higher.

For the considered model, the coefficient of multiple determination R^2 = 0.709, i.e., 71% is explained by the considered factors; the standard error of the model σ = 3.21%; the condition number, as an indicator of the solution stability of the system, equals *cond* = 143, i.e., the system is sufficiently well-conditioned and when the right side of equation (1) is perturbed by δX, the left side will change by no more than $\delta Y = cond\ \delta X$.

For applied tasks, it is important to know the changes in CR intensity J at Earth's orbit outside the magnetosphere, which are related to the found CR variations ν. For such a transition, the "NM network" detector was calibrated for the base period of 2009 using PAMELA data ("Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics") [Yanke et al. 2023]. By definition $\nu = J / J_{2009} - 1$, where the intensity at Earth's orbit equals $J = J_{2009}(\nu + 1)$ for the intensity in the base period $J_{2009} \equiv J_{PAMELA}$ = 26.869 p /(m^2 $s\ sr\ GV$) for 10 GV with an error of

~4%.

It is possible to transform the bases and transition from the base level J_{2009} to the level of interstellar intensity J_{LIS} : intensity variations J_{2009} relative to CR intensity J_{LIS} at the heliosphere boundary equal $\Delta_{2009} = J_{2009} / J_{LIS} - 1$. From which $J_{2009} = J_{LIS}(\Delta_{2009} + 1)$ and

$$J = J_{2009}(v+1) = J_{LIS}(\mathit{\Delta}_{2009} + 1)(v+1)\,, \tag{2}$$

here J_{LIS} = 33.2 p/(m $^2 sr\ GV$) [Bisschoff et al., 2019] is the intensity of 10 GV particles at the heliosphere boundary with an error of ~25%, $\mathit{\Delta}_{2009} = -0.19$ [Yanke et al. 2023] is the amplitude of residual modulation of galactic CR in the heliosphere for the base period of 2009.

6. FORCE-FIELD MODEL FOR COSMIC RAY INTENSITY RECONSTRUCTION

If the solar activity forecast is made for the □ cosmic ray solar modulation potential, then the assessment of expected variations can be performed using the force-field approximation. To describe the effect of solar modulation in this approximation, a simplified parametric force-field model was developed [Gleeson et al., 1968; Moraal, 2013; Vas et al., 2015]. Its basis in the quasi-stationary regime is the diffusion equation, which describes the propagation of particles in a spherically symmetric heliosphere with a diffusion coefficient depending only on the particle rigidity *R* and distance from the Sun *r* ; with no cosmic ray sources present. The analytical expression connecting the measured spectrum and the spectrum of particles of type *i* at the heliosphere boundary is given by the following one-parameter formula:

$$J_i(r, E_i) = J_{LIS,i}(r \to \infty, E + \Phi_i)\frac{E^2 - m^2}{(E + \Phi_i)^2 - m^2}\ , \tag{3}$$

where *E* is the total particle energy, *m* is the particle mass, *J(r, E+Φ)* is the measured cosmic ray flux, *J(r→ ∞ , E+Φ)* is the local interstellar cosmic ray flux, $\Phi_i(r, t, E, Z/A) = \varphi \cdot (eZ_i / A_i)$ - are the average energy losses of particles with charge Z_i and mass number A_i in the heliosphere, determined by the φ cosmic ray solar modulation potential.

This approximation provides a good description of long-term modulation effects of particles with rigidity from ~0.6 GV to tens of GV and includes the effective (10 GV) particle rigidity for neutron monitors [Caballero-Lopez at al., 2004]. The methodology for estimating the cosmic ray modulation potential from cosmogenic data is described in [Poluianov et al., 2016].

7. DISCUSSION OF RESULTS

Our task is to predict cosmic ray intensity for the next century based on known long-term solar activity forecasts. Moreover, as noted, we have used two types of solar activity data – sunspot numbers and cosmic ray solar modulation potential. This allows us to obtain the planned result as independently as possible. All solar activity forecasts are obtained from millennial time series, although when creating the cosmic ray variation model, we had to limit ourselves to the space era period, i.e., the period of ground-based cosmic ray monitoring.

In Section 3, three least contradictory forecasts of solar activity were identified (Fig.1) with uncertainty up to 100% Barnard et al., 2011]. The assessment of the expected CR intensity based on the predicted sunspot number R_z was performed using a linear pair correlation model (equation 1) between CR variations and sunspot number R_z.

Comparison of expected CR variations or corresponding CR intensities based on three sunspot number forecasts [Herrera et al., 2021; Nasirpour et al., 2021; Vinos, 2022] is shown in Fig.4. Like the original sunspot number data (Fig. 1), the resulting CR intensities for different data series are in phase at the beginning but gradually diverge by about 4 years by the end of the century. The continuous line connects the points of variation maxima (intensity minima) of CR.

In works [Steinhilber et al., 2013; Barnard et al., 2011] forecasts of the φ solar modulation potential of CR were made. To estimate the intensity of $J_i(r,K,\varphi)$ galactic CR near Earth for rigidities ~10 GV, it is necessary to use the force-field approximation (3). In [Steinhilber et al., 2013], forecasts of the solar modulation potential of CR were performed using two methods: Fourier analysis and wavelet decomposition, which predict significantly different temporal changes in cosmic ray intensity (Fig.5). Moreover, the Fourier analysis method gives CR intensity for the end of the 21st century characteristic of the Dalton minimum, which clearly contradicts all sunspot number forecasts. The wavelet decomposition method provides a more realistic forecast. The CR intensity forecast based on the solar modulation potential prediction performed in Barnard et al., 2011] qualitatively agrees with the forecast [Steinhilber et al., 2013] only until the middle of the 21st century and diverges strongly by its end.

8. CONCLUSIONS

All long-term forecasts of solar activity for the 21st century are based on the same experimental material - the observed 400-year series of sunspot numbers or the 9400-year series of solar activity indices reconstructed from cosmogenic radionuclide data [Stuiver et al., 1998; Steinhilber et al., 2008], but forecasts are made using different methods. The uncertainty of solar activity parameter forecasts increases and reaches ~100% by the end of the century Barnard et al., 2011; Steinhilber et al., 2013]. The error in converting from solar activity parameter to cosmic ray intensity does not exceed 10%.

1) A joint analysis of nine sunspot number forecasts made by different authors over the past decade has been conducted. Three mutually consistent forecasts of solar activity for the current century (Fig. 1) made by different methods have been identified. According to these forecasts, the maximum number of sunspots by the middle of the 21st century reaches values of 200 and, overall, decreases to values of 150 at the turn of the century.

2) A joint analysis of three forecasts of the solar modulation potential of cosmic rays, performed by different methods, has been carried out. The forecasts are in satisfactory agreement in the first half of the 21st century but are contradictory at the end of the century (Fig. 2).

3) It has been established that, according to the sunspot number forecast data and the employed cosmic ray variation model (1), some decrease in cosmic ray intensity (~6%) is expected in the middle of the 21st century during solar activity maxima (Fig. 4), but by the end of the century, the intensity will reach contemporary values. No global minimum of solar activity is observed at the end of the century.

4) It has also been established that for the three forecasts of the solar modulation potential of cosmic rays, only qualitative agreement for cosmic ray intensity (Fig. 5) is observed until the middle of the 21st century. By the end of the century, the forecasts diverge significantly, and in the case of the forecast [Steinhilber et al., 2013, FFT model], even a Dalton minimum is expected, which contradicts the forecasts based on sunspot numbers.

5) It is proposed to conduct long-term forecasts directly for cosmogenic data series to reduce uncertainties. The evaluation of cosmic ray intensity in this case should be done directly, without intermediaries.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest with other researchers in this field.

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REFERENCES

– *Belov A.V., Eroshenko E.A., Yanke V.G., Oleneva V.A., Abunina M.A., Abunin A.A.* Global Survey Method for the World Network of Neutron Monitors // Geomagnetism and Aeronomy. Vol. 58. No.3. P. 374–389. 2018. https://doi.org/10.7868/S0016794018030082 .

– *Veretenenko S.V., Ogurtsov M.G.* Study of the Spatial-Temporal Structure of Long-Period Effects of Solar Activity and Cosmic Ray Variations in the Lower Atmosphere Circulation // Geomagnetism and Aeronomy. Vol. 52. No.5. P. 626–638. 2012.

– *Kudryavtsev I.V., Yunger H.* Variations in Atmospheric Transparency under the Influence of Galactic Cosmic Rays as a Possible Cause of Their Impact on Cloud Formation // Geomagnetism and Aeronomy. Vol. 51. No. 5. P. 668–676. 2011.

– *Lushnikov A.A., Zagainov V.A., Lyubovtseva Y.S., Gvishiani A.D.* Formation of Nanoaerosols in the Troposphere under the Influence of Cosmic Radiation // Izv. RAS. Atmospheric and Oceanic Physics. Vol. 50. No. 2. P. 175–184. 2014. https://doi.org/10.7868/S0002351514020072

– *Yanke V.G., Belov A.V., Gushchina R.T., Kobelev P.G., Trefilova L.A.* On the Residual Modulation of Galactic Cosmic Rays in the Heliosphere // Cosmic Research. Vol. 61. No.1. 2023. P. 43–51. https://doi.org/10.31857/S0023420622060115

– *Yanchukovsky V.L.* Mid-Latitude Atmospheric Response to Sporadic Variations of Cosmic Rays // Solar-Terrestrial Physics. Vol. 10. No.4. P. 1–8. 2024. https://doi.org/10.12737/szf-101101101

– *Abreu, J. A., Beer J., Ferriz-Mas A.* , Past and future solar activity from cosmogenic radionuclides. // In Astronomical Society of the Pacific Conference Series: Soho-23: Understanding a Peculiar Solar Minimum. V. 428. P. 287–295. 2010.

– *Barnard L., Lockwood M., Hapgood M. A., Owens M. J., Davis C. J., Steinhilber F.* Predicting space climate change // Geophys. Res. Lett. V. 38. L16103. 2011. https://doi.org/10.1029/2011GL048489

– *Belov A., Eroshenko E., Yanke V., Oleneva V., Abunin A., Abunina M., Papaioannou A., Mavromichalaki E.* The Global Survey Method applied to Ground Level Cosmic Ray Measurements // Solar Physics. V. 293. Article number 68. 2018. https://doi.org/10.1007/s11207-018-1277-6.

– *Bisschoff D., Potgieter M.S., Aslam O.P.M.* New very local interstellar spectra for electrons, positrons, protons and light cosmic ray nuclei // The Astrophysical Journal . V. 878. № 1. Article number 59. 2019. https://doi.org/10.3847/1538-4357/ab1e4a

– *Biswas A.,· Karak B. B.,· Usoskin I., Weisshaar E.* Long-Term Modulation of Solar Cycles // Space Science Reviews. V. 219. Article number 19. 2023. https://doi.org/10.1007/s11214-023-00968-w

– *Bonev B. P., Penev K. M., Sello S.* Long-term solar variability and the solar cycle in the 21st century // The Astrophysical Journal . V. 605. № 1. L81–L84. 2004. https://doi.org/10.1086/420695

– *Caballero-Lopez R. A., Moraal H.* // Limitations of the force field equation to describe cosmic ray modulation. // JGRA. V. 109. Issue A1. A01101. 2004. https://doi.org/10.1029/2003JA010098

– *Gleeson L.J., Axford W.I.* Solar Modulation of Galactic Cosmic Rays // Astrophys. J. V. 154. P.1011. 1968. https://doi.org/10.1086/149822

– *Clilverd M.A., Clarke E., UlichT., Rishbeth H., Jarvis M.J.* Predicting Solar Cycle 24 and beyond // Space Weather. V. 4. S09005. 2006. https://doi.org/10.1029/2005SW000207

– *Gray L.J., Beer J., Geller M., J.D. Haigh J.D., Lockwood M., Matthes K., Cubasch U.,* *Fleitmann D., Harrison G., Hood L. et.al.* Solar influences on climate // Reviews of Geophysics V. 48. Issue 4. RG4001. https://doi.org/10.1029/2009RG000282

– *Gulev S.K., Thorne P.W.* Climate change 2021: The physical science basis. // 6th AR IPCC. London: Cambridge University Press P. 422. 2023. https://doi.org/10.1017/9781009157896.004

– *Herrera V.M.V., Soon W., Legate D.R.* Does Machine Learning reconstruct missing sunspots and forecast a new solar minimum? // Advances in Space Research. V. 68. Issue 3. P. 1485–1501. 2021. https://doi.org/10.1016/j.asr.2021.03.023

– *Hiremath K.,* 2008. Prediction of solar cycle 24 and beyond // Astrophys. Space Sci. V. 314. P. 45–49. https://doi.org/10.1007/s10509-007-9728-9

– *Karak B.B., Miesch M.* "Solar Cycle Variability Induced by Tilt Angle Scatter in a Babcock–Leighton Solar Dynamo Model" // The Astrophysical Journal. V. 847. Article number 69. 2017. https://doi.org/10.3847/1538-4357/aa8636

– *Kniveton D.R.* Precipitation, cloud cover and Forbush decreases in galactic cosmic rays // J. Atmos. Solar Terr. Phys. V. 66. Issues 13–14. P. 1135–1142. 2004. https://doi.org/10.1016/j.jastp.2004.05.010

– *Knudsen M. F., Riisager P., Jacobsen B. H., Muscheler R., Snowball I., Seidenkrantz M. S.* Taking the pulse of the Sun during the Holocene by joint analysis of (14)C and (10)Be // Geophys. Res. Lett. V. 36. L16701. 2009. https://doi.org/10.1029/2009GL039439

– *Lockwood M.* Solar change and climate: An update in the light of the current exceptional solar minimum. // Proc. R. Soc. A.V. 466. Issue 2114. P. 303–329. 2010. https://doi.org/10.1098/rspa.2009.0519

– *Lockwood M., Owens M. J., Barnard L., Davis C. J., and Steinhilber F.* The persistence of solar activity indicators and the descent of the Sun into Maunder Minimum conditions // Geophys. Res. Lett. V. 38. Issue 22. L22105. 2011. https://doi.org/10.1029/2011GL049811

– *Moraal H.* Cosmic-ray modulation equations // Space Sci. Rev. V. 176. P. 299–319. 2013. https://doi.org/10.1007/s11214-011-9819-3

– *Morner N.A.* The approaching new grand solar minimum and little ice age climate conditions // Natural Science. // V. 7. P. 510–518. 2015. http://dx.doi.org/10.4236/ns.2015.711052

– *Nasirpour M.H., Sharifi A., Ahmadi M.* Revealing the relationship between solar activity and COVID-19 and forecasting of possible future viruses using multi-step autoregression (MSAR) //

Environmental Science and Pollution Research V. 28. P. 38074–38084. 2021. https://doi.org/10.1007/s11356-021-13249-2

– *Palle E., Butler C.J., O'Brien K.* The possible connection between ionization in the atmosphere by cosmic rays and low level clouds // J. Atm. Solar-Terr. Phys. V. 66. P. 1779–1790. 2004. https://doi.org/10.1016/j.jastp.2004.07.041

– *Poluianov S.V., Kovaltsov G.A., Mishev A.L., Usoskin I.G.* Production of cosmogenic isotopes 7Be, 10Be, 14C, 22Na, and 36Cl in the atmosphere: Altitudinal profiles of yield functions // J. Geophys. Res. Atmos. V. 121. P. 8125-8136. 2016. https://doi.org/10.1002/2016JD025034

– *Rigozo N.R., Nordemann D.J.R., Echer E., Echer M.P.S., Silva H.E.* Prediction of solar minimum and maximum epochs on the basis of spectral characteristics for the next millennium // Planetary and Space Science. V. 58. P. 1971–1976. 2010. https://doi.org/10.1016/j.pss.2010.09.020

– *Salvador R.J.* A mathematical model of the sunspot cycle for the past 1000 yr // Pattern Recogn. Phys. V. 1. P. 117–122. 2013. https://doi.org/10.5194/prp-1-117-2013

– *Steinhilber F., Abreu J.A., Beer J.* Solar modulation during the Holocene // Astrophys. Space Sci. Trans. V.4. P. 1–6. 2008. https://doi.org/10.5194/astra-4-1-2008

– *Steinhilber, F., Beer J.* Prediction of solar activity for the next 500 years // J. Geophys. Res. Space Physics. V. 118. P. 1861–1867. 2013. https://doi.org/10.1002/jgra.50210

– *Stuiver M., Reimer P.J., Braziunas T.* High-precision radiocarbon age calibration for terrestrial and marine samples // Radiocarbon, V. 40. № 3. P. 1127-1151. 1998. https://doi.org/10.1017/S0033822200019172

– *Tinsley B.A., Zhou L.* Initial results of a global circuit model with stratospheric and tropospheric aerosols // J. Geophys. Res. V. 111. D16205. 2006. https://doi.org/10.1029/2005JD006988

– *Tinsley B.A.* A working hypothesis for connections between electrically-induced changes in cloud microphysics and storm vorticity, with possible effects on circulation // Adv. Space Res. 2012. V. 50. Issue 6. P. 791–805. https://doi.org/10.1016/j.asr.2012.04.008

– *Usoskin I.G.* A history of solar activity over millennia // Living Rev. Sol. Phys. V.14. Article number 3. 2017. https://doi.org/10.1007/s41116-017-0006-9

– *Vinos J.* Climate of the Past, Present and Future. A Scientific Debate // Madrid: Critical Science

Press. 279 p. 2022. https://judithcurry.com/wp-content/uploads/2022/09/Vinos-CPPF2022.pdf

– *Vos E.E., Potgieter M.S.* New Modeling of Galactic Proton Modulation during the Minimum of Solar Cycle 23/24 // Astrophys. J. V. 815. № 2. Article number 119. 2015 . https://doi.org/doi:10.1088/0004-637X/815/2/119

– *Xepapadeas A.* Uncertainty and climate change: The IPCC approach vs decision theory" // Journal of Behavioral and Experimental Economics. V. 109. 102188. 2024. https://doi.org/10.1016/j.socec.2024.102188

– *Yanke V.G., Belov A.V., Gushchina R.T., Kobelev P.G., Trefilova L.A.* "Forecast of Modulation of Cosmic Rays with Hardness of 10 GV in the 25th Solar Activity Cycle". // Geomagnetism and Aeronomy. V. 64. No. 2. P. 201–210. 2024 . https://doi.org/10.1134/S0016793223601072

Table 1. List of used works for forecasting the number of sunspots R_z or the potential φ of cosmic ray solar modulation.

	Forecasting method and predicted SA near 2100 (~31st cycle).
	Solar activity index – sunspot number R_z (Fig. 1).
1	The solar activity forecast was modeled as a low-frequency harmonic oscillator (periods of 22, 53, 88, 106, 208, and 420 years). By the middle of the current century, solar activity of ~200 is predicted, and after 2100 – a lower one, close to the Dalton minimum [Clilverd et al., 2006].
2	Based on an autoregressive model (modeling of forced and damped harmonic oscillators from the previous 22 cycles), a forecast of the future 15 cycles until 2175 has been made. For the 31st SA cycle (2087-2098), low solar activity of ~60 is predicted [Hiremath, 2008].
3	Epignosis resulting from spectral analysis of time series of the average number of sunspots over 10 years and a 1000-year forecast of solar activity. For the current century, moderate SA of ~120 is predicted, and for the end of the century – very low solar activity of ~25 [Rigozoet al., 2010].
4	The model is based on the theory of solar dynamo perturbations, founded on the interaction of planets with the Sun and represents four interacting frequencies (orbital parameters of planets 22.14, 19.528, 19.858, 21.005 years), modulated by waves of 178.8 and 1253 years. The model predicts that the Sun is entering a grand minimum, and this minimum may persist for an extended period of time: low solar activity throughout the century: maximum ~80 and very low SA ~40 by 2100 [Salvador, 2013].
5	Forecast until 3500, based on the dynamo model; randomness and nonlinearity in the emergence of bipolar magnetic regions in the model lead to variable magnetic cycles. The forecast satisfactorily agrees with later forecasts, for example, [Herrera et al., 2021] except for the beginning of the current century. [Karak et al., 2017].
6	Epignosis and forecast using algorithms and machine learning analysis applied to sunspot time series (1700-2019). For the current century, relatively high SA is predicted, around 200, which does not decrease by 2100 [Herrera et al., 2021].
7	Forecasts from the ARIMA method to modern approaches of long short-term memory (LSTM) with various neural network architectures. For the current century, high SA is predicted with some decrease toward the 31st SA cycle [Nasirpour et al., 2021].
8	Forecast by spectral analysis method of time series of the average number of sunspots for five different frequency series. By the middle of the current century, solar activity of ~200 is predicted, with a downward trend toward the end and after 2100. [Vinos, 2022; https://judithcurry.com/2023/07/04/solar-activity-solar-cycle-25-surpasses-cycle-24].
9	For the current century, relatively low SA of about ~100 is predicted [Barnard et al., 2011].
	The solar activity index – potential φ of solar modulation of CR (Fig.2).
1	Forecasting the potential of solar modulation of cosmic rays for the period 2000-2080 using polynomial extrapolation method. By the end of the century, a moderate value of the potential of solar modulation of cosmic rays of ~400 MV is predicted, close to the current value Barnard et al.,

	2011].
2	Forecasting the potential of solar modulation of cosmic rays for the period 2000-2500 using Fourier analysis and wavelet decomposition methods. By the end of ~2100, a fairly low value of the potential of solar modulation of cosmic rays of ~200 MV and below is predicted [Steinhilber et al., 2013].

Figure captions

Fig. 1. Long-term forecasts of sunspot number R_z according to Table 1. The maxima of sunspot numbers for each forecast are connected by a continuous line. For the forecast [Barnard et al., 2011], an error corridor is also shown. The upper panel shows all 9 available forecasts, the lower panel shows 3 selected forecasts.

Fig. 2 . Reconstructed up to ~2000 and predicted potential φ of solar modulation of cosmic rays for two forecast models (FFT and WTAR) for the period 2000-2500 (22-year smoothing) [Steinhilber et al., 2013] and potential forecast from [Barnard et al., 2011]. The inset shows the annual average values of the solar modulation potential of CR according to the ground-based neutron monitor network [Usoskin et al., 2017].

Fig. 3. Comparison of experimentally determined by GSM method amplitude variations v of galactic cosmic rays with rigidity of 10 GV outside the magnetosphere and variations determined by the model v_{model}(1). The model residuals v v v_{model} are also shown.

Fig. 4. Comparison of predicted variations/intensity for 10 GV (left/right scale) in Earth orbit V_{1AE} in the current century for three sunspot number forecasts [Herrera et al., 2021; Nasirpour et al., 2021; Vinos, 2022]. The maxima of variations (minima of intensity) for the forecast [Vinos, 2022] are connected by a continuous line. The CR intensity in the interstellar medium J_{LIS} *is shown schematically.*

Fig. 5 . Reconstructed cosmic ray intensity J_{1AU} (left scale) and its variations (right scale) at Earth orbit based on the predicted potential of solar modulation of cosmic rays for three forecast

models: [Steinhilber et al., 2013] − (FFT and WTAR) and [Barnard et al., 2011] (Fig. 2).

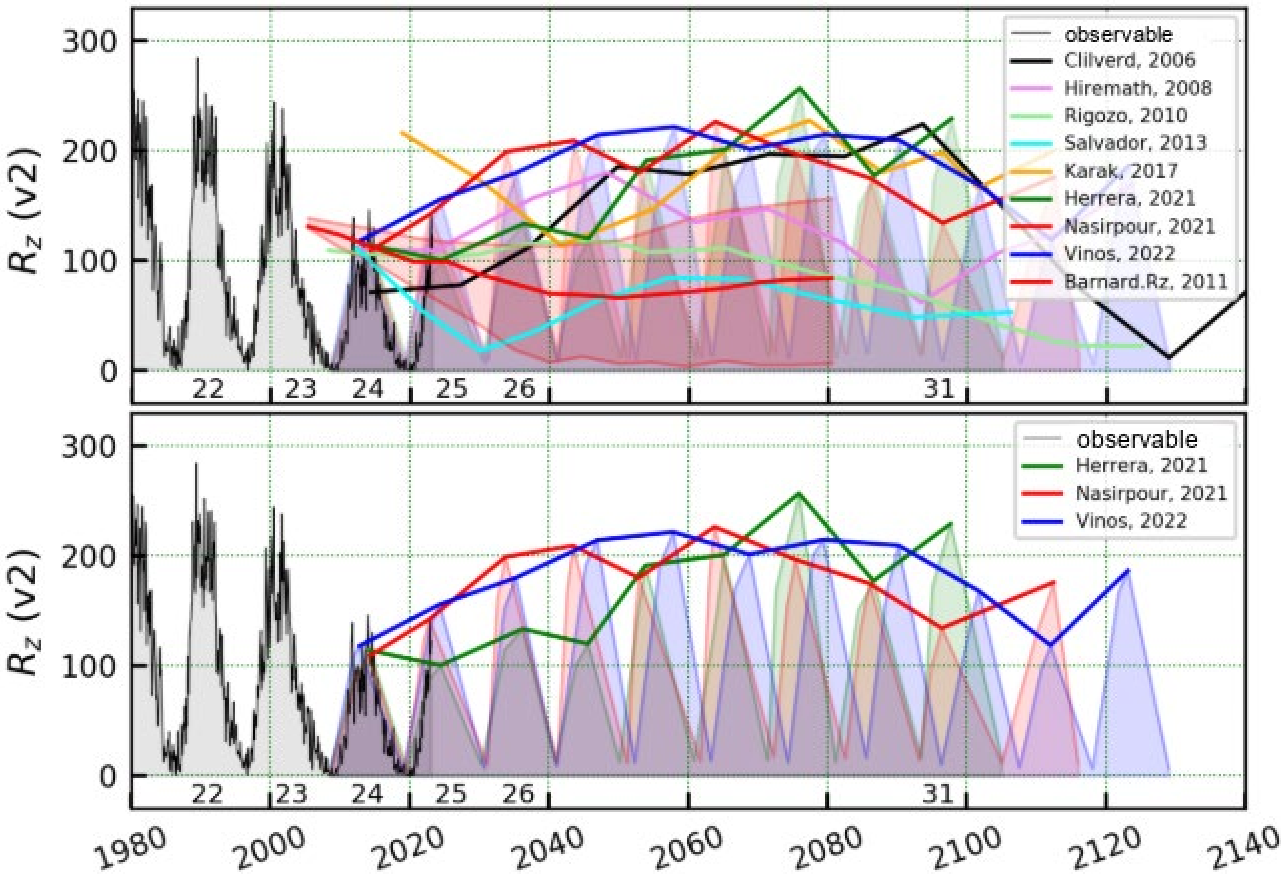
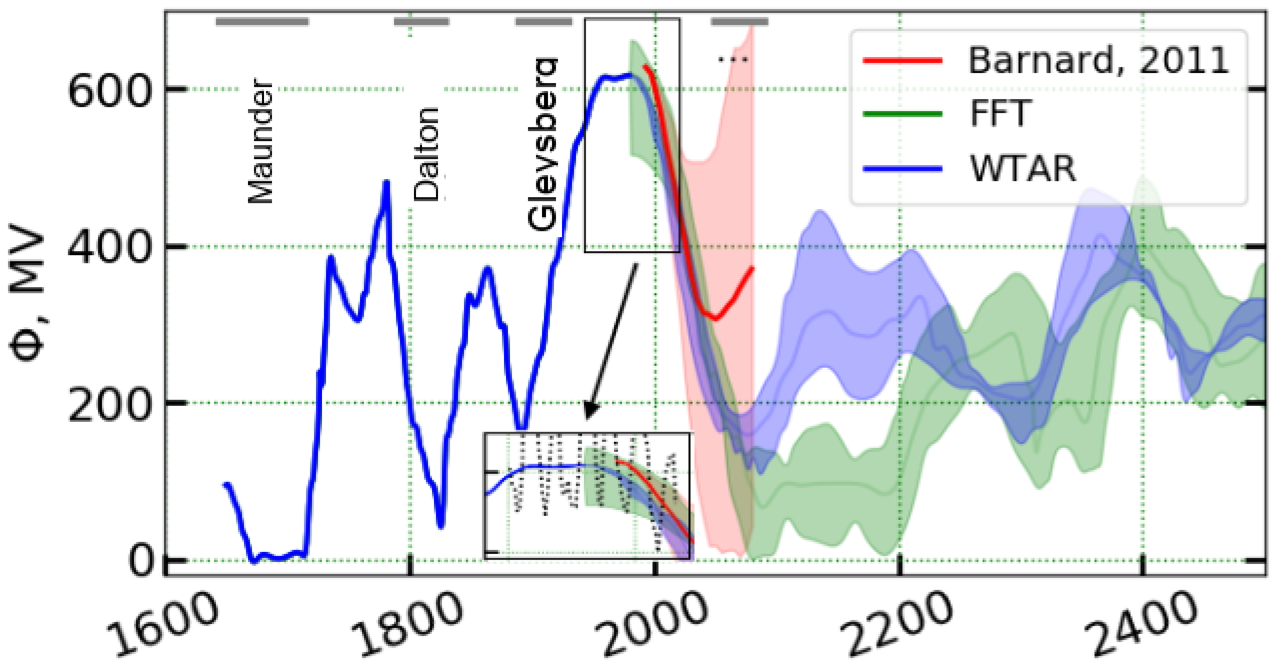
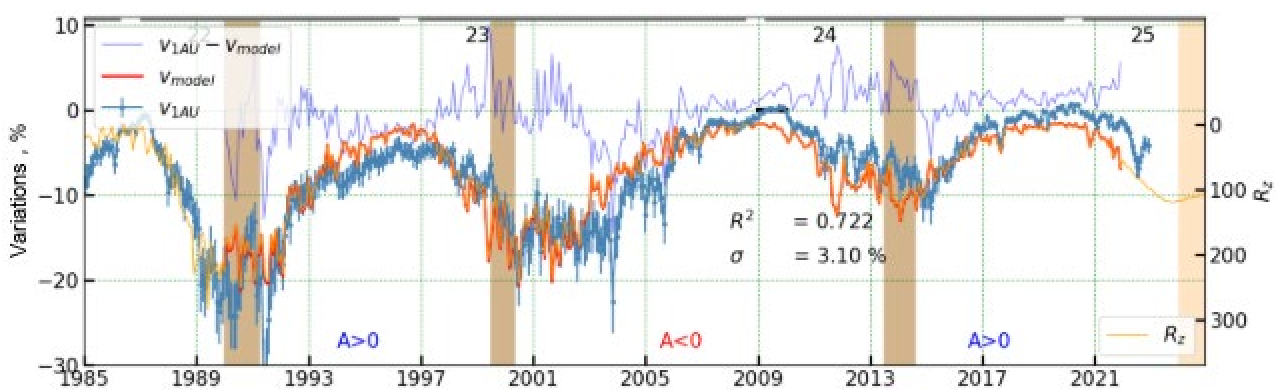


Fig.1.



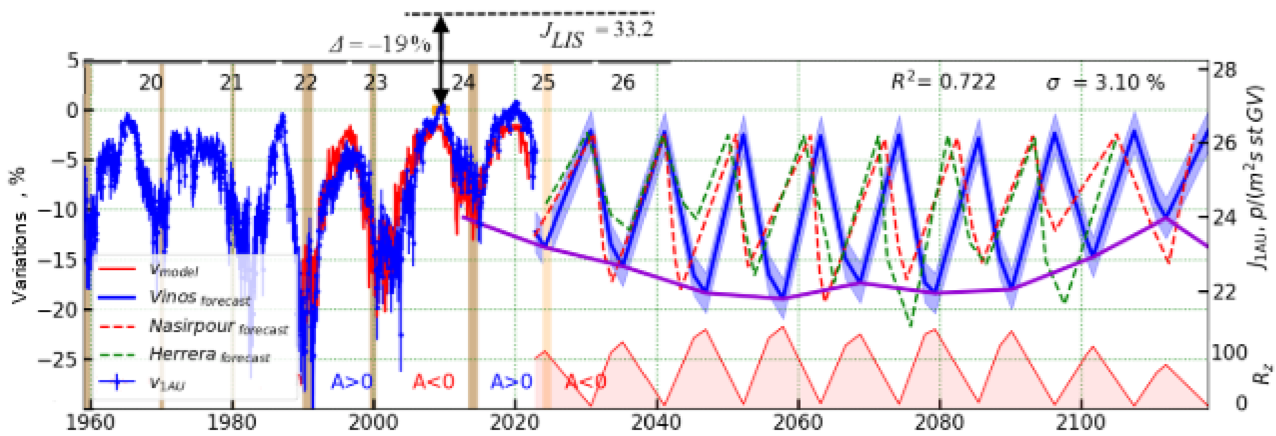
Φ, MV
600
400
200
0
1600
1800
2000
2200
2400
Maunder
Dalton
Gleysberg
...
Barnard, 2011
FFT
WTAR

Fig.2.



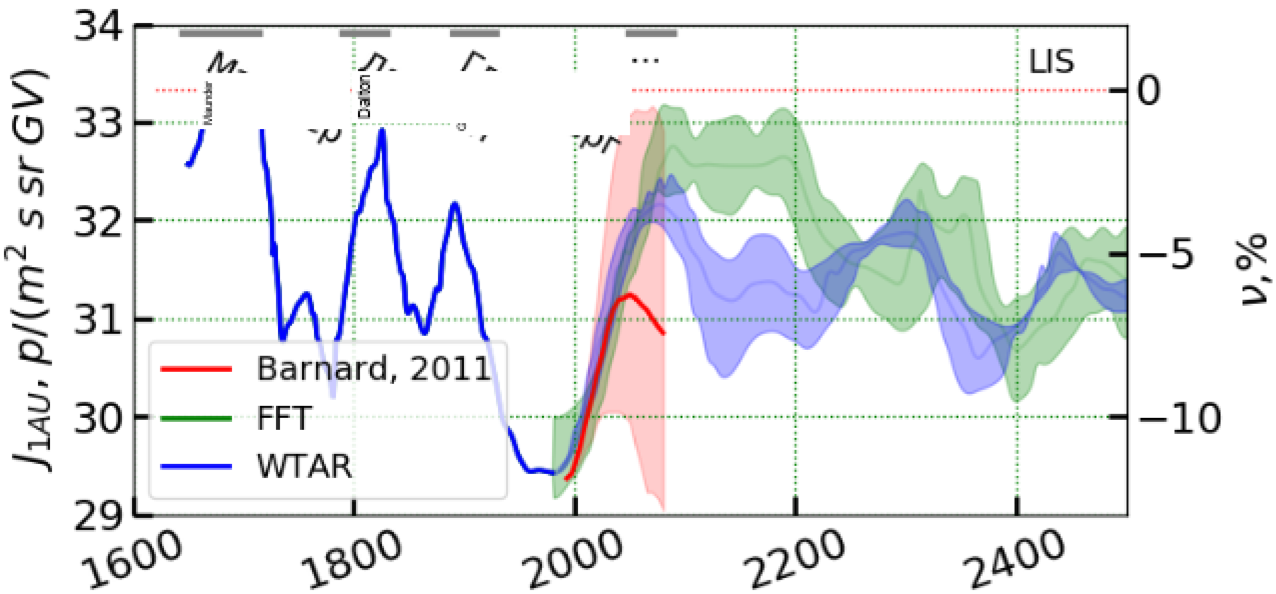
$v_{1AU} - v_{model}$
v_{model}
v_{1AU}
Variations , %
22
23
24
25
R^2 = 0.722
σ = 3.10 %
A>0
A<0
A>0
R_z
1985
1989
1993
1997
2001
2005
2009
2013
2017
2021

Fig.3.



Δ = −19%
J_{LIS} = 33.2
R^2= 0.722
σ = 3.10 %
20
21
22
23
24
25
26
Variations , %
J_{1AU}, p/(m²s st GV)
R_z
v_{model}
$Vinos_{forecast}$
$Nasirpour_{forecast}$
$Herrera_{forecast}$
v_{1AU}
A>0
A<0
A>0
A<0
1960
1980
2000
2020
2040
2060
2080
2100

Fig.4.



LIS
Dalton
J_{1AU}, p/(m² s sr GV)
ν,%
Barnard, 2011
FFT
WTAR
1600
1800
2000
2200
2400

Fig.5.